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Doctoral Thesis



Space-Time-Coding Modulation in Digital Metasurfaces: Applications to Multi-Frequency Syntheses and Diagnostics

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ABSTRACT

This Thesis deals with a study of space-time modulation applied to digital metasurfaces and reconfigurable intelligent surfaces (RISs), with a particular focus on their synthesis and diagnostics. The investigations revolve around two fundamental challenges: the efficient design of multi-frequency scattering responses, and the reliable detection of faulty elements in large-scale RISs.

On the synthesis side, we explore semi-analytical and machine-learning-based strategies that enable the independent control of harmonic equivalent currents in both magnitude and phase. This enables the realization of advanced multi-frequency functionalities, which have been experimentally validated at microwave frequencies.

On the diagnostics side, we investigate space-time modulation approaches that allow for accurate and efficient fault identification. Two different strategies are analyzed: one based on orthogonal coding, where distinct code channels are assigned to each element, and another based on non-uniform modulation, where a one-to-one mapping is established between harmonic amplitudes and RIS elements. Both methods are validated through theoretical analysis, simulations, and experiments, demonstrating robustness, scalability, and practicality for real-world implementations.

Overall, the Thesis illustrates how space-time modulation and machine learning can be leveraged to expand both the functional capabilities and the operational reliability of RISs, paving the way toward their deployment in future electromagnetic platforms for wireless communications, radar, and imaging.

Keywords: digital metasurfaces, fault diagnostics, machine learning, multi-frequency synthesis, reconfigurable intelligent surfaces, space-time-coding.

PUBLICATIONS

Some ideas and figures have appeared previously in the following publications:

JOURNAL PAPERS

- [1] M. Di Renzo, A. Mohamed, A. Zappone, G. Gradoni, **M. Rossi**, M. Moccia, G. Castaldi, and V. Galdi. "Effects of realistic reradiation models in digital reconfigurable intelligent surfaces." In: *Journal of Information and Intelligence* 1.3 (2023). (Invited), pp. 238–252. DOI: [10.1016/j.jiixd.2023.06.005](https://doi.org/10.1016/j.jiixd.2023.06.005).
- [2] **M. Rossi**, L. Zhang, X. Q. Chen, C. Liu, G. Castaldi, T. J. Cui, and V. Galdi. "Machine-learning-enabled multi-frequency synthesis of space-time-coding digital metasurfaces." In: *Advanced Functional Materials* 34.40 (2024), p. 2403577. DOI: [10.1002/adfm.202403577](https://doi.org/10.1002/adfm.202403577).
- [3] Y. N. Zheng, L. Zhang, X. Q. Chen, **M. Rossi**, G. Castaldi, S. Liu, T. J. Cui, and V. Galdi. "Rapid diagnostics of reconfigurable intelligent surfaces using space-time-coding modulation." In: *Advanced Functional Materials* 36.1 (2025), e07430. DOI: [10.1002/adfm.202507430](https://doi.org/10.1002/adfm.202507430).
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- [1] L. Zhang, **M. Rossi**, X. Q. Chen, G. Castaldi, T. J. Cui, and V. Galdi. "Some emerging concepts in the design of space-time-coding digital metasurfaces." In: *Eighteenth International Congress on Artificial Materials for Novel Wave Phenomena (Metamaterials)*. (Invited). 2024, pp. 1–3. DOI: [10.1109/Metamaterials62190.2024.10703321](https://doi.org/10.1109/Metamaterials62190.2024.10703321).
- [2] **M. Rossi**, G. Castaldi, and V. Galdi. "Machine-learning-based design of space-time-coding digital metasurfaces." In: *IEEE International Symposium on Antennas and Propagation and INC/USNC-URSI Radio Science Meeting (AP-S/INC-USNC-URSI)*. 2024, pp. 2011–2012. DOI: [10.1109/AP-S/INC-USNC-URSI52054.2024.10687026](https://doi.org/10.1109/AP-S/INC-USNC-URSI52054.2024.10687026).

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ACRONYMS

AE	Autoencoder
AI	Artificial Intelligence
ANN	Artificial Neural Network
ASTC-DM	Asynchronous Space-Time-Coding Digital Metasurface
BF	Brute-Force
BPSO	Binary Particle Swarm Optimization
CDMA	Code-Division Multiple Access
CNN	Convolutional Neural Network
CPPN	Compositional Pattern-Producing Network
CS	Compressed Sensing
DER	Diagnostics Error Rate
DL	Deep Learning
DNG	Double Negative
DNN	Deep Neural Network
DOA	Direction of Arrival
DPS	Double Positive
EM	Electromagnetic
ENG	Epsilon Negative
ENZ	Epsilon Near Zero
FIR	Finite-Impulse-Response
FT	Fourier Transform
FPGA	Field-Programmable Gate Array
GAN	Generative Adversarial Network
GA	Genetic Algorithm
GRU	Gated Recurrent Unit
ISAC	Integrated Sensing and Communication
LBFGS	Limited-Memory Broyden–Fletcher–Goldfarb–Shanno Quasi-Newton Algorithm
LCM	Least Common Multiple
LFSR	Linear Feedback Shift Register
LHCP	Left-Handed Circular Polarization
LHM	Left-Handed Medium

LSTM	Long Short-Term Memory
MD	Minimum Distance
MIM	Metal-Insulator-Metal
MLP	Multilayer Perceptron
ML	Machine Learning
MMA	Matrix Method Algorithm
MNG	Mu Negative
MNZ	Mu Near Zero
MSE	Mean Squared Error
NA	Numerical Aperture
OAM	Orbital-Angular-Momentum
PB	Pancharatnam–Berry
PCA	Principal Component Analysis
PCB	Printed Circuit Board
PDE	Partial Differential Equation
PIN	Positive-Intrinsic-Negative
PN	Pseudorandom Noise
RCS	Radar Cross Section
RHCP	Right-Handed Circular Polarization
RHM	Right-Handed Medium
RNN	Recurrent Neural Network
ReLU	Rectified Linear Unit
SGD	Stochastic Gradient Descent
SLL	Sidelobe Level
SNR	Signal-to-Noise Ratio
SRR	Split-Ring Resonator
STC-DM	Space-Time-Coding Digital Metasurface
STC	Space-Time Coding
TDC	Time-Domain Coding
USRP	Universal Software Radio Peripheral
VAE	Variational Autoencoder

Part I

RESEARCH FRAMEWORK AND OBJECTIVES

This part introduces the general context in which the present research has been carried out, outlining the scientific background and motivation behind the study. It provides a structured overview of the Thesis organization and defines the main research objectives within the framework of electromagnetic wave manipulation and metasurface technology.

INTRODUCTION

1.1 THESIS ORGANIZATION

This Thesis presents the research conducted at the **Fields & Waves Lab**, Department of Engineering, University of Sannio (Benevento, Italy), within the framework of the PhD program in Information Technologies for Engineering. Some parts of this work were developed in collaboration with Prof. Tie Jun Cui's group (Southeast University, China).

The research activities focus on the multi-frequency synthesis and diagnostic methods for Space-Time-Coding Digital Metasurfaces (**STC-DMs**). This chapter, along with [Chapter 2](#), provides an overview of fundamental concepts and background information, beginning with an introduction to metamaterials and metasurfaces.

[Chapter 3](#) introduces the fundamental concepts of metasurfaces, covering their principles, modeling techniques, and a broad spectrum of applications. This chapter sets the stage for the more specialized discussions that follow, particularly those focused on digital metasurfaces.

[Chapter 4](#) discusses the multi-frequency synthesis procedures for STC-DMs, including both analytical approaches and machine-learning-enabled strategies.

[Chapter 5](#) addresses diagnostic techniques for these metasurfaces. It examines methods based on uniform and non-uniform modulation schemes to accurately characterize their Electromagnetic (**EM**) behavior and experimentally validate the proposed approaches.

Finally, [Chapter 6](#) presents some concluding remarks, summarizing the key findings of this work and outlining potential directions for future research.

1.2 BACKGROUND AND GENERALITIES

Metamaterials [1] and metasurfaces [2] are synthetic structures specifically designed to exhibit properties that are not typically present in natural materials. These structures are created by arranging dielectric or metallic inclusions, periodically or aperiodically, within a host medium [3]. These inclusions are characterized by small dimensions and spacing relative to the operating wavelength so that, on a macroscopic level, the metamaterial can be considered a homogeneous medium. In this way, the microscopic properties of each meta-atom become invisible to EM radiation, and the material's response is characterized by the collective behavior of all the meta-atoms. The ability to artificially engineer such meta-atoms has been widely exploited in

recent years to control EM waves, allowing for the investigation of numerous phenomena.

Compared to metamaterials, metasurfaces are easier to fabricate and integrate with other devices; therefore, they have attracted greater interest in the research field [4–6]. In particular, the generalized Snell’s laws [4] have encouraged the study and development of metasurface applications. This type of metasurface exhibits spatial phase gradient discontinuities at the interface separating two media, allowing the manipulation of EM wavefronts in the microwave and visible spectral regions. However, conventional metasurfaces do not exploit the temporal dimension and are generally constrained by Lorentz reciprocity.

Recently, space-time modulated metasurfaces have attracted significant attention as a promising research field [7–10]. Time-varying metasurfaces provide an additional degree of freedom, arising from the ability to modulate their constitutive parameters in the time domain. By combining this aspect with spatial modulation, space-time metasurfaces are described by constitutive parameters that vary in both space and time, such as electric permittivity [11], conductivity [7], and characteristic impedance [8]. Space-time modulated metasurfaces have been investigated to enable new applications, including optical isolators [12], overcoming Lorentz reciprocity [13], and designing nonreciprocal antennas [7]. Notably, these characterizations of space-time metasurfaces can be regarded as an form of *analog modulation*, where the descriptive parameters vary continuously.

A new class of metasurfaces, featuring digital coding, was introduced by Cui *et al.* in 2014 [14]. The concept behind these metasurfaces is based on the implementation of a finite set of coding elements. Specifically, in a 1-bit coded digital metasurface implemented on a substrate terminated with a perfect electric conductor, the logical state “0” is assigned when the phase of the reflection coefficient is 0, while “1” is assigned when it is π . Using these building blocks, digital metasurfaces enable the manipulation of EM waves through an appropriate coding sequence, simplifying the design and optimization process [15–17]. This digital coding of metasurfaces, understood as a form of *digital modulation* achieved through discrete coding sequences, can be physically implemented and is well-suited for integration with other components, such as diodes. By independently managing the active components embedded in the digital coding elements through a Field-Programmable Gate Array (FPGA), programmable digital metasurfaces can be developed to dynamically regulate EM waves [14, 18, 19]. This concept has been applied in various domains, including polarization control [20], imaging techniques [21–24], and wireless communications [25–27].

In early studies, digital coding was defined only in the spatial domain, allowing control over the spatial distribution of EM waves [14, 16–20, 24, 26]. By incorporating digital coding into the temporal domain, it becomes possible to control the spectral distribution of waves reflected by the metasurface by dynamically adjusting the temporal coding sequences [25]. In 2018, the

general theory of STC-DMs was proposed by Zhang *et al.* [28]. The phase of the reflection coefficients describing STC-DMs is encoded in both space and time, allowing EM waves to be controlled in both the spatial and spectral domains. In other words, it is possible to manipulate the field distribution at the harmonic frequencies of the operating frequency and the propagation direction of the reflected waves. STC-DMs have been applied to control the scattering pattern of the reflected field at harmonic frequencies [28], perform scattering reduction [28], generate programmable nonreciprocal effects [29], and in the context of wireless communications [30].

MATERIALS CLASSIFICATION

Materials derive their properties from the arrangement of electrons in their atomic structure. Most natural materials do not exhibit pronounced electrical or magnetic behavior because the movement of electrons is random. However, in the presence of EM waves, the electrical and magnetic properties of most materials are altered based on the direction of the electric and magnetic fields of the EM wave, reorganizing the orientation of the dipoles at the atomic level.

An EM field incident on a material sample causes, at the microscopic level, charge displacements that determine the response of individual atoms. The material's response to the EM field is given by the sum of the individual atomic contributions. Accordingly, a distinction is drawn between a material's response at the *microscopic* level and at the *macroscopic* level. If the material is homogeneous, the macroscopic response is obtained as the average of the microscopic behaviors within the unit volume. Macroscopic constitutive parameters are then defined to describe the material's response. The *constitutive relations* mathematically express the material's response to an EM field and are necessary to solve an EM problem [31].

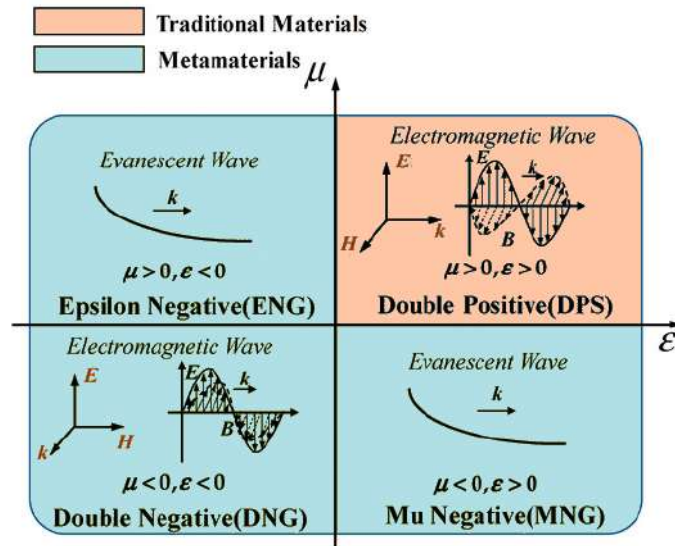


Figure 1: Plane of the EM properties of materials, characterized by the electric permittivity ϵ and the magnetic permeability μ .

The EM properties of materials are described, through Maxwell's equations, by two fundamental quantities: the dielectric permittivity ϵ and the magnetic permeability μ . Dielectric permittivity and magnetic permeability

are parameters that provide a measure of how electric and magnetic fields interact with matter, polarizing it. From a macroscopic perspective, it is possible to categorize all materials in nature based on the values of ϵ and μ , as shown in Figure 1. Therefore, by referring to the plane of the EM properties of materials, the following classes of materials can be defined [32]:

- Double Positive (DPS) materials. The first quadrant contains the most common class of materials. These materials exhibit both positive permittivity and positive permeability, encompassing most dielectric substances.
- Epsilon Negative (ENG) materials. The second quadrant hosts materials with negative permittivity and positive permeability. This class includes metals, doped semiconductors, and ferroelectric materials, which exhibit this property at certain frequency ranges, typically below the plasma frequency.
- Double Negative (DNG) materials. The third quadrant, representing materials with both negative permittivity and permeability, is of high interest. Since no naturally occurring materials with these properties have been discovered, they must be artificially engineered and realized.
- Mu Negative (MNG) materials. The fourth quadrant includes materials characterized by negative permeability and positive permittivity. While less common, examples include certain ferrite-based materials that can exhibit negative permeability, typically via magnetic resonances whose effectiveness often fades beyond the microwave frequency range.

The EM behavior of these materials is rigorously described by Maxwell's equations. In particular, assuming a linear, temporally homogeneous, spatially non-dispersive, and isotropic medium, Maxwell's curl equations can be written in the frequency domain as follows:

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}, \quad (1a)$$

$$\nabla \times \mathbf{H} = j\omega\epsilon\mathbf{E}, \quad (1b)$$

obtained in the absence of currents $\mathbf{J}(\mathbf{r}, t)$ and under the assumption that the fields $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{H}(\mathbf{r}, t)$ are harmonic with respect to space $\mathbf{r}$ and time t , that is

$$\begin{aligned} \mathbf{E}(\mathbf{r}, t) &= \Re \left\{ \mathbf{E} e^{j(\omega t - \mathbf{k} \cdot \mathbf{r})} \right\}, \\ \mathbf{H}(\mathbf{r}, t) &= \Re \left\{ \mathbf{H} e^{j(\omega t - \mathbf{k} \cdot \mathbf{r})} \right\}, \end{aligned} \quad (2)$$

where $\mathbf{k} = k_x \hat{\mathbf{x}} + k_y \hat{\mathbf{y}} + k_z \hat{\mathbf{z}}$ is the wave vector and ω represents the angular frequency. For a plane wave, the electric field $\mathbf{E}$ and the magnetic field $\mathbf{H}$ satisfy the following relationships:

$$\mathbf{k} \times \mathbf{E} = \omega\mu\mathbf{H}, \quad (3a)$$

$$\mathbf{k} \times \mathbf{H} = -\omega\epsilon\mathbf{E}. \quad (3b)$$

From these relations, it is deduced that, in a plane wave, $\mathbf{k}$, $\mathbf{E}$, and $\mathbf{H}$ form a set of orthogonal vectors. Furthermore, considering the case of a plane wave propagating in a medium described by constitutive relations of the form $\mathbf{D} = \epsilon\mathbf{E}$ and $\mathbf{B} = \mu\mathbf{H}$, the following *dispersion relation* is obtained:

$$k^2 = \omega^2 \epsilon \mu, \quad (4)$$

where $k = \sqrt{k_x^2 + k_y^2 + k_z^2}$ is the magnitude of $\mathbf{k}$.

By solving Equation (4), two possible solutions are found: $k = \pm\omega\sqrt{\epsilon\mu}$. The correct choice of the square root determination must be made based on physical considerations. For example, considering a DPS material, also known as Right-Handed Medium (RHM), it can be stated that $\mathbf{E}$, $\mathbf{H}$, $\mathbf{k}$ form a right-handed set; this occurs if $\Re(\epsilon), \Re(\mu) > 0$. This implies that, in a DPS material, the correct determination is the one for which $\Re(k) > 0$.

Another consideration for DPS materials concerns the Poynting vector, which, in the frequency domain, is defined as:

$$\mathbf{S} = \frac{1}{2} \mathbf{E} \times \mathbf{H}^*, \quad (5)$$

where $*$ indicates the complex conjugate. From this relation, it is easily noted that in a DPS material, the Poynting vector is also orthogonal to the plane formed by the vectors $\mathbf{E}$ and $\mathbf{H}$, meaning it is parallel to the wave vector $\mathbf{k}$. If a DNG material, also known as Left-Handed Medium (LHM), is considered, from the relations in Equation (3), it is deduced that the vectors $\mathbf{k}$, $\mathbf{E}$, and $\mathbf{H}$ form a left-handed set, while the vectors $\mathbf{S}$, $\mathbf{E}$, and $\mathbf{H}$ continue to form a right-handed set. Therefore, in a DNG material, the Poynting vector is antiparallel to the wave vector, meaning that the energy flux of the EM radiation is antiparallel to the motion of the wavefronts, as shown in Figure 2. To understand the consequences of this phenomenon, it is useful to express the wave vector as follows:

$$\mathbf{k} = nk_0 \hat{\mathbf{s}}_0, \quad (6)$$

where $k_0 = \omega\sqrt{\epsilon_0\mu_0}$ is the wavenumber in vacuum, with ϵ_0 and μ_0 representing the permittivity and permeability of free space, respectively; n is the refractive index of the medium, and $\hat{\mathbf{s}}_0$ is the unit vector parallel to the Poynting vector.

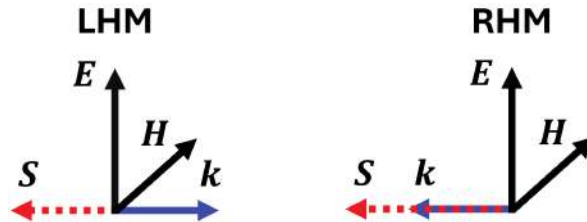


Figure 2: Vector set $\{\mathbf{E}, \mathbf{H}, \mathbf{k}\}$ in a LHM and an RHM.

By defining the wave vector in this way and keeping in mind the considerations made earlier, it follows that in a DPS material, the refractive index must be positive, whereas it is negative in a DNG material. Therefore, the refractive index can be defined as follows:

$$n = \begin{cases} \sqrt{\epsilon_r \mu_r}, & \text{in a DPS material,} \\ -\sqrt{\epsilon_r \mu_r}, & \text{in a DNG material,} \end{cases} \quad (7)$$

where $\epsilon_r = \epsilon/\epsilon_0$ and $\mu_r = \mu/\mu_0$ represent the relative electric permittivity and relative magnetic permeability, respectively. From a mathematical perspective, DPS and DNG materials correspond to the two possible determinations of the square root of the refractive index.

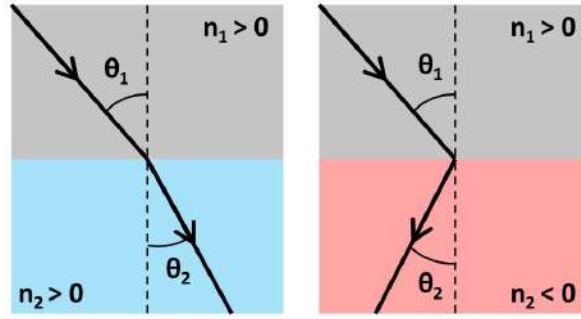


Figure 3: Refraction of light in materials with positive refractive index (left) and negative refractive index (right).



Figure 4: Refraction of light in water (left) and how light refraction might appear in a medium with a negative refractive index (right).

To understand the effects of such phenomena, one can consider, for example, the refraction of light in materials with a positive refractive index, where the behavior of the radiation is well described by conventional optical laws. When an EM wave propagates from a medium with refractive index n_1 to a medium with refractive index n_2 , its propagation direction θ_1 with respect to the normal is deviated along the direction θ_2 according to Snell's law:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2. \quad (8)$$

In the case where light propagates from a medium with a positive refractive index to one with a negative refractive index, the incident angle and exit angles must have opposite signs for their sine to continue satisfying Snell's law. That is, the refracted light propagates at negative angles, on the same side as the incident light, as shown in Figures 3 and 4.

2.1 METAMATERIALS

After the work by Veselago [33], where the possibility of EM wave propagation through materials with simultaneously negative permittivity and permeability was studied mathematically, the term “metamaterials” was introduced to refer to materials that go beyond natural materials.

Metamaterials are artificially engineered structures designed to exhibit properties not necessarily available in natural materials. These artificial structures are created by periodically arranging resonant metallic inclusions in a host medium, which is typically a dielectric or magneto-dielectric material. When exposed to an EM field, metamaterials alter the EM properties of the host medium due to the characteristics of the inclusions. Figure 5 conceptually shows a possible realization of a metamaterial structure.

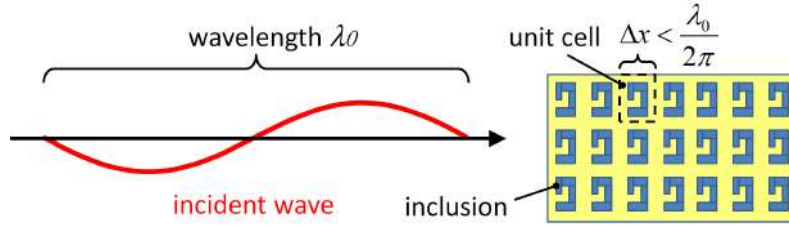


Figure 5: An example of a generic metamaterial. The unit cells have dimensions small compared to the wavelength, so that, at the macroscopic level, the metamaterial can be considered a homogeneous medium.

It is possible to recreate in laboratory materials with engineered EM properties by starting from common materials, and organizing them into individual elements (known as *meta-atoms*) distributed periodically or randomly, with dimensions and spacings small compared to the wavelength of the EM radiation they interact with. In this way, the material's response is influenced by the collective behavior of all meta-atoms. In other words, the inhomogeneous set of meta-atoms can be described, from a macroscopic perspective, as a homogeneous medium characterized by electric permittivity ϵ and magnetic permeability μ .

2.2 DEFINITION OF METAMATERIAL

A possible definition of a metamaterial is as follows [34]:

A metamaterial is an artificial material designed to exhibit certain specific properties. On a macroscopic scale, a metamaterial can be homogeneous, whereas on a microscopic scale it consists of unit cells arranged according to a precise geometry.

This definition is quite general and can be applied to almost all artificial materials. A unit cell is considered “microscopic” if its characteristic length Δx is much smaller than the wavelength λ_0 of the EM radiation with which it interacts:

$$\Delta x \ll \lambda_0. \quad (9)$$

In general, the particular properties of a metamaterial are determined by its microscopic structure rather than its chemical composition. When the condition described by Equation (9) is satisfied, a lumped-element model can be used to describe metamaterials, where an equivalent resonant circuit can be used to provide a qualitative description of the physical behavior of such artificial materials [35].

2.2.1 Epsilon Negative Metamaterial

A natural medium that exhibits negative permittivity ε is *plasma*. Examples of plasma are metallic conductors and the ionosphere, which is an ionized gas created by solar radiation. The constitutive relations of a cold, isotropic, and non-collisional plasma are [31]:

$$\mu(\omega) = \mu_0, \quad (10a)$$

$$\varepsilon(\omega) = \varepsilon_0 \left(1 - \frac{\omega_p^2}{\omega^2} \right), \quad (10b)$$

where

$$\omega_p = \sqrt{\frac{N_i q_e^2}{\varepsilon_0 m_e}} \quad (11)$$

is the plasma frequency, N_i is the number of ionized particles per unit volume, q_e is the electron charge, and m_e is the electron mass.

From Equation (10b), it can be observed that when $\omega > \omega_p$, the dielectric function assumes positive values, allowing the propagation of EM waves through the medium. In contrast, when $\omega < \omega_p$, the dielectric function assumes negative values, meaning that the EM wave decays exponentially and does not propagate in the medium. For such materials, the wavenumber is purely imaginary and is given by $k = j\omega\sqrt{|\varepsilon|\mu}$. Finally, when the incident wave has a frequency $\omega = \omega_p$, the dielectric function vanishes, and the EM wave remains confined within the material, inducing oscillation modes of the electrons.

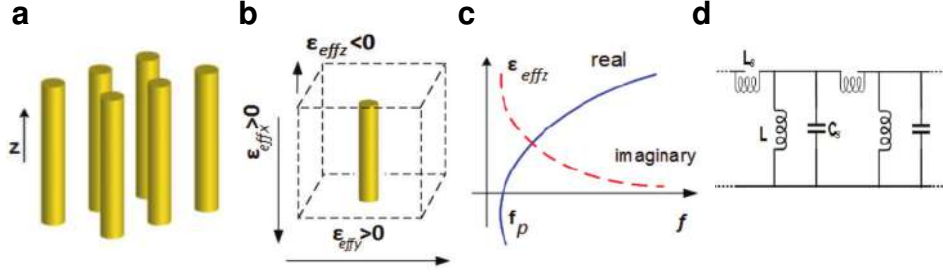


Figure 6: Artificial ENG metamaterial [36]. **a)** ENG metamaterial realized through an array of thin metallic wires. **b)** Unit cell of the ENG metamaterial. **c)** Behavior of the real and imaginary parts of the relative permittivity of the ENG metamaterial. **d)** Equivalent circuit of the ENG metamaterial.

The realization of resonant inclusions modeled in a host medium allowed for the achievement of negative values of electric permittivity. Pendry *et al.* [36] showed that a series of thin metallic bars or wires arranged in a cubic lattice can exhibit, in the microwave regime, a negative dielectric response given by the Drude function:

$$\epsilon_r(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega - j\zeta)}, \quad (12)$$

where ζ is a dissipative factor accounting for collisions occurring in a plasma. If losses are negligible ($\zeta \approx 0$), from Equation (12) it is evident that EM waves oscillating at a frequency lower than ω_p are non-propagating, since $\epsilon < 0$ and $\mu > 0$. The relative dielectric permittivity function expressed by Equation (12) can be used to describe the response of a homogeneous medium made through a series of very thin metallic wires. Notably, in the realization of the periodic lattice, the diameter of the wires is much smaller than the wavelength of interest, so the structure is perceived by the EM radiation as a homogeneous medium.

2.2.2 Mu Negative Metamaterial

This category includes a few natural materials, such as *ferrites*. Ferrites are ceramic materials made from a mixture of metals and iron oxides. Below is the permeability matrix of a magnetized ferrite [31]:

$$\boldsymbol{\mu} = \mu_0 \begin{pmatrix} \mu_1 & j\mu_2 & 0 \\ -j\mu_2 & \mu_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (13)$$

where

$$\begin{aligned}\mu_1 &= 1 + \frac{\omega_0 \omega_m}{\omega_0^2 - \omega^2}, \\ \mu_2 &= \frac{\omega_m \omega}{\omega_0^2 - \omega_m^2},\end{aligned}\quad (14)$$

with $\omega_0 = \gamma_{\text{gm}} H_0$, H_0 being the magnitude of the magnetic field, $\gamma_{\text{gm}} = 2.21 \cdot 10^5$ m/A the gyromagnetic ratio, $\omega_m = \gamma_{\text{gm}} M_s / \mu_0$, and M_s the saturation magnetization density per unit volume.

Among the engineered materials with negative permeability, artificial magnetic materials have been of interest for many years. This is due to their characteristics, including the ease of low-cost integration with radio frequency/microwave circuits. A wide variety of artificial magnetic inclusions have been proposed and implemented. One of the most popular and widely applied artificial magnetic materials is the Split-Ring Resonator (SRR) [37], consisting of two concentric metallic rings printed in a dielectric host medium. The distance separating the inner ring from the outer one creates a capacitive effect, while the circular structure of the rings acts as an inductor. Thus, the combination of the two rings behaves like an LC resonant circuit. The relative magnetic permeability of the MNG metamaterial is given by:

$$\mu_r(\omega) = 1 - \frac{\omega_{\text{mp}}^2 - \omega_0^2}{\omega^2 - \omega_0^2 - j\zeta\omega}, \quad (15)$$

where ω_{mp} is the magnetic plasma frequency, i. e., the frequency at which $\mu_r(\omega) = 0$ in the absence of losses, ω_0 is the resonance frequency, at which $\mu_r(\omega)$ diverges, and ζ is a factor representing losses. The frequencies ω_{mp} and ω_0 depend on the spacing between the SRRs making up the metamaterial and geometric parameters such as the inner and outer ring radii and the distance separating the two rings.

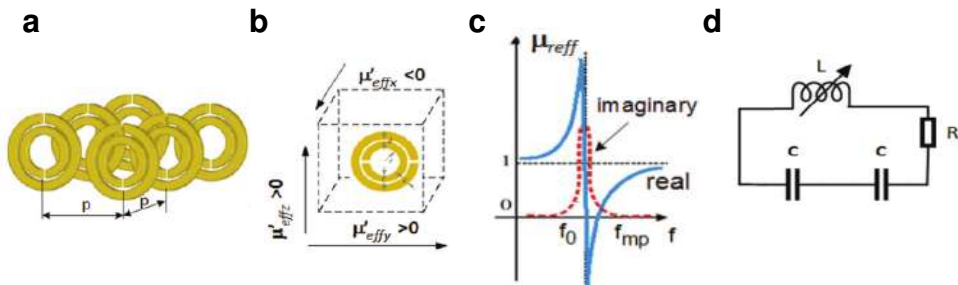


Figure 7: Artificial MNG metamaterial [37]. **a)** MNG metamaterial realized using an array of SRRs. **b)** Constitutive unit cell of the MNG metamaterial. **c)** Behavior of the real and imaginary parts of the relative magnetic permeability of the MNG metamaterial. **d)** Equivalent circuit of the MNG metamaterial.

2.2.3 Double Negative Metamaterial

The properties of DNG metamaterials were obtained by combining the structure of thin metallic wires, used to create ENG metamaterials, with the structure based on SRRs, used to create MNG metamaterials. The overall structure resulting from this combination, proposed by Smith *et al.* [38], exhibits negative values of both dielectric permittivity and magnetic permeability in the microwave region.

The relative dielectric permittivity $\epsilon_r(\omega)$ and relative magnetic permeability $\mu_r(\omega)$ responses of this metamaterial are described by Drude–Lorentz models and follow the relations [38]:

$$\epsilon_r(\omega) = 1 - \frac{\omega_{ep}^2 - \omega_{e0}^2}{\omega^2 - \omega_{e0}^2 + j\zeta_c\omega}, \quad (16a)$$

$$\mu_r(\omega) = 1 - \frac{A_c\omega^2}{\omega^2 - \omega_{m0}^2 + j\tilde{\zeta}_c\omega}, \quad (16b)$$

where ω_{ep} and ω_{mp} are the electric and magnetic plasma frequencies, ω_{e0} and ω_{m0} are the electric and magnetic resonance frequencies, ζ_c is a factor accounting for collisions, A_c is an amplitude factor, and $\tilde{\zeta}_c$ is a damping factor.

2.3 PERFECT LENS

After the development of the first metamaterials, attention shifted to studying and realizing some of their revolutionary potentials. In particular, the possibility of creating superlenses capable of surpassing the diffraction limit of light certainly represented one of the first and most significant examples of metamaterial applications.

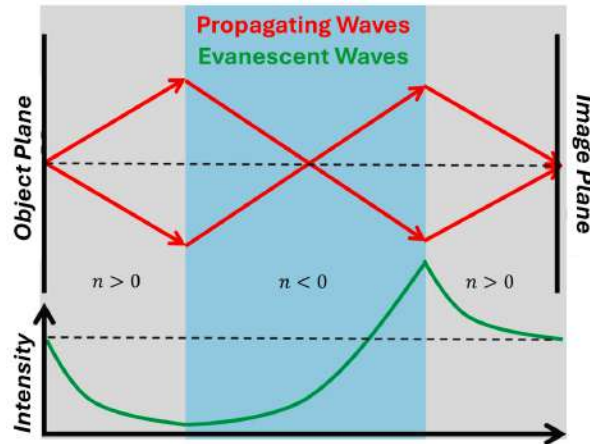


Figure 8: Operation diagram of a superlens. Propagating waves are focused by the negative refractive index, while evanescent waves are amplified.

In conventional optical systems, it is not possible to distinguish two points if they are at a distance smaller than $\lambda/2n$, where n is the refractive index of the surrounding medium. This is the so-called diffraction limit. This intrinsic limit arises because information about structures with dimensions comparable to the wavelength of light propagates optically in the form of evanescent waves, i. e., EM waves that decay exponentially as they propagate. As a result, the image of these objects is lost before it can be collected and recreated by a lens or objective. Pendry later proposed the use of a negative-refractive-index metamaterial as a superlens (or perfect lens), capable of reconstructing an object's image without loss of structural information, even below the diffraction limit.

This property arises from the fact that, in a material with a negative refractive index, evanescent waves are no longer exponentially decaying but, on the contrary, are amplified. At the same time, propagating waves are automatically focused due to the negative refractive index, as schematically shown in Figure 8.

By appropriately choosing the thickness of a plate made of a material with a refractive index of $n = -1$, radiation can be focused to a point, unlike what happens with a traditional lens.

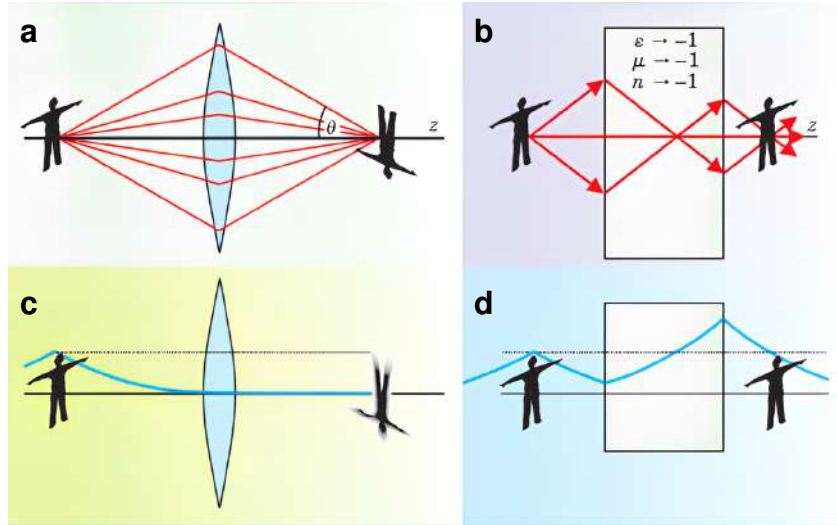


Figure 9: a,b) Comparison between the behavior of a lens with a positive refractive index and one with a negative refractive index, respectively. c,d) Comparison between evanescent waves in a right-handed lens and a left-handed lens, respectively.

In the panels of Figure 9, two lenses are compared: a traditional one and one made with metamaterials. Figure 9a illustrates the path of light rays as they encounter a traditional lens, whereas Figure 9b schematically depicts the trajectory of light when using a metamaterial-based lens. Figure 9c shows the behavior of evanescent waves in a traditional lens. The rapid decrease in intensity can be observed, which results in a deterioration of image quality

beyond the lens. Figure 9d highlights how evanescent waves are amplified when using a lens with a refractive index of $n = -1$.

2.4 OPTICAL INVISIBILITY

The following analysis examines the conditions required for an invisibility cloak or cloaking device. In general, to make an object invisible, the light field outside the volume occupied by the object must remain undisturbed. In other words, an *ideal* cloaking device should meet the following criteria:

- *Absence of reflection:* Waves impinging on the object must not be reflected.
- *Absence of refraction:* No detectable deviation or phase shift should occur due to refraction.
- *No radiation:* The cloaked system must not emit any waves.
- *No shadowing:* The system must not cast a shadow and it must be absorption-less.

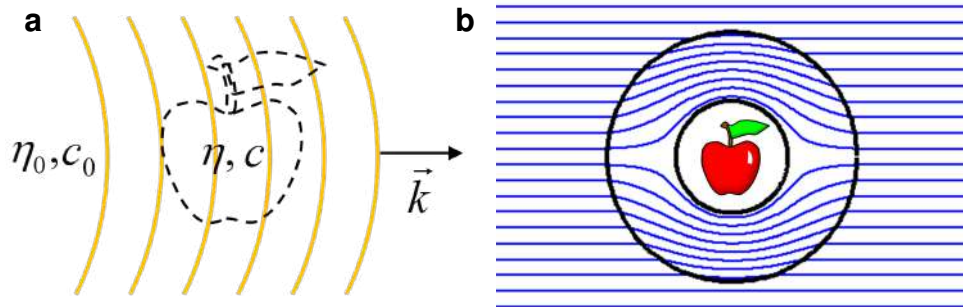


Figure 10: The two main methods for rendering an object invisible. **a)** Invisibility achieved through transparency. **b)** Invisibility achieved by bending light.

Thus, an “invisible” object should not produce any detectable effect outside the cloaking device. There are two main approaches to achieving invisibility:

1. Making the object globally transparent so that light passes through it without disturbance.
2. Bending light in such a way that it bypasses the object.

2.4.1 Transparency-Based Invisibility

To achieve invisibility through transparency, the incident light must pass through the object without alteration. For simplicity, assuming the surrounding medium is vacuum, this is possible only if:

- The characteristic impedance η of the object matches that of the surrounding medium:

$$\eta = \eta_0 \quad (17)$$

This ensures the absence of reflection.

- The refractive index n of the object matches that of the surrounding medium:

$$n = n_0. \quad (18)$$

Since the refractive index is related to the wave propagation speed in the medium, this condition can be replaced by an equivalent one:

$$c = c_0. \quad (19)$$

Thus, the wave should propagate at the same speed both inside and outside the object. This ensures the absence of refraction.

These two conditions can also be extended to other types of waves, such as acoustic and mechanical waves, when the characteristic impedance η and the propagation speed c are well defined.

A classic example of invisibility achieved through transparency is that of a glass bottle immersed in glycerin. When a glass bottle is submerged in glycerin, it appears to vanish. This occurs because the two media have nearly the same refractive index, $n \approx 1.47$, and, since they are non-magnetic ($\mu \approx \mu_0$), they also share the same characteristic impedance η .

2.4.2 Invisibility Through Optical Transformations

Theoretically, deflection-based invisibility can be achieved by bending light around an object. The possibility of guiding light through a metamaterial to achieve this effect was considered by Pendry *et al.* [39]. The proposed method is based on a coordinate transformation and can be summarized as follows:

1. Consider the propagation of an EM wave.
2. Transform the spatial domain by applying a coordinate change, allowing light to follow the desired path.
3. Apply the same coordinate transformation to Maxwell's equations to determine the required values of permittivity ϵ and permeability μ at each point in the considered domain.

Supposing one wants to cloak an object enclosed within a spherical shell designed to guide incident light, it is necessary to determine the appropriate metamaterial for constructing the shell.

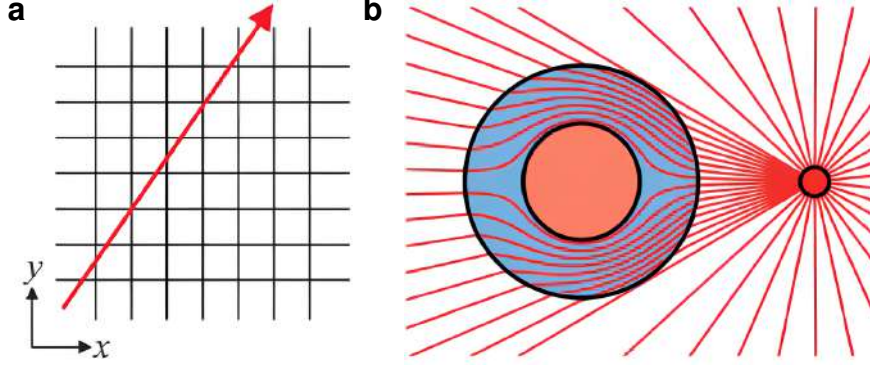


Figure 11: Example of optical transformation [39]. **a)** A field line in an homogeneous medium with the background Cartesian coordinate grid shown. **b)** The field radiated from a point source is excluded from the cloaked region, but emerges from the cloaking sphere undisturbed.

As mentioned, a plane wave propagating in free space is considered, where $\varepsilon = \varepsilon_0$ and $\mu = \mu_0$. Then, the original domain is transformed by introducing a circle of radius R_1 within which light cannot propagate. A proposed linear transformation is [39]:

$$\begin{aligned} \rho' &= \begin{cases} R_1 + \frac{R_2 - R_1}{R_2} \rho, & 0 \leq \rho \leq R_2, \\ \rho, & \rho > R_2, \end{cases} \\ \theta' &= \theta, \\ \phi' &= \phi, \end{aligned} \quad (20)$$

where ρ' , θ' , and ϕ' are the transformed spherical coordinates. Note that for $\rho > R_2$, i.e., outside the spherical shell, the domain remains undistorted since $\rho' = \rho$.

After this coordinate transformation, EM waves experience curvature when approaching the spherical shell. For the considered case, the permittivity within the spherical shell, in the region where $R_1 < \rho < R_2$, is given by [39]:

$$\begin{aligned} \varepsilon'_{\rho'} &= \frac{R_2}{R_2 - R_1} \frac{(\rho' - R_1)^2}{\rho'}, \\ \varepsilon'_{\theta'} &= \frac{R_2}{R_2 - R_1}, \\ \varepsilon'_{\phi'} &= \frac{R_2}{R_2 - R_1}, \end{aligned} \quad (21)$$

The permeability $(\mu'_{\rho'}, \mu'_{\theta'}, \mu'_{\phi'})$ follows the same functional expression in Equation (21). This is just a possible transformation, since other solutions are available. In practice, this cloaking technique is based on two main requirements:

1. The cloaked object should not be hit by the waves impinging from outside, so that it is completely concealed.
2. The incident waves enter into the coating without reflection, then they are guided through the coating itself, curving around the cavity, and finally emerge outside returning to their original trajectory.

To avoid reflection, the outer surface of the shell must be perfectly matched to the surrounding environment, thus the impedance matching condition η must be satisfied.

The light rays that would strike the object must be deflected. This effect can be achieved by decreasing the refractive index of the inner layers of the coating. Since $n = c/v_p$, if the refractive index n is less than 1, a superluminal phase velocity v_p is required. Indeed, the waves that should hit the coating must travel a longer path in the same amount of time to remain in phase with the other waves. If a superluminal phase velocity is required, then permittivity and permeability values lower than 1, or close to 0, might be needed.

$$v_p > c \quad \longrightarrow \quad |\epsilon_r \mu_r| < 1. \quad (22)$$

For that reason, an Epsilon Near Zero (ENZ) or a Mu Near Zero (MNZ) metamaterial could be useful for the construction of such an invisibility cloak.

2.5 METASURFACES

A metasurface [2] can be defined as a metamaterial where one of the three dimensions is negligible compared to the others, and where the characteristic length Δx of the unit cell is small relative to the EM wavelength λ_0 . The main advantage of metasurfaces is that they are thinner, making them less bulky and easier to fabricate and integrate with other devices compared to 3-D metamaterials. For instance, a metasurface can be directly printed on a substrate, whereas constructing a 3-D metamaterial, which requires arranging all inclusions in a specified geometry, is a more complex operation.

The manipulation of EM waves using conventional optical components, such as lenses and prisms, is achieved through a process in which light accumulates a phase delay. However, the integrability and miniaturization of such optical components are significantly limited by their dimensions. Metasurfaces can modify the amplitude and introduce phase delays in the incident wave, allowing for a more efficient manipulation of EM waves.

2.5.1 Generalized Snell Laws

When a wave propagating in a medium encounters a discontinuity in the refractive index n at the interface with another medium, it is partially reflected back into the first medium and partially transmitted through the interface. If the phase of the transmission coefficient remains constant at the interface between the two media, the directions of reflection and refraction remain unchanged. On the other hand, when the phase of the transmission coefficient is not constant, specific phase variations can be achieved to effectively control the propagation direction of the wave and the shape of the wavefront.

In general, if a metasurface is added at the interface to vary the phase of the reflection coefficient as a function of space, Snell's laws are generalized as [4]:

$$\begin{cases} n_t \sin \theta_t - n_i \sin \theta_i = \frac{1}{k_0} \frac{d\Phi}{dx}, \\ \sin \theta_r - \sin \theta_i = \frac{1}{n_i k_0} \frac{d\Phi}{dx}, \end{cases} \quad (23)$$

where n_i and n_t are the refractive indices of the two media separated by the interface, k_0 is the wavenumber in vacuum, and θ_i , θ_r , θ_t are the angles of incidence, reflection, and transmission, respectively, as shown in Figure 12. The term $d\Phi/dx$ represents the phase variation at the interface due to the metasurface. The relations in Equation (23) are the generalized laws of refraction and reflection, as the direction of the reflected wave depends on the optical properties of the surrounding media and the phase response of the artificial interface separating these media.

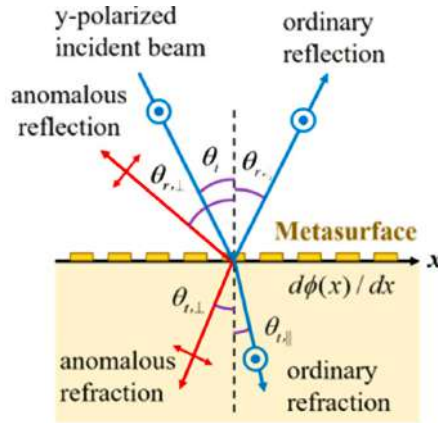


Figure 12: Conceptual representation of the generalized Snell's laws, where an anomalous refraction and reflection phenomenon is observed.

To experimentally validate the generalized Snell's laws, Yu *et al.* [4] designed a metasurface composed of V-shaped antennas, made of two arms of equal length h connected at the ends to form an angle Δ , as shown in Figure 13a. To orient the V-shaped antennas, two unit vectors were defined: a first

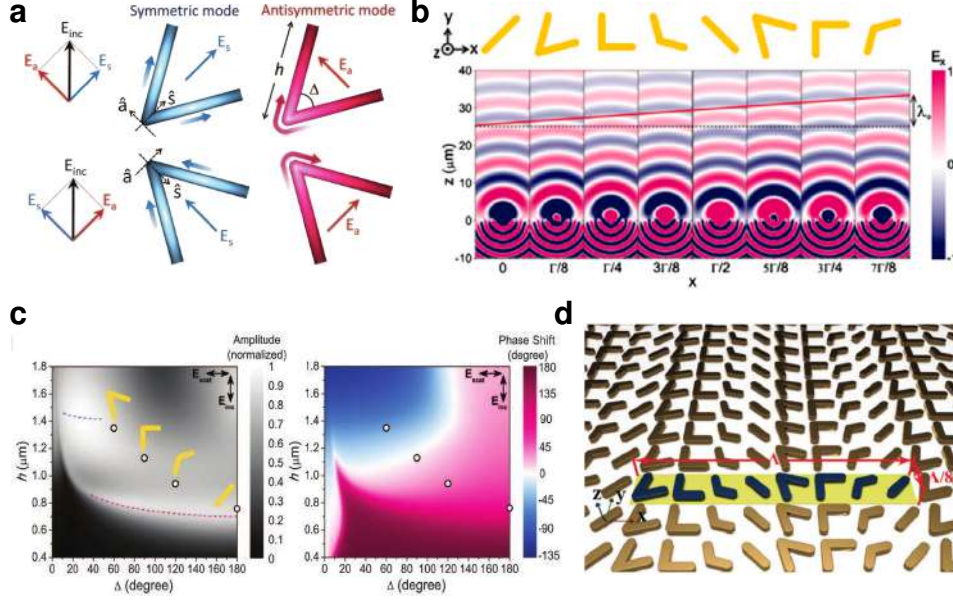


Figure 13: **a)** V-shaped antenna supporting symmetric and asymmetric modes, with current intensity shown by color and direction indicated by faded arrows. **b)** Schematic of the V-shaped antenna array forming the metasurface from [4], designed for a phase difference $\Delta\Phi = \pi/4$ between adjacent elements. **c)** Amplitude and phase distributions of the reflected wave from the antennas, as a function of h and Δ , at a fixed wavelength $\lambda_0 = 8 \mu\text{m}$, with points representing experimental values from Ref. [4]. Dashed curves indicate resonance peaks for symmetric and asymmetric modes. **d)** Metasurface used to experimentally verify anomalous reflection and refraction by generalized Snell's laws.

unit vector $\hat{s}$ along the symmetry axis of the antenna and a second unit vector $\hat{a}$ orthogonal to the first. The V-shaped antennas allow the propagation of “symmetric” and “asymmetric” modes, which are excited by the components of the electric field along the axes identified by $\hat{s}$ and $\hat{a}$, respectively.

In the symmetric mode, the current distribution in each arm of the antenna approximates that of a single dipole of length h . In the asymmetric mode, the current distribution in each arm of the antenna approximates that of half of a dipole antenna of length $2h$. As a result of the modal properties of the V-shaped antennas and the degrees of freedom in selecting the antenna geometry, namely the length h and the angle Δ , the reflected wave can assume different phase and amplitude values for a given wavelength λ_0 . The numerical simulations of the amplitude and phase responses of the V-shaped antennas are shown in Figure 13c.

The geometry and arrangement of the antennas were appropriately chosen for an incident wave, polarized along the x -axis and with a wavelength of $\lambda_0 \approx 8 \mu\text{m}$. With this configuration, the component of the wave polarized along the y -axis undergoes a phase increment of $\pi/4$ between each adjacent

antenna that constitutes the unit cell of the metasurface. The amplitude of the component polarized along the y -axis is adjusted to be uniform across the entire antenna array. The metasurface induces reflected and transmitted beams at angles that comply with generalized laws over a wide range of wavelengths.

2.5.2 Metasurfaces Based on Geometric Phase

The modulation of the amplitude and phase of the reflected wave, as discussed earlier, can be achieved by appropriately designing the unit cell of the metasurface. An alternative method for introducing discontinuities in the phase response is based on the Pancharatnam–Berry (PB) phase [40, 41], also known as the geometric phase. This approach exploits the polarization conversion obtained through anisotropic media, with dimensions smaller than the wavelength of interest, that are geometrically identical but oriented differently in space.

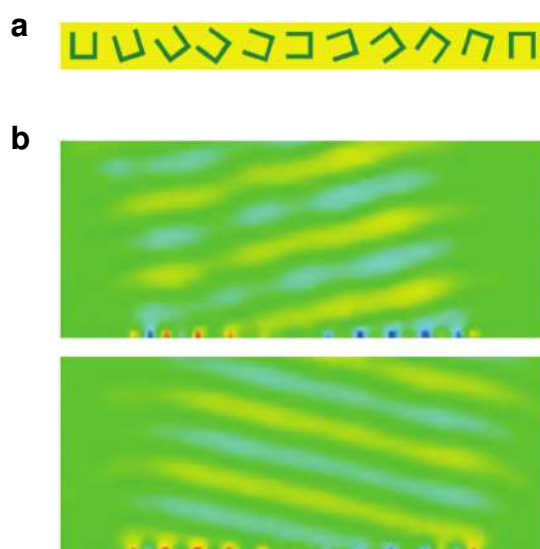


Figure 14: **a)** Unit cell consisting of U-shaped antennas, identical in geometry but differently oriented. **b)** Numerical simulation results showing that the metasurface from Kang *et al.* [42] directs maximum radiation based on the polarization direction of the incident wave.

One way to understand the relationship between polarization and phase is to use the Jones formalism [43]. Generally, the Jones matrix of an anisotropic medium can be written as:

$$\mathbf{M} = \mathbf{R}(-\alpha) \begin{pmatrix} S_o & 0 \\ 0 & S_e \end{pmatrix} \mathbf{R}(\alpha), \quad (24)$$

where S_o and S_e are the scattering coefficients associated with the two orthogonal components of the incident linearly polarized wave,

$$\mathbf{R}(\alpha) = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \quad (25)$$

is the rotation matrix associated with the angle α . For a given incident wave with Right-Handed Circular Polarization (RHCP) or Left-Handed Circular Polarization (LHCP) $\mathbf{E}_i^{R/L}$, the reflected wave from the anisotropic medium $\mathbf{E}_r^{R/L}$ can be written as [42]:

$$\begin{aligned} \mathbf{E}_r^{R/L} &= \mathbf{M} \cdot \mathbf{E}_i^{R/L} \\ &= \frac{S_o + S_e}{2} \mathbf{E}_i^{R/L} + \frac{S_o - S_e}{2} e^{j2\zeta\alpha} \mathbf{E}_i^{L/R}, \end{aligned} \quad (26)$$

From Equation (26), the reflected wave consists of two circular polarization states: the first component has the same polarization direction as the incident wave and shows no phase delay; the second component has the opposite polarization direction to that of the incident wave and exhibits a phase delay of $2\zeta\alpha$, where

$$\zeta = \begin{cases} 1, & \text{RHCP incident wave,} \\ -1, & \text{LHCP incident wave,} \end{cases} \quad (27)$$

By exploiting this principle, Kang *et al.* [42] showed that a phase-gradient metasurface allows directing the maximum of the scattering pattern along different directions, depending on the polarization direction of the incident wave. The unit cell of the metasurface consists of a series of U-shaped antennas, geometrically identical but differently oriented, as shown in Figure 14.

2.5.3 Metalenses

Metasurfaces can introduce abrupt phase shifts in incident EM waves over sub-wavelength scales. As discussed in the previous section, these sharp phase variations along an interface cause abnormal refraction and reflection. By controlling the metasurface's phase profile, the light beam can be twisted or focused, and special optical beams like the optical vortex [44–46] and Bessel beams [47] can be generated. Since phase profile engineering is the fundamental tool for shaping the incident wavefront, this section provides a discussion on creating the phase profile for metalenses.

To focus the EM energy at a distance d_{focus} (as shown in Figure 15), the phase distribution (in the x - y plane) for the reflected or transmitted wave must be arranged to compensate for the optical phase difference:

$$\varphi_L(x, y) = \frac{2\pi}{\lambda} \left(\sqrt{x^2 + y^2 + d_{\text{focus}}^2} - d_{\text{focus}} \right). \quad (28)$$

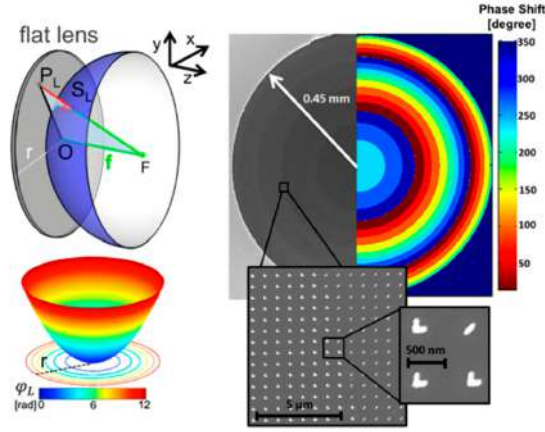


Figure 15: Distribution of the phase profile φ_L of a metalens composed of V-shaped nanoantennas with different lengths and opening angles [48].

Exploiting the concept of optical phase discontinuities, Aieta *et al.* [48] demonstrated a metalens operating at telecommunication wavelengths using V-shaped nanoantennas (as illustrated in Figure 15). However, the device suffered from a low focusing efficiency of approximately 1%. Complementary V-shaped apertures were subsequently proposed to realize metalenses in the optical regime, but without any notable improvement in efficiency.

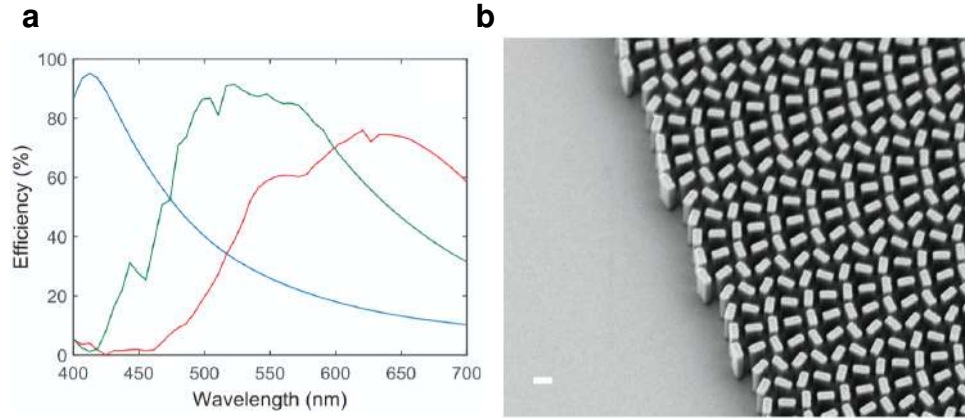


Figure 16: Dielectric metalens composed of high-aspect-ratio TiO_2 nanobricks with different orientations [6]. **a)** Simulated polarization conversion efficiency as a function of wavelength. **b)** Scanning electron microscope micrograph of the fabricated metalens. Scale bar: 300 nm.

To address this limitation, Metal-Insulator-Metal (MIM) structures were introduced, which successfully improved the focusing efficiency in the near-infrared [49] and mid-infrared regimes [50]. Nevertheless, the significant ohmic loss inherent to metals at optical frequencies still fundamentally limits efficiency. Consequently, many metallic metalenses are designed to operate

at larger wavelengths, such as the terahertz, millimeter-wave, or microwave regimes [51–53].

Dielectric metasurfaces were recently developed to circumvent the ohmic losses inherent to metallic structures at optical frequencies [54–57].

In 2014, Lin *et al.* [58] introduced a transmissive metalens composed of thousands of nanobeam nanoantennas, leveraging the spin-dependent properties of the PB phase. Through rational structural tuning, the metalens was designed for operation at a 500 nm wavelength, exhibiting a 100 μm focal length and a Numerical Aperture (NA) of 0.43. When illuminated with right-hand circularly polarized RHCP light through the substrate, the transmitted light was converted into an LHCP focal spot measuring 670 nm, which closely approached the diffraction limit.

Subsequently, Khorasaninejad *et al.* [6] successfully fabricated high-aspect-ratio titanium dioxide (TiO_2) metasurfaces (as shown in Figure 16). Based on the geometric PB phase, they demonstrated three metalenses with a high NA of 0.8 working at 660, 532, and 405 nm with corresponding efficiencies of 66, 73, and 86%. Their later work [59] achieved a polarization-insensitive metalens operating at these same three wavelengths, boasting a large NA of 0.85 and high efficiency (over 60%). Crucially, this work demonstrated promising diffraction-limited focusing spots ($\approx 0.64\lambda$).

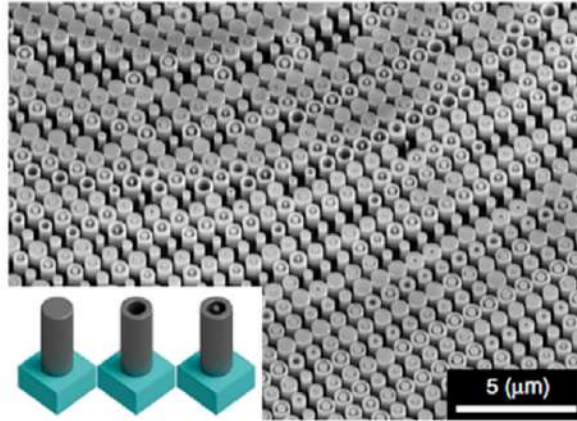


Figure 17: Dielectric metalens working in a large wavelength range (1200–1650 nm) with meta-atoms composed of multi-resonators [60].

Because these metalenses rely on a phase jump governed by resonance conditions, their operation is intrinsically limited across a broad frequency range by the dispersion effect. Consequently, the minimization or removal of chromatic aberration has become a critical concern in metalens design and fabrication.

Various approaches have been developed to address this. Avayu *et al.* [61] designed a multi-layer structure by vertically stacking independent metasurfaces to demonstrate a triply working wavelength achromatic metalens in the visible range (450, 550, and 650 nm). Similarly, Khorasaninejad *et*

al. [62] combined multiple dielectric resonators into meta-atoms to realize an achromatic metalens at three discrete wavelengths (1300, 1550, and 1800 nm). Shrestha *et al.* [60] further utilized this combining method, employing new meta-unit geometries that achieve a much greater space-filling ratio, resulting in a polarization-independent metalens with focusing efficiencies over 20% across a broadband of 1200–1650 nm (as shown in Figure 17).

Furthermore, an achromatic metalens working in a large wavelength band (1200–1680 nm) with an efficiency of $\approx 12\%$ was experimentally demonstrated by employing both low-Q-factor resonance-induced phase and geometric phase that compensated for each other [63]. A significant development was the design and fabrication of an achromatic metalens composed of gallium nitride (GaN)-based integrated resonator meta-atoms, which successfully eliminated chromatic aberration in the visible spectrum (400–660 nm) with an average efficiency of $\approx 40\%$ [64]. More recently, tunable dielectric meta-lenses were also proposed [65, 66], offering greater freedom and potential usage for meta-devices.

2.5.4 Metasurface-Based Invisible Cloak

A cloak is a device capable of rendering objects invisible to incident waves. Transformation optics and metamaterials provide powerful tools for developing such cloaking devices [67–69]. While metamaterials successfully demonstrated the concept of an invisible cloak, the practical development of a cloak using 3-D metamaterials remains extremely challenging.

An alternative approach was first introduced with waveguide-based cloaking. This played a crucial role by leading to a “quasi-conformal mapping technique,” where the cloak concealed an object by restoring a wavefront to appear as if it were reflected from a flat surface. This concept, termed “carpet cloaking,” was subsequently demonstrated [70].

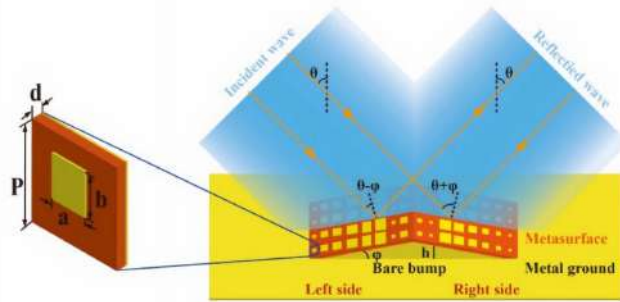


Figure 18: Schematic representation of the metasurface based carpet cloak [71].

The realization of ultra-thin carpet cloaking became feasible through the use of an ultra-thin, engineered metasurface-based structure. Metasurfaces offer an efficient way to design arbitrarily shaped 3-D carpet cloaks with minimal thickness and low loss. A multi-layer metasurface structure was

proposed to achieve broadband performance of the cloak [72]. Specifically, a metallic resonator with a discontinuous phase profile was fabricated onto a polyimide substrate. This structure demonstrated excellent wavefront manipulation when applied to a surface with a triangular bump [71].

The schematic representation of the metasurface carpet cloak is shown in Figure 18. The cloaking layer was fabricated using a standard lithography process, with a low-loss, flexible polyimide layer serving as the spacer medium. The terahertz cloak demonstration used time-domain spectroscopy with an angular scanning system.

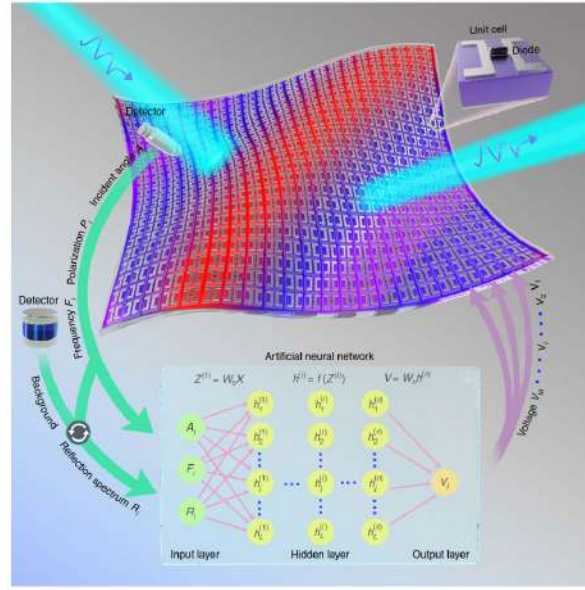


Figure 19: Deep-learning-enabled self-adaptive metasurface cloak [73].

Material dispersion poses the primary challenge for achieving broadband carpet cloaking in optical components. To address this, a novel metasurface was recently proposed for carpet cloaking that operates with linear polarization across the visible spectrum, specifically from 650 to 800 nm. This design introduced a thin slot of a waveguide-based metasurface [74].

More recently, the field has advanced with the use of a metasurface that incorporates a machine learning algorithm to create an adaptive cloaking system. This work represents a significant step toward developing the next generation of invisible cloaks powered by artificial intelligence.

A tunable metasurface—effectively loaded with varactor diodes—was employed to fabricate an invisible cloak operating in the microwave range, as illustrated in Figure 19. Crucially, this metasurface cloak can respond swiftly, on a millisecond timescale, to changes in the incident wave or surrounding medium. This rapid adaptability is managed by a pre-trained artificial neural network, which serves as a fundamental building block of Deep Learning (DL). For further technical details, the reader is referred to Ref. [73].

Part II

OVERVIEW OF DIGITAL METASURFACES

This part offers a comprehensive introduction to digital metasurfaces, outlining their physical principles, electromagnetic properties, and functional classifications. Core concepts, such as wavefront engineering, are explored to establish the theoretical foundation for advanced topics. Particular emphasis is placed on space-time-coding digital metasurfaces, which are crucial for practical applications and represent a central focus of this Thesis.

DIGITAL METASURFACES

3.1 BACKGROUND AND GENERALITIES

Digital metasurfaces consist of meta-atoms whose EM response is discretized into a finite number of quantized states. In a B -bit digital metasurface, each element supports $S = 2^B$ phase states. Ideally, the phases of the corresponding reflection (or transmission) coefficients are uniformly spaced over a 2π interval, with a phase step of $2\pi/S$.

Figure 20a schematically illustrates a 1-bit digital metasurface, described by two distinct coding elements, “0” and “1,” characterized by a phase difference of 180° . Figure 20b shows the geometry of the meta-atom and the phase response of the associated reflection coefficient. The meta-atoms of the metasurface are realized using a square metallic patch, with dimensions smaller than the wavelength, printed on a dielectric substrate terminated by a ground plane. Cui *et al.* [14] have shown that, from 8.1 to 12.7 GHz, two subwavelength square metallic patches of different widths can realize the binary coding elements.

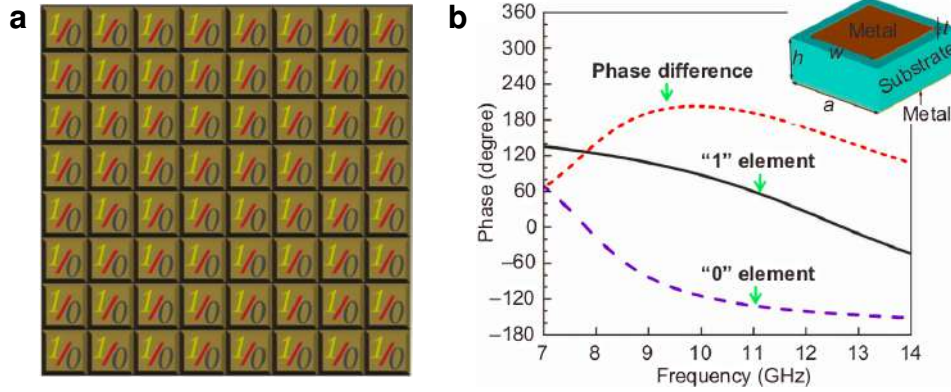


Figure 20: 1-bit coding digital metasurface [14]. **a)** 1-bit coding digital metasurface, composed of the elements “0” and “1”. **b)** Binary element realized with a metallic patch printed on a dielectric substrate.

By properly designing the coding sequence, it is possible to control the EM field reflected by the metasurface. For example, with a coding scheme in which the “0” and “1” elements alternate only along the x -axis, the incident wave is reflected by the metasurface in two directions; whereas with a coding scheme where the binary elements alternate along both axes, the incident wave is reflected in four directions, as shown in Figure 21.

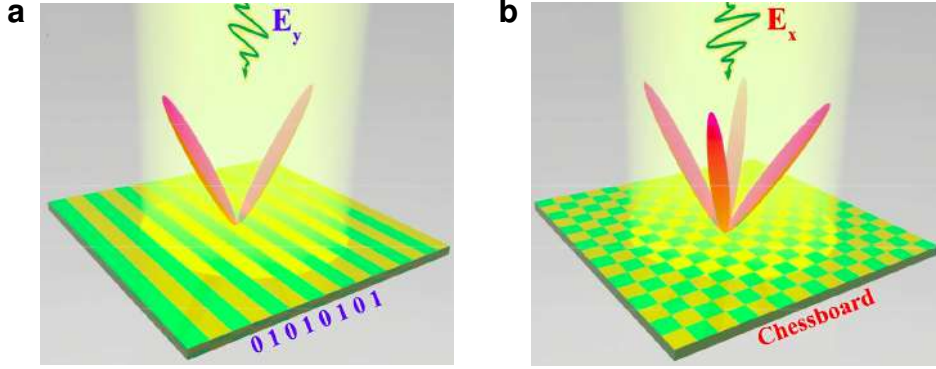


Figure 21: Digital metasurfaces with two different periodic coding schemes, used to control the reflection of an incident wave [14]. **a)** Coding scheme in which the binary elements alternate along the x -axis. **b)** Coding scheme in which the binary elements alternate along both axes.

To understand why this control is possible, without loss of generality, one can consider a 1-bit digital metasurface composed of $N \times M$ unit cells, each with dimensions d_x along the x -axis and d_y along the y -axis. Each unit cell is occupied by either a “0” or “1” element, according to an arbitrarily designed coding sequence. The phase response φ_{nm} of the (n, m) -th meta-atom can take values of either 0 or π . Considering the case of normally incident plane waves and a time-harmonic dependence $\exp(j2\pi f_c t)$, the far-field scattering pattern of the metasurface can be expressed as [14]:

$$\mathbf{f}(\theta, \phi) = \sum_{n=1}^N \sum_{m=1}^M \mathbf{E}_{nm}(\theta, \phi) e^{j(\varphi_{nm} + \frac{2\pi}{\lambda_c} \hat{\mathbf{r}} \cdot \mathbf{r}_{nm})}, \quad (29)$$

where θ and ϕ are the elevation and azimuth angles, respectively, λ_c denotes the wavelength at the carrier frequency f_c , and $\mathbf{E}_{nm}(\theta, \phi)$ represents the field scattered by the (n, m) -th meta-atom of the metasurface. Recalling that the radial unit vector can be expressed as

$$\hat{\mathbf{r}} = \sin \theta \cos \phi \hat{\mathbf{x}} + \sin \theta \sin \phi \hat{\mathbf{y}} + \cos \theta \hat{\mathbf{z}}, \quad (30)$$

and that the position of the (n, m) -th element of the metasurface, lying on the plane $z = 0$, can be identified as

$$\mathbf{r}_{nm} = (n-1)d_x \hat{\mathbf{x}} + (m-1)d_y \hat{\mathbf{y}} + 0 \hat{\mathbf{z}}, \quad (31)$$

it follows that Equation (29) becomes

$$\mathbf{f}(\theta, \phi) = \sum_{n=1}^N \sum_{m=1}^M \mathbf{E}_{nm}(\theta, \phi) e^{j\{\varphi_{nm} + \frac{2\pi}{\lambda_c} \sin \theta [(n-1)d_x \cos \phi + (m-1)d_y \sin \phi]\}}. \quad (32)$$

Furthermore, the directivity $\text{Dir}(\theta, \phi)$ of the metasurface can be expressed as

$$\text{Dir}(\theta, \phi) = \frac{4\pi |\mathbf{f}(\theta, \phi)|^2}{\int_0^{2\pi} \int_0^{\pi/2} |\mathbf{f}(\theta, \phi)|^2 \sin \theta d\theta d\phi}. \quad (33)$$

From Equations (32) and (33), it follows that the scattering pattern of the reflected field can be shaped by modifying the metasurface coding. Consequently, the direction of maximum radiation can be controlled.

The concept of digital metasurfaces can be extended from 1-bit coding to 2-bit coding, and so on. In 2-bit coding, four unit cells are required to realize the “00,” “01,” “10,” and “11” elements, for which the phase exhibits a step difference of 90° and a possible coding alphabet would be $\{0^\circ, 90^\circ, 180^\circ, 270^\circ\}$. Clearly, 2-bit coding offers more degrees of freedom in manipulating EM waves compared to 1-bit coding, since more coding states are available. Figure 22 shows a possible realization of the unit cells of a 2-bit digital metasurface [14]. By varying the width of the metallic patches, the four coding states of the 2-bit scheme can be obtained.

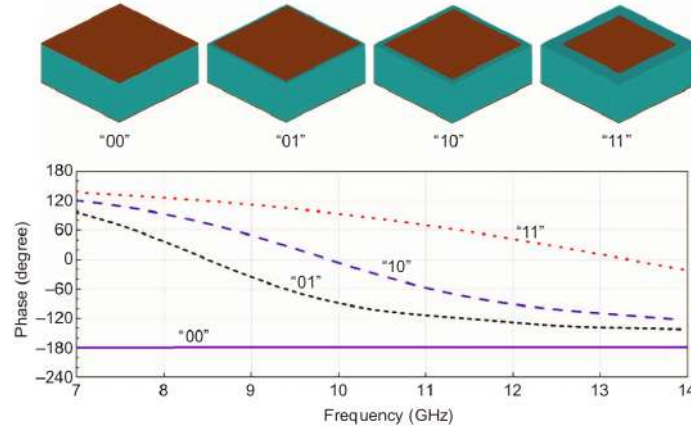


Figure 22: Constitutive elements of a metasurface with 2-bit coding and the corresponding phase responses [14].

3.1.1 Information-Theoretic Aspects

A common scenario in information theory involves transmitting information from multiple transmitters to multiple receivers through various channels. As illustrated in Figure 23a, a pair of horn antennas (for transmitting and receiving) and a metasurface can be considered as an information system. Inspired by Shannon’s information entropy, which quantifies the information in each message, Cui *et al.* [75] proposed a similar metric to measure the information associated with a coding metasurface.

Figure 23b shows that a simple 1-bit coding scheme provides an effective way to study the information entropy of a metasurface. In this scheme, only four combinations— $G(0,0)$, $G(0,1)$, $G(1,0)$, and $G(1,1)$ —are possible in adjacent coding elements. The 2-D information entropy is determined by the probabilities of these four combinations. Based on the definition of normalized Shannon entropy, a geometric information entropy H_1 was introduced to quantify the coding pattern, while a physical information entropy H_2 was

proposed to estimate the information carried by the far-field pattern, computed via a Fourier Transform (FT) of the coding pattern.

The relationship between the geometric (coding pattern) and physical (far-field pattern) information entropies, as a function of iteration number, is shown in Figure 23c. As seen, the physical information entropy gradually increases, approaching the geometric entropy. This demonstrates that the physical information entropy can be controlled by the diffusion depth of the random coding pattern.

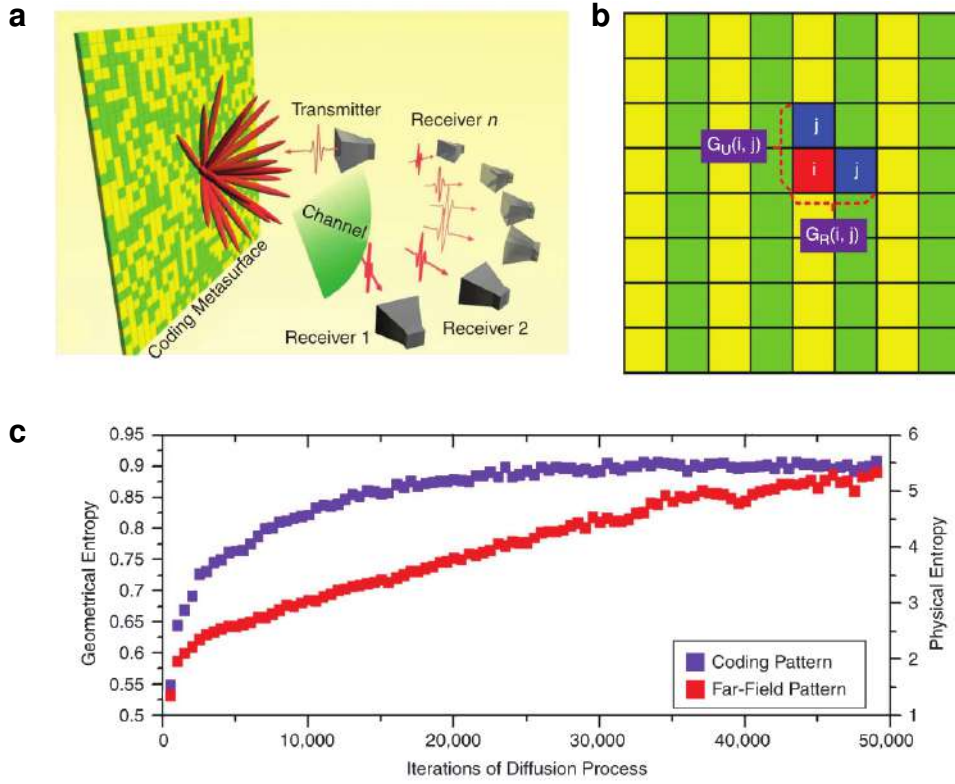


Figure 23: Information-theoretic aspects [75]. **a)** Schematics of possible information systems based on metasurfaces. **b)** A conceptual calculation of the physical information entropy. **c)** Geometrical and physical information as a function of iterations in the diffusion process.

3.1.2 Convolution-Based Operations

To steer a scattering pattern toward an arbitrarily predesigned direction, convolution-type operations can be exploited [76]. In signal processing, as illustrated in Figure 24g-i, convolution implies a shift in the spectrum, namely

$$f(t)e^{j\omega_0 t} \xLeftrightarrow{FT} F(\omega) * \delta(\omega - \omega_0) = F(\omega - \omega_0), \quad (34)$$

with $\delta(\cdot)$ denoting the Dirac delta impulse function. Likewise, we can define the principle of scattering-pattern shift as follows:

$$E(x_\lambda)e^{jx_\lambda \sin \theta_0} \xleftrightarrow{FT} E(\sin \theta) * \delta(\sin \theta - \sin \theta_0) = E(\sin \theta - \sin \theta_0), \quad (35)$$

where $x_\lambda = x/\lambda$ is the electrical length, and $\exp(jx_\lambda \sin \theta_0)$ denotes the electric field distribution generated by a gradient phase pattern. Hence, by multiplying the coding pattern $E(x_\lambda)$ by the gradient coding sequence associated with $\exp(jx_\lambda \sin \theta_0)$, a steering of a quantity $\sin \theta_0$ (in angular coordinates) is obtained. As shown in Figure 24a–c, Ref. [76] considers a 2-bit coding metasurface with the four digits “0,” “1,” “2,” and “3” represented with different shades of gray. Figure 24a,b shows that a cross-shaped coding pattern (consisting of digits “0” and “2”) is multiplied by a gradient coding sequence. The result in Figure 24c clearly shows that, after the convolution operation with the single-beam scattering pattern, the scattering pattern (Figure 24d) is steered from the original direction, with no apparent distortion. This is analogous to the convolution in the frequency domain schematized in Figure 24g–i.

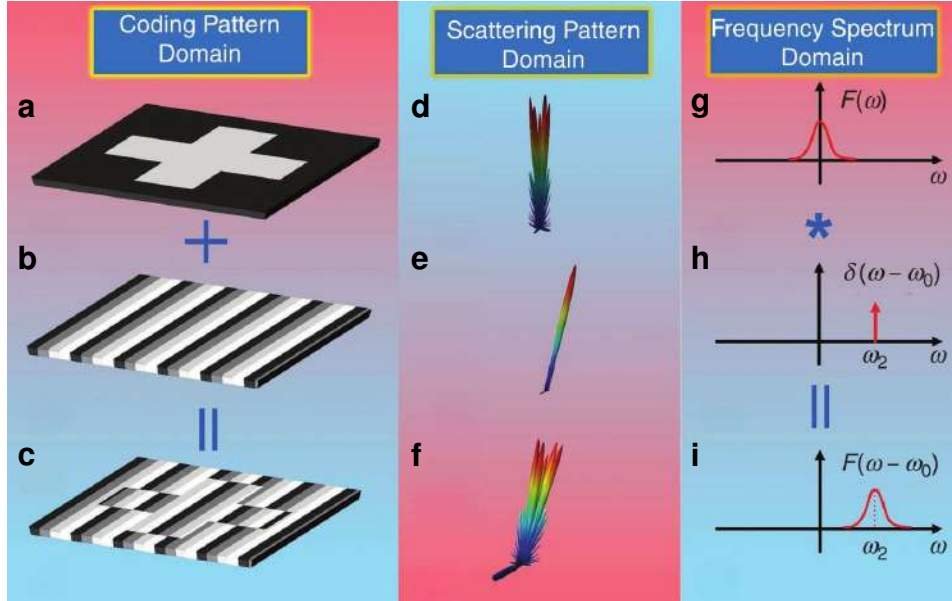


Figure 24: Convolution-based operations [76]. **a–c)** Coding patterns with a cross shape, phase gradient, and their modulus, respectively. **d–f)** Corresponding scattering patterns. **g–i)** Analogy in the frequency domain.

3.2 PROGRAMMABLE METASURFACES

The metasurfaces described so far allow the engineering of certain properties of EM waves through spatial modulation of the meta-atoms. In fact, by using patches of different sizes, it is possible to obtain the various elements

of a digital coding scheme. However, once the geometry of the constituent elements and their arrangement are fixed, it is not possible to vary the properties of the metasurface thus created. Assuming that the properties of the metasurface are not time-varying, the device is capable of manipulating the EM wave spectrum in the wave vector domain, leaving the frequency domain behavior unchanged.

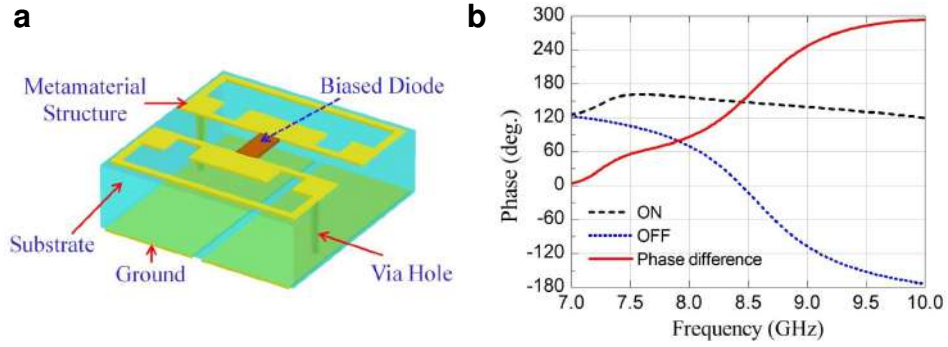


Figure 25: Device designed to achieve temporal modulation [14]. **a)** The structure exhibits the coding states “0” and “1” when the PIN diode is in the “OFF” and “ON” states, respectively. **b)** Phase response of the device when the PIN diode is in the “OFF” or “ON” state.

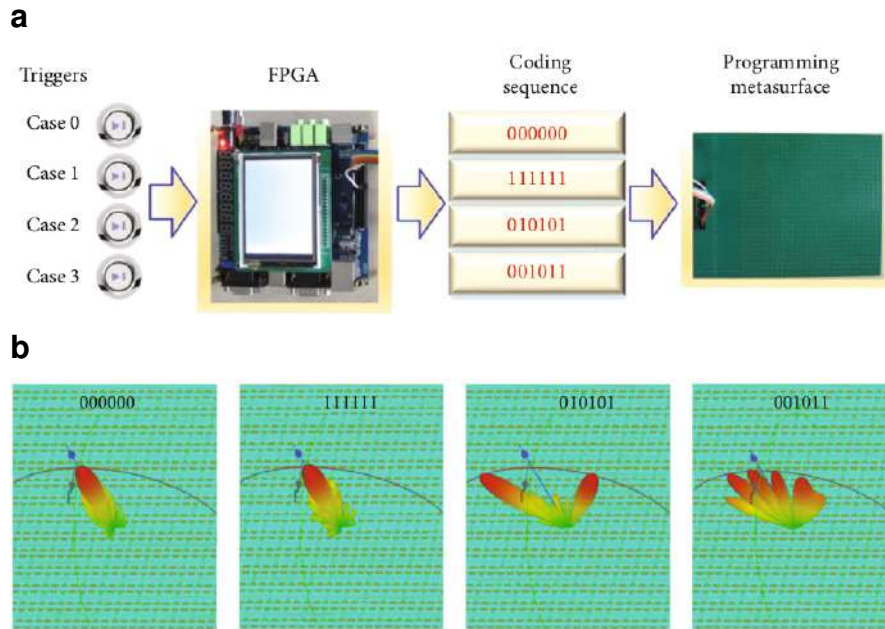


Figure 26: **a)** Flowchart of a 1-bit programmable metasurface controlled by FPGA hardware. **b)** Scattering patterns, obtained numerically, of the field reflected by the metasurface for different coding sequences [14].

In order to introduce a temporal modulation, Cui *et al.* [14] proposed implementing active devices, such as diodes, to design metasurfaces capable of controlling EM fields in real-time. Figure 25 shows the geometry and phase behavior of the reflection coefficient of the 1-bit programmable element, which behaves like the “0” and “1” elements of a binary coding scheme when the Positive-Intrinsic-Negative (PIN) diode is in the “OFF” and “ON” states, respectively. The coding states of a programmable metasurface can be generalized from the binary case to an B -bit configuration. For example, a 2-bit programmable metasurface can be realized by integrating multiple PIN diodes into each unit cell. In this way, a more versatile programmable metasurface can be designed and characterized.

Figure 25a schematically shows the process by which a digital metasurface can be controlled via an FPGA, allowing for different behaviors of the reflected field depending on the chosen coding sequence, as shown in Figure 25b.

3.3 ANALYTICAL MODEL

In this section, the main theoretical aspects of the STC-DMs are presented, with reference to a metasurface composed of $N \times M$ programmable elements, shown in Figure 27. The programmable elements (represented by the yellow patches) have the same geometry, and each of them is integrated with active devices, such as PIN diodes. By applying different voltages to the active devices, the reflection coefficient of the programmable elements can be dynamically adjusted, allowing it to take on quantized values in either amplitude or phase.

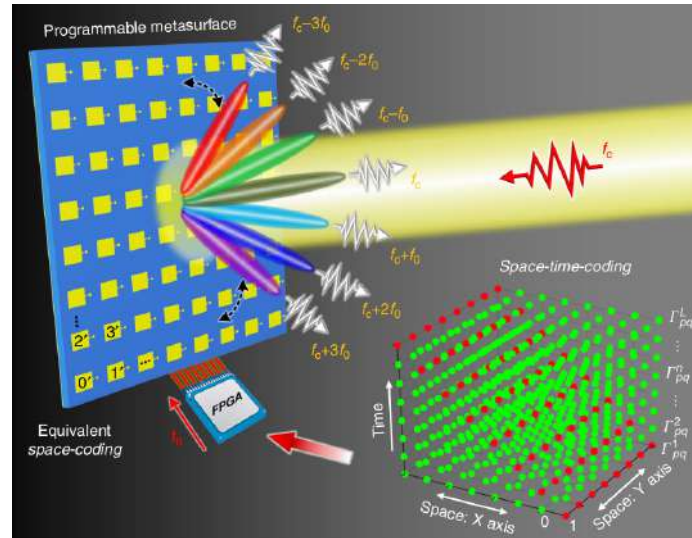


Figure 27: Conceptual illustration of a STC-DM [28].

For the 1-bit coding scheme, the behavior of each element alternates between two states, “0” and “1,” corresponding to which the reflected wave is either in-phase or out-of-phase with the incident wave. Moreover, it is assumed that the states of the programmable elements change in both the spatial and temporal domains through an FPGA, in line with the Space-Time Coding (STC) matrix represented in Figure 27, where each element is periodically modulated over time through a set of temporal coding sequences.

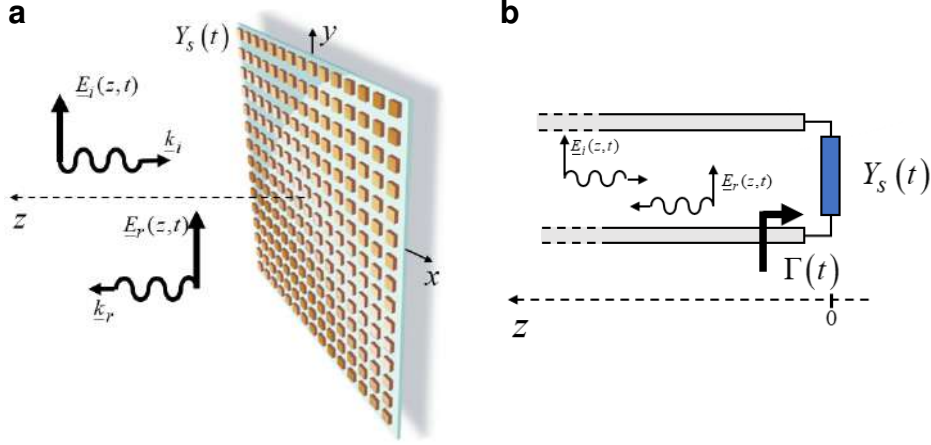


Figure 28: **a)** A metasurface with time-varying admittance, illuminated by an EM wave oscillating at frequency f_c . **b)** Equivalent transmission line model [10].

In order to obtain information about the reflected EM field from the considered metasurface, one can focus on the case of normal incidence of a plane wave with harmonic time dependence of the form $\exp(j2\pi f_c t)$. This allows for deriving the general relationships between the incident and reflected waves in the time-varying case, and between the reflection coefficient and the admittance that describes the metasurface [10]. If the characteristic dimension of the unit cell is much smaller than the wavelength of interest, the array composed of such elements can be described by an equivalent admittance Y_s . Furthermore, assuming that the modulation frequency f_0 is much lower than the central frequency f_c of the EM wave, the mathematical analysis is simplified, and the metasurface admittance becomes a time-dependent function, $Y_s = Y_s(t)$. Under these conditions, the system can be studied using an equivalent transmission line model in which the incident and reflected waves are related by a time-varying reflection coefficient

$$\Gamma(t) = \frac{Y_0 - Y_s(t)}{Y_0 + Y_s(t)}, \quad (36)$$

where Y_0 is the admittance of free space. The scattered EM field at a large distance from the digital metasurface under consideration can be expressed as [28]

$$\mathbf{f}(\theta, \phi, t) = \sum_{n=1}^N \sum_{m=1}^M \mathbf{E}_{nm}(\theta, \phi) \Gamma_{nm}(t) e^{j \frac{2\pi}{\lambda_c} \sin \theta [(n-1)d_x \cos \phi + (m-1)d_y \sin \phi]}, \quad (37)$$

where $\mathbf{E}_{nm}(\theta, \phi)$ is the field scattered by the (n, m) -th element, λ_c is the operating wavelength, d_x and d_y are the dimensions of the unit cell, and $\Gamma_{nm}(t)$ represents the time-varying reflection coefficient of the (n, m) -th element, which is a periodic function of time, defined over a period T_0 as a linear combination of shifted time windows

$$\Gamma_{nm}(t) = \sum_{l=1}^L \Gamma_{nm}^{(l)} W^{(l)}(t), \quad 0 < t < T_0, \quad (38)$$

where $W^{(l)}(t)$ denotes a time window with modulation period T_0

$$W^{(l)}(t) = \begin{cases} 1, & (l-1)\tau \leq t \leq l\tau, \\ 0, & \text{otherwise,} \end{cases} \quad (39)$$

where $\tau = T_0/L$ is the time duration of the window $W^{(l)}(t)$, L is a positive integer representing the length of the temporal coding sequence, and $\Gamma_{nm}^{(l)} = A_{nm}^{(l)} \exp(j\varphi_{nm}^{(l)})$ is the reflection coefficient associated with the (n, m) -th coding element during the time interval $(l-1)\tau \leq t \leq l\tau$. The function $W^{(l)}(t)$ can be expressed through the Fourier series as

$$W^{(l)}(t) = \sum_{v=-\infty}^{+\infty} w^{(v,l)} e^{j2\pi v f_0 t}, \quad (40)$$

where $f_0 = 1/T_0$ and the Fourier coefficients $w^{(v,l)}$ are given by

$$w^{(v,l)} = \frac{1}{T_0} \int_0^{T_0} W^{(l)}(t) e^{-j2\pi v f_0 t} dt. \quad (41)$$

At this point, the reflection coefficient can also be expressed through its Fourier series

$$\Gamma_{nm}(t) = \sum_{v=-\infty}^{+\infty} a_{nm}^{(v)} e^{j2\pi v f_0 t}, \quad (42)$$

where the coefficients $a_{nm}^{(v)}$ are given by

$$a_{nm}^{(v)} = \frac{1}{T_0} \int_0^{T_0} \Gamma_{nm}(t) e^{-j2\pi v f_0 t} dt. \quad (43)$$

By using Equation (38) for the reflection coefficient $\Gamma_{nm}(t)$ in Equation (43), the following expression for the coefficients $a_{nm}^{(v)}$ is obtained:

$$\begin{aligned} a_{nm}^{(v)} &= \frac{1}{T_0} \sum_{l=1}^L \Gamma_{nm}^{(l)} \int_{(l-1)\tau}^{l\tau} e^{-j2\pi v f_0 t} dt \\ &= \frac{1}{L} \text{sinc}\left(\frac{\pi v}{L}\right) \sum_{l=1}^L \Gamma_{nm}^{(l)} e^{-j\frac{\pi v(2l-1)}{L}}, \end{aligned} \quad (44)$$

where $\text{sinc}(x) = \sin x/x$. Furthermore, by substituting Equation (44) into Equation (37), the scattered field at a large distance from the digital metasurface can be expressed as follows:

$$\mathbf{f}(\theta, \phi, t) = \sum_{\nu=-\infty}^{+\infty} \mathbf{F}^{(\nu)} e^{j2\pi(f_c + \nu f_0)t}, \quad (45)$$

where $\mathbf{F}^{(\nu)}(\theta, \phi)$ is the far-field scattering pattern corresponding to the ν -th harmonic order, whose frequency is $f_c + \nu f_0$. This results in a harmonic expansion where each term corresponds to a distinct equivalent scattering pattern

$$\mathbf{F}^{(\nu)}(\theta, \phi) = \sum_{n=1}^N \sum_{m=1}^M \mathbf{E}_{nm}(\theta, \phi) a_{nm}^{(\nu)} e^{j\frac{2\pi}{\lambda_c} \sin \theta [(n-1)d_x \cos \phi + (m-1)d_y \sin \phi]}. \quad (46)$$

Thus, given an arbitrary STC sequence, it is possible to determine the scattered field at a large distance from a STC-DM corresponding to each harmonic frequency.

It is useful to note that the complex coefficients $a_{nm}^{(\nu)}$ defined by Equation (44) can be synthesized to obtain equivalent amplitudes $|a_{nm}^{(\nu)}|$ and phases $\angle a_{nm}^{(\nu)}$ through linear combinations of the reflection coefficients $\Gamma_{nm}^{(l)}$. In other words, it is possible to individually control the amplitudes and phases of each coding element, at a specific frequency, by acting on the STC sequences. Therefore, for an arbitrary 3-D STC matrix derived from Equation (38), the corresponding feeding currents can be calculated through Equation (44).

3.3.1 Equivalent Phase Coverages

The equivalent phases $\psi_{nm}^{(\nu)} = \angle a_{nm}^{(\nu)}$ of each coding element can be designed *independently* at any harmonic order ν through a suitable selection of the corresponding time-coding sequence. A comprehensive theoretical investigation into the phase coverage achievable at different harmonic frequencies was previously reported in Ref. [28]. In the 1-bit scenario, where the coding elements are restricted to only 0° or 180° phase responses, the resultant equivalent phase coverages are illustrated in Figure 29a. It can be readily observed that the phase coverage spans a full 360° at all harmonic frequencies, with the sole exception of the central frequency ($\nu = 0$). This observation can be rigorously explained by noting that the equivalent phase $\psi_{nm}^{(0)}$ at the central frequency is fundamentally determined by the expression:

$$\psi_{nm}^{(0)} = \arg \left(\frac{1}{L} \sum_{l=1}^L \Gamma_{nm}^{(l)} \right), \quad (47)$$

which clearly dictates that $\psi_{nm}^{(0)}$ can only assume values of either 0° or 180° . To mitigate this constraint, the authors extended their study to the phase

coverages of 2-bit digital coding elements, which possess four distinct phase states: 0° , 90° , 180° , and 270° . As depicted in Figure 29b, the equivalent phase coverages at the central frequency are shown to achieve nearly 360° , provided that the time-coding sequences associated with each element are of sufficient length.

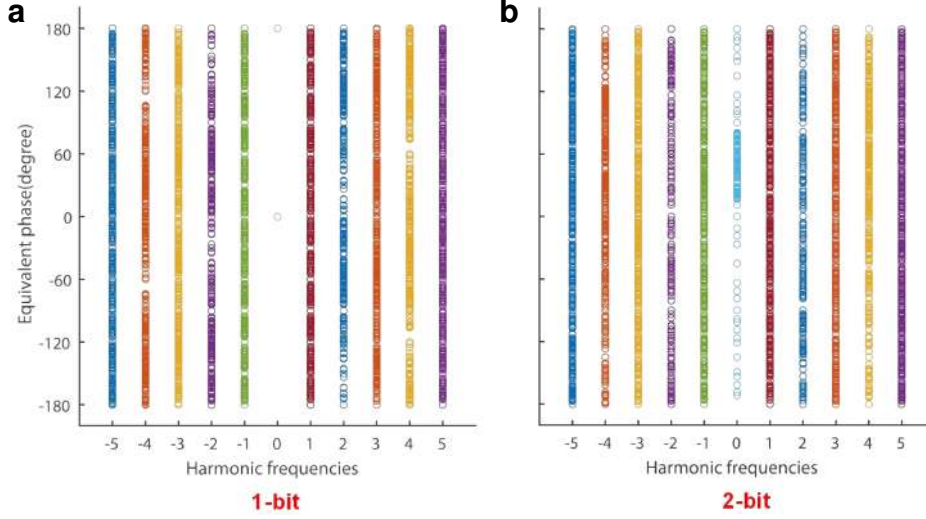


Figure 29: Equivalent phase coverages of digital coding elements, at different harmonic frequencies [28]. **a)** 1-bit case. **b)** 2-bit case.

In contrast, the control systems and biasing circuitry required for conventional programmable metasurfaces with more than 2 bits are inherently complex. To realize non-modulated elements with higher-bit coding, such as 3-bit resolution, additional diodes must typically be incorporated into the element design. This necessity leads to a significantly complicated layout and poses considerable challenges to the control system's architecture. However, the proposed STC-DMs offer an elegant, simplified approach for achieving nearly 360° phase coverage at both the central and all harmonic frequencies. This capability means that high-bit programmable metasurfaces can be implemented simply by suitably tailoring the time-coding sequences, entirely circumventing the need for intricate layouts and complex control systems.

3.4 REPRESENTATIVE APPLICATIONS

Using the theoretical model introduced earlier, STC-DMs can be employed to manipulate EM waves, both in the spectral and spatial domains. Furthermore, by carefully designing the STC matrix, information processing can be performed not only in the spatial domain but also in the temporal domain. In this section, we will primarily focus on the phase modulation scheme of the reflection coefficient, where the amplitudes $A_{nm}^{(l)}$ of each coding element remain constant, while the phases $\varphi_{nm}^{(l)}$ take discrete values in accordance

with a STC matrix, in order to discuss representative and significant applications of STC-DMs.

3.4.1 Harmonic Beam Manipulations

3.4.1.1 Harmonic Beam Steering

An application inspired by STC-DMs is the harmonic beam steering, through which the main beam of the reflected field is directed along different directions corresponding to different harmonics [28], as shown in Figure 27. In Ref. [28], a simple STC matrix is proposed, shown in Figure 30, through which harmonic beam steering can be realized. However, this scheme presents unbalanced power distributions. To address the problem, the Binary Particle Swarm Optimization (BPSO) algorithm has been used to optimize the STC matrix, in order to equalize power levels at different frequencies [28].

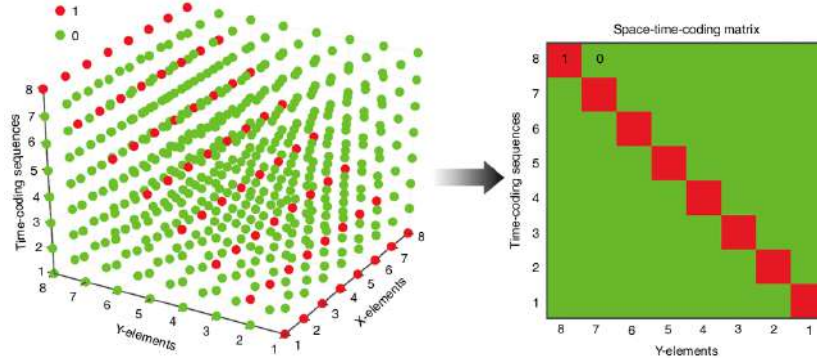


Figure 30: The 3-D STC matrix and its corresponding 2-D coding matrix [28].

In this case, a STC-DM consisting of 8×8 elements with dimensions equal to half the wavelength was considered, along with a 1-bit temporal coding sequence of length $L = 10$. The elements of the metasurface belonging to the same column share the same temporal coding sequence, which allows the original 3-D coding matrix to be reduced to a 2-D matrix, shown in Figure 31a. According to this coding matrix, the scattering patterns obtained for the various harmonics, shown in Figure 31b–d, were numerically calculated using Equation (46). As observed, the radiation peak approximately maintains the same amplitude value across different harmonics and lies along different directions. To experimentally verify this behavior, a STC-DM was designed to operate at a frequency of 10 GHz [28]. When the PIN diode integrated into the coding element is in the “ON” and “OFF” states, the reflection coefficient exhibits a phase difference of 180° . In the case of 1-D beam steering, the eight coding elements in the same column of the metasurface are supplied with the same voltage via FPGA hardware, which provides eight control voltages in accordance with the coding matrix in Figure 31a.

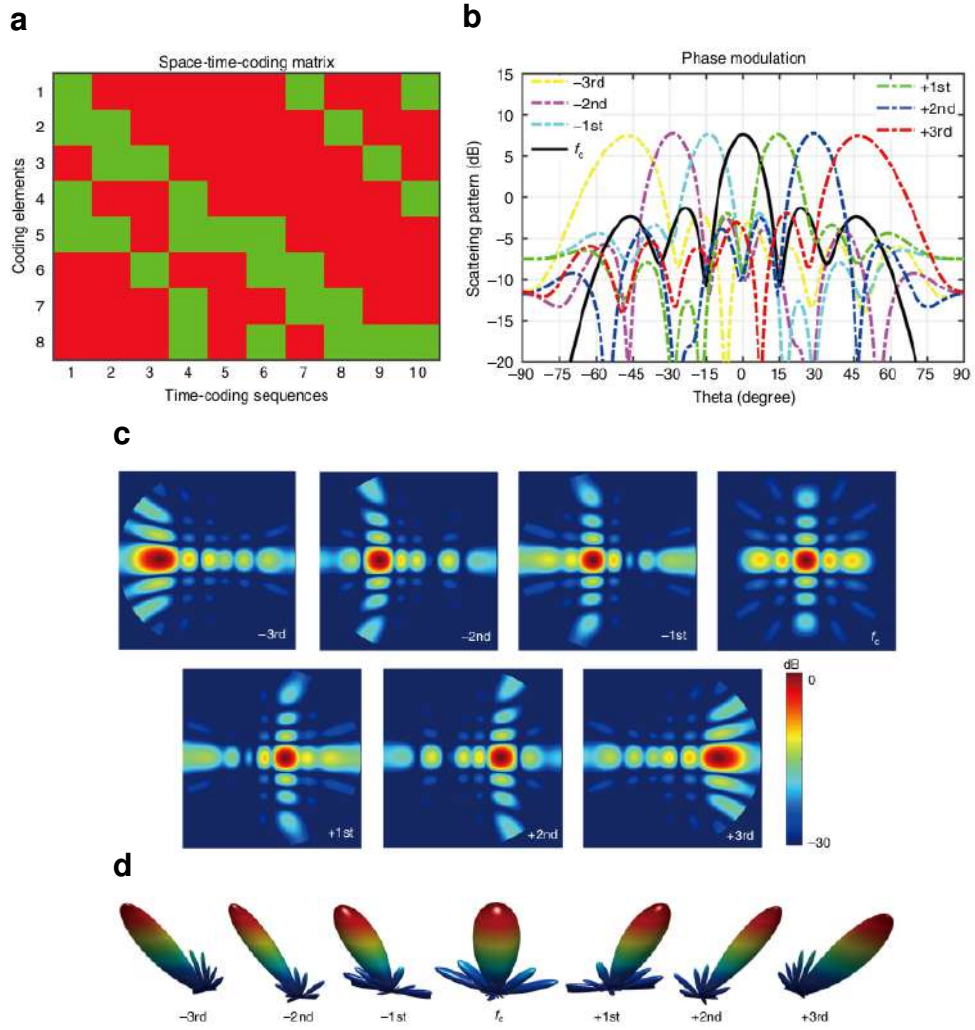


Figure 31: a) Optimized 2-D STC matrix. b–d) Corresponding 1-D scattering patterns, obtained for $\phi = 90^\circ$, 2-D, and 3-D, respectively, for different harmonics [28].

Using a modulation frequency of $f_0 = 0.5$ MHz, the measured scattering patterns are in agreement with the theoretical ones.

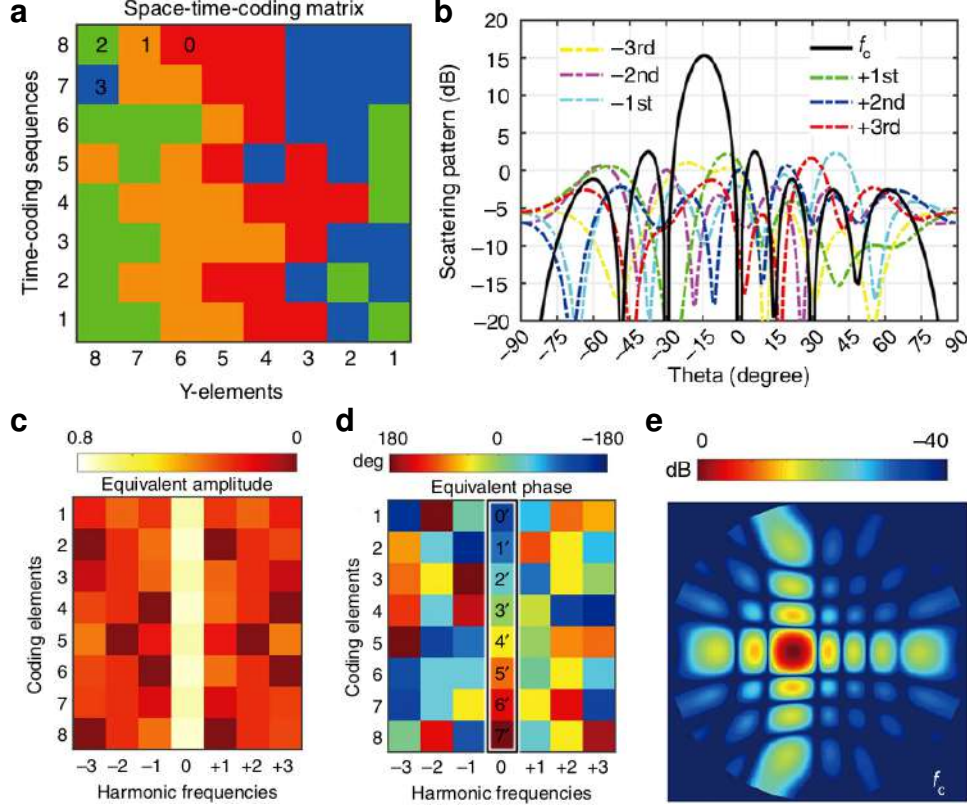


Figure 32: a) STC matrix. b) Corresponding 1-D scattering patterns, obtained for $\phi = 90^\circ$. c,d) Amplitude and phase distributions corresponding to different harmonics. e) Corresponding 2-D scattering pattern at the central frequency [28].

From the beam steering example just discussed, it is noted that for harmonic orders ($\nu \neq 0$), the radiation peak can be oriented along a direction different from the broadside direction, while at the central frequency ($\nu = 0$), the radiation peak always lies along the broadside when using a 1-bit STC matrix. This limitation is a consequence of Equation (44), where 1-bit temporal coding sequences can produce equivalent phases of 0° and 180° at the central frequency. To relax this constraint, a multi-bit coding scheme has been adopted in the physical realization of the metasurface [28]. Therefore, an example of beam steering at the central frequency has been proposed, where the STC-DM is analogous to the one used in the previous example, but the length of the temporal coding sequences is $L = 8$ and a 2-bit coding is employed.

Figure 32a shows the 2-bit STC matrix used. The amplitudes and equivalent phases of these eight coding elements at different harmonics are shown in Figure 32c,d. It is observed that the proposed STC matrix exhibits a re-

sponse equivalent to a 3-bit coding and a phase gradient at the central frequency. In accordance with the generalized Snell's laws [4], this phase discontinuity allows the radiation peak to be steered along a direction different from the broadside. Figure 32b shows the 1-D scattering patterns at different harmonics, while Figure 32e shows the 2-D scattering pattern at the central frequency f_c .

3.4.1.2 Beam Shaping at the Central Frequency

Using the eight temporal coding sequences shown in Figure 33a, an example of beam shaping at the central frequency was proposed [28], through which it was possible to shape the main beam of the scattering pattern. The STC-DM was segmented into eight sectors, with each sector using the same temporal coding sequence. Figure 33a,b show the amplitudes and phases corresponding to the central frequency, where the phases equivalent to a 3-bit coding follow a spiral pattern and can generate a vortex beam. The corresponding 2-D scattering pattern is shown in Figure 33c, where the typical behavior of a vortex beam with Orbital-Angular-Momentum (OAM) of order $\ell = 1$ [77] can be recognized at the central frequency.

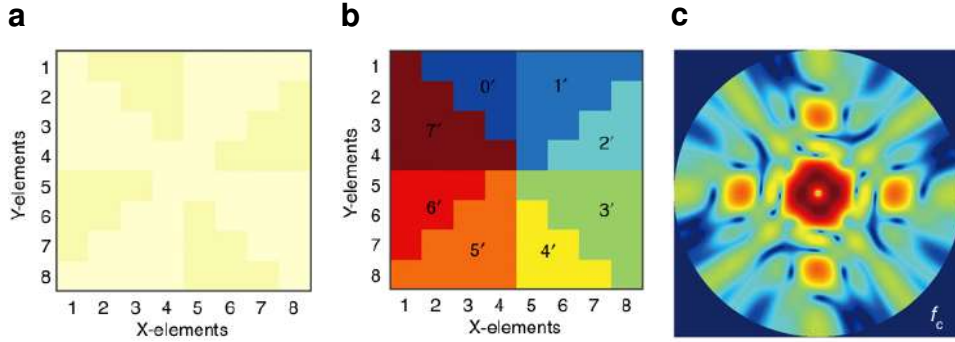


Figure 33: Application of STC-DMs in beam shaping [28]. **a,b)** Amplitude and equivalent phase distributions to generate the vortex beam at the central frequency. **c)** Corresponding 2-D scattering pattern at the central frequency.

3.4.1.3 Multi-Frequency Beam Shaping and Steering

In Section 3.4.1.1, a harmonic beam steering method was introduced, enabling the orientation of the radiation maximum at different frequencies through numerical optimization using the BPSO algorithm. However, the applicability of optimization methods is constrained by their computational complexity. A limitation of STC schemes is the intrinsic coupling of multi-frequency synthesis [28], derived from Equation (44). From this equation, it can be seen that the equivalent amplitudes $|a_{nm}^{(v)}|$ and the equivalent phases $\angle a_{nm}^{(v)}$ depend on all the reflection coefficients $\Gamma_{nm}^{(l)}$ of the temporal coding

sequence. Therefore, conventional STC schemes do not allow independent multi-frequency synthesis of scattering patterns. To overcome this limitation, a new coding technique has been proposed based on temporal coding sequences designed from appropriate subsequences, to decouple the multi-frequency synthesis [78].

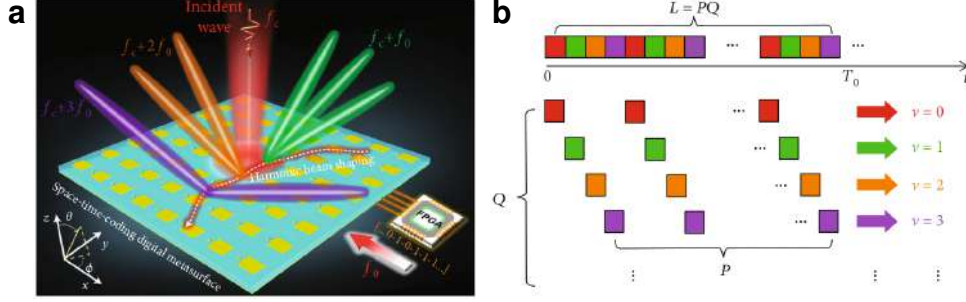


Figure 34: a) Conceptual representation of multi-frequency synthesis using a STC-DM. b) Diagram of the coding subsequences [78].

As shown in Figure 34a, a digital metasurface encoded with the approach proposed in Ref. [78] can independently and simultaneously synthesize assigned scattering patterns at multiple frequencies. In Ref. [78], the phase values of the reflection coefficient of the STC-DM are quantized by the number of available coding states S over the interval $(0^\circ, 360^\circ)$. Thus, for the 1-bit case ($S = 2$), we have $\Gamma_{nm}^{(l)} \in \{1, -1\}$, while for the 2-bit case, $\Gamma_{nm}^{(l)} \in \{1, j, -1, -j\}$, and so on. Figure 34b schematically shows the proposed strategy, based on coding subsequences. As hypothesized in Ref. [78], the problem was formulated in terms of Q coupled harmonic syntheses ($v = 0, \dots, Q - 1$). To address this, each temporal coding sequence is broken down into Q coding subsequences, each of length P . Thus, the total length of the temporal coding sequence over a period T_0 is $L = PQ$. Omitting the nm subscripts for simplicity of notation, the temporal coding sequence composed of the subsequences can be expressed as [78]

$$\Gamma_{pQ+q+1} = \Omega_{pq} \Gamma_{q+1}, \quad p = 0, \dots, P - 1, \quad (48)$$

$$q = 0, \dots, Q - 1,$$

where Ω_{pq} are coefficients that introduce a phase delay. Therefore, the complex quantities expressed by Equation (44) can be rewritten as [78]

$$a^{(v)} = c^{(v)} \sum_{q=0}^{Q-1} \Gamma_{q+1} \alpha_q^{(v)} e^{-j \frac{2\pi v q}{L}}, \quad (49)$$

where

$$c^{(v)} = \frac{1}{L} \text{sinc} \left(\frac{\pi v}{L} \right) e^{-j \frac{\pi v}{L}} \quad (50)$$

are coefficients that describe how the spectrum decays at the ν -th harmonic order, while the terms

$$\alpha_q^{(\nu)} = \sum_{p=0}^{P-1} \Omega_{pq} e^{-j \frac{2\pi \nu p}{P}} \quad (51)$$

can be interpreted as digital filters to be appropriately designed by adjusting the coefficients Ω_{pq} to decouple the syntheses at different frequencies. Decoupling the syntheses is equivalent to requiring that

$$\alpha_q^{(\nu)} \propto \tilde{\delta}_{q\nu}, \quad (52)$$

where

$$\tilde{\delta}_{q\nu} = \begin{cases} 1, & q = \nu \\ 0, & q \neq \nu \end{cases}, \quad q, \nu = 0, \dots, Q-1, \quad (53)$$

denotes a reduced Kronecker delta, defined over a restricted subset of indices. In this context, each coding subsequence acts on a specific harmonic, enabling decoupled multi-frequency synthesis. To determine the coefficients Ω_{pq} such that $\alpha_q^{(\nu)} \propto \tilde{\delta}_{q\nu}$, the following minimization problem was solved:

$$\min_{\Omega_{pq}} \left\| \sum_{p=0}^{P-1} \Omega_{pq} e^{-j \frac{2\pi \nu p}{P}} - \chi \tilde{\delta}_{q\nu} \right\|, \quad \nu = 0, \dots, Q-1, \quad (54)$$

where $\|\cdot\|$ denotes an appropriate norm and $\chi \neq 0$ is a constant. This problem admits a straightforward solution when the number of harmonics of interest Q does not exceed the number of available quantized phase states S that the phase of the reflection coefficient can assume—i. e., when $Q \leq S$. Therefore, by choosing $P = S$, one obtains

$$\Omega_{pq} = e^{j \frac{2\pi pq}{S}}. \quad (55)$$

To demonstrate the effectiveness of this approach, in Ref. [78], a STC-DM with 2-bit encoding ($S = 4$) consisting of 16×16 programmable elements was considered. The proposed technique was applied to perform a synthesis at $Q = 4$ harmonic frequencies, using coding subsequences of length $P = 4$. Consequently, the total length of the temporal coding sequence fitting within one period is $L = PQ = 16$. Figure 35a shows the synthesized sequences for the complex-valued equivalent amplitudes $a^{(\nu)}$ corresponding to the first four harmonic orders, while Figure 35b shows the corresponding 2-D scattering patterns of the field reflected by the metasurface. It can be seen that the four scattering patterns corresponding to the harmonic frequencies were synthesized independently.

Due to the inherent periodicity of the digital-filter terms in Equation (51), the syntheses performed at the first Q harmonic orders repeat periodically, with progressively decreasing amplitudes following the spectral decay described in Equation (50). This behavior leads to reduced spectral efficiency,

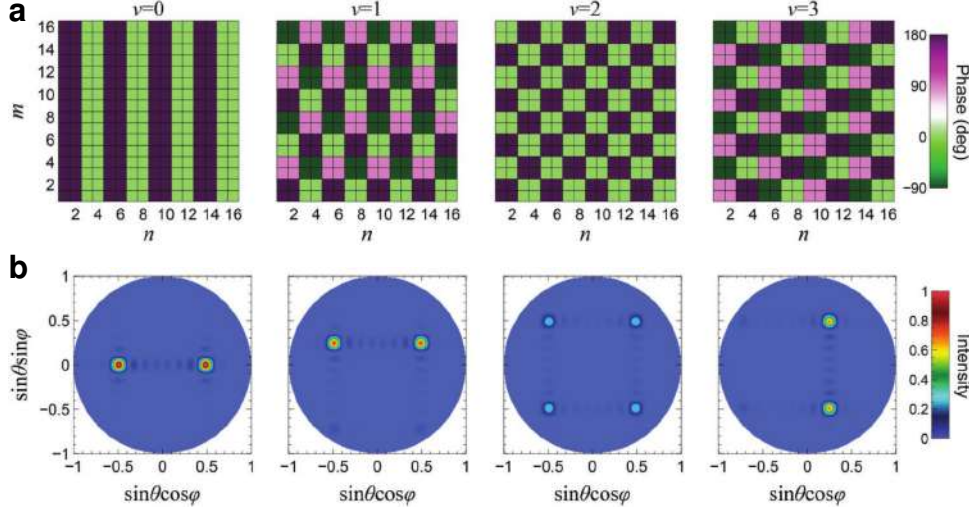


Figure 35: a) Synthesized sequences for the complex-valued equivalent amplitudes $a^{(v)}$ corresponding to the first four harmonic frequencies. b) Corresponding 2-D scattering patterns, determined numerically [78].

since energy is periodically spread across replicated harmonic bands, limiting the effective utilization of available spectral resources.

Overall, the proposed approach for performing multi-frequency scattering pattern synthesis in Ref. [78] provides greater versatility to STC-DMs and enables promising applications in the fields of wireless communications, radar, imaging, and other more advanced processing systems. Finally, it is worth highlighting that this encoding technique inherits the programmability feature in the context of digital metasurfaces, meaning that scattering patterns can be dynamically modified by adjusting the coding sequences loaded from an FPGA.

3.4.2 Scattering Reduction

In radar and stealth technologies, reducing the scattering contribution from a target, that is, reducing its Radar Cross Section (RCS), is a fundamental problem. Many methods have been devised for RCS reduction. For example, some conventional materials are capable of absorbing the EM energy incident on them, while metasurfaces based on the phase cancellation principle can scatter the incident wave in multiple directions in the spatial domain [14, 15, 79]. Moreover, a phase-switching screen has been proposed to redistribute the scattered power to the odd harmonic orders in the frequency domain [80]. In this context, a new method has been proposed, enabled by STC-DMs, to redistribute the energy of the reflected wave both in the spatial and spectral domains [28], deviating from conventional approaches and allowing for a greater reduction of RCS.

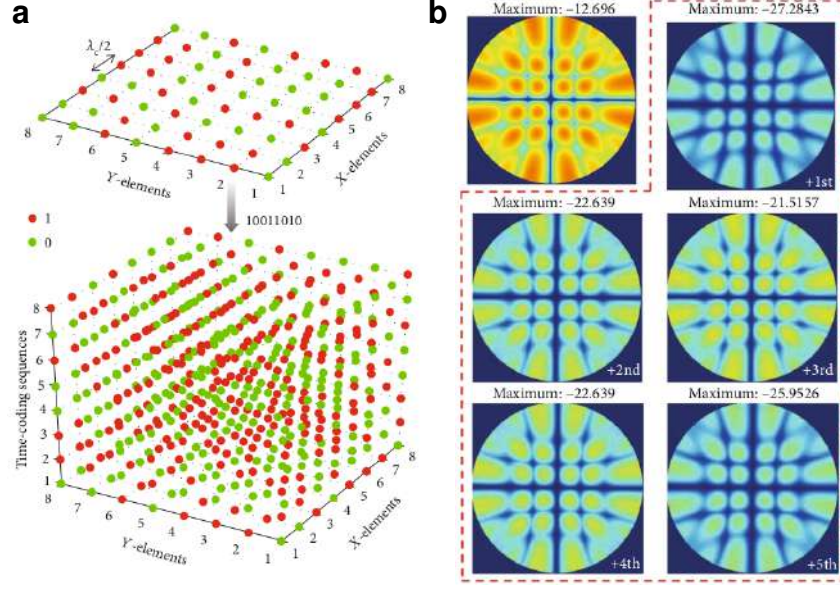


Figure 36: Control of scattering through a STC-DM [28]. **a)** Optimized 2-D spatial coding matrix and 3-D STC matrix to achieve RCS reduction. **b)** Corresponding scattering patterns at various harmonics.

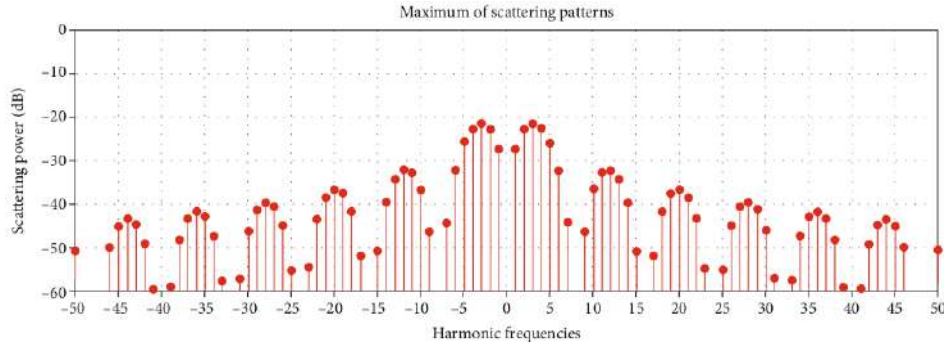


Figure 37: Maxima of the reflected field scattering patterns corresponding to the 3-D coding matrix in Figure 36a, for harmonic orders between -50th and +50th [28].

Considering the spatial coding matrix proposed in Ref. [28], shown at the top of Figure 36a, the corresponding reflected power is distributed more uniformly over the spatial domain, and the maximum reflected power is reduced by ≈ 12.7 dB compared to the result obtained using a metal screen, as shown in Figure 36b. Subsequently, an arbitrary temporal coding sequence was applied to optimize the spatial coding matrix, resulting in the 3-D STC matrix shown at the bottom of Figure 36a. In this case, the incident power can be redistributed more evenly, both in the spatial and spectral domains, and the maximum reflected power across the considered harmonic orders is further reduced by ≈ 8.82 dB, when compared to the result obtained using

the matrix characterized by spatial coding alone. The scattering patterns at the various harmonic orders are shown in Figure 36b.

Furthermore, Figure 37 displays the maxima of the scattering patterns from the metasurface at harmonic orders ranging from the -50 th to the $+50$ th, corresponding to the 3-D STC matrix presented in Figure 36a. It can be observed that the maximum intensity gradually diminishes as the harmonic frequency increases. Thus, by utilizing STC, the reflected power can be effectively reduced in both the spatial and spectral domains, leading to a more significant reduction in RCS.

3.4.3 Generation of Multi-bit Programmable Phases

Multi-bit programmable metasurfaces offer low quantization error and provide more precise control over EM waves, enhancing versatility and enabling more complex functionalities. 1-bit and 2-bit programmable metasurfaces have been developed for integration with active devices and FPGA hardware. However, creating multi-bit programmable metasurfaces using PIN diodes presents significant challenges. To overcome this, alternative methods have been explored, such as the use of varactor diodes, liquid crystals, and micro-electromechanical systems. Despite these efforts, these approaches still fall short of achieving continuous phase modulation within the range $[0, 2\pi]$.

In Ref. [81], a 2-bit programmable metasurface was designed to achieve nearly continuous phase modulation, as conceptually shown in Figure 38a. Figure 38b shows a schematic representation of the vector synthesis method proposed to analyze the phase coverage obtained. With the help of this temporal coding strategy, it was possible to synthesize the phase values corresponding to a 4-bit coding and those corresponding to a higher-order coding. The reflection coefficients, i. e., $\exp(j\varphi_{00})$, $\exp(j\varphi_{01})$, $\exp(j\varphi_{10})$, $\exp(j\varphi_{11})$, of the original 2-bit coding are represented in the complex plane by four base vectors (in red), as shown in Figure 38b. Assuming unit-amplitude reflection coefficients of the 2-bit programmable metasurface, the reflection coefficients obtained at the central frequency f_c and the first harmonic frequency $f_c + f_0$ can be obtained from Equation (44) and are given by:

$$\begin{aligned} a^{(0)} &= \frac{1}{L} \sum_{l=1}^L \Gamma^{(l)} = A_0 e^{j\psi^{(0)}}, \quad \text{where } \Gamma^{(l)} \in \{e^{j\varphi_{00}}, e^{j\varphi_{01}}, e^{j\varphi_{10}}, e^{j\varphi_{11}}\}, \\ a^{(1)} &= \frac{1}{L} \text{sinc}\left(\frac{\pi}{L}\right) \sum_{l=1}^L \Gamma^{(l)} e^{-j\frac{\pi(2l-1)}{L}} = A_1 e^{j\psi^{(1)}}. \end{aligned} \quad (56)$$

By appropriately designing the temporal coding sequence, new vectors (in green) with arbitrary phases $\psi^{(v)}$ can be synthesized through a linear combination of the base vectors [81]. It can be observed from Figure 38c,d that nearly continuous phase modulation can be obtained in the range $[0, 2\pi]$ at both the central frequency and the first harmonic frequency by properly designing the 2-bit temporal coding sequence. Furthermore, some beam steer-

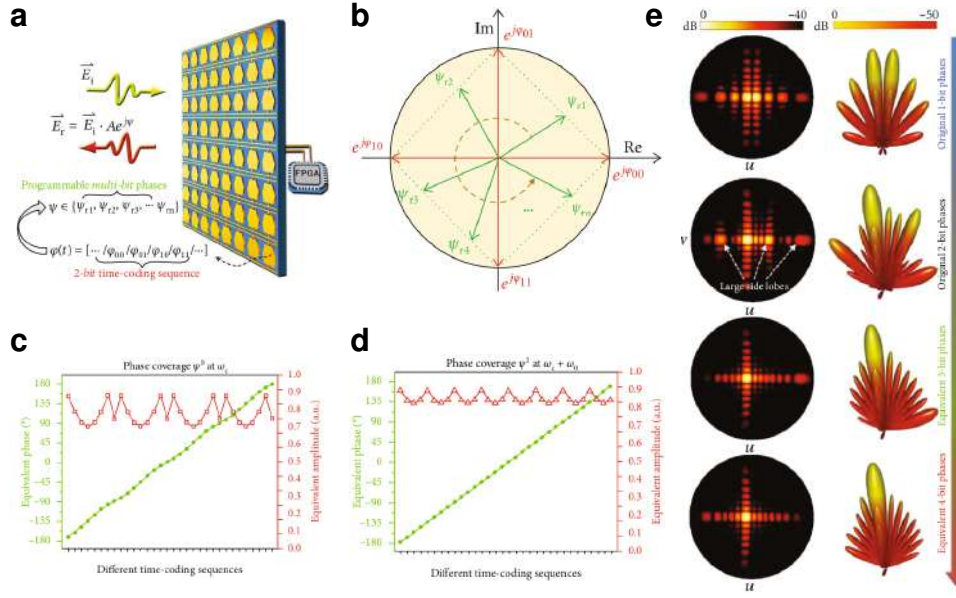


Figure 38: Generation of multi-bit programmable phases [81]. **a)** Conceptual representation of a multi-bit programmable metasurface controlled by an FPGA. **b)** Synthesis of the vectors in the complex plane to generate multi-bit programmable phases. **c,d)** Distribution of the equivalent phase at the central frequency and the first harmonic frequency, respectively. **e)** Comparison of the scattering patterns corresponding to the original codings (1-bit and 2-bit) with those corresponding to the equivalent codings (3-bit and 4-bit).

ing examples have been proposed to validate the effectiveness of the temporal coding method. Figure 38e shows the scattering patterns corresponding to the original 1-bit and 2-bit codings, as well as those corresponding to the equivalent 3-bit and 4-bit codings. In this figure, it can be observed that, in the scattering patterns associated with the original codings, the main lobes are prominent, whereas, in the case of the equivalent codings, the side lobes are attenuated. For experimental validation, a prototype of a programmable metasurface controlled by an FPGA was designed and tested, with the measured results showing strong agreement with the numerical predictions. Further details and additional results can be found in Ref. [81].

Therefore, the temporal coding strategy presented in Ref. [81] provides an alternative approach for designing multi-bit programmable metasurfaces, avoiding the need for complex control systems.

3.4.4 Nonreciprocal Effects

Based on the work of Yu *et al.* [4], regarding the generalized Snell's laws, metasurfaces have enabled the possibility of engineering the wavefront and other properties of EM waves. However, metasurfaces that only introduce a

spatial discontinuity of the phase gradient are constrained by Lorentz reciprocity. This implies that, for instance, the time-reversed version of a reflected wave propagates in the same direction as the original incident wave, maintaining the same frequency.

Overcoming the limits imposed by reciprocity has attracted attention mainly for the potential applications in communication systems. In wireless communication systems, a nonreciprocal antenna could radiate an EM field with high directivity without being limited in receiving the reflected component. One way to achieve nonreciprocal effects, especially at microwave frequencies, is to use magnetic materials such as ferrites. These materials, however, are bulky, expensive, and difficult to integrate and scale to optical wavelengths. For this reason, new approaches based on temporally modulated devices have been proposed, which are less costly and offer greater prospects for integration.

In 2015, Shaltout *et al.* [9] proposed to exploit the temporal discontinuity of the phase gradient, achieved on time-varying metasurfaces, in order to break Lorentz reciprocity. In this work, a plane wave with phase $\psi(\mathbf{r}, t) = \mathbf{k}_c \cdot \mathbf{r} - 2\pi f_c t$ was considered, impinging on a time-varying metasurface. For a plane wave, the frequency and the wave vector can be expressed as:

$$f_c = -\frac{1}{2\pi} \frac{\partial \psi}{\partial t}, \quad (57a)$$

$$\mathbf{k}_c = \nabla \psi. \quad (57b)$$

As a result of the interaction with the metasurface, the phase $\psi_r(\mathbf{r}, t)$ of the reflected wave experiences a shift relative to the phase $\psi(\mathbf{r}, t)$ of the incident wave:

$$\psi_r(\mathbf{r}, t) = \psi(\mathbf{r}, t) + \psi_{ms}(\mathbf{r}, t), \quad (58)$$

where $\psi_{ms}(\mathbf{r}, t)$ represents the phase shift introduced by the metasurface. By applying the relations in (57), it follows that the frequency and wavenumber of the wave reflected by the time-varying metasurface satisfy the following relations:

$$f_r = f_c - \frac{1}{2\pi} \frac{\partial \psi_{ms}}{\partial t}, \quad (59a)$$

$$\mathbf{k}_r = \mathbf{k}_c + \nabla \psi_{ms}. \quad (59b)$$

This highlights how, through spatial modulation of the metasurface alone, it is possible to manipulate the spectrum in the wave vector domain. By introducing temporal modulation to the metasurface, it becomes possible to also control the behavior of the incident wave in the frequency domain.

3.4.4.1 Programmable Nonreciprocal Effects

In recent years, several experimental works based on STC-DMs have been proposed to overcome Lorentz reciprocity by introducing an appropriate space-time phase gradient [29, 82].

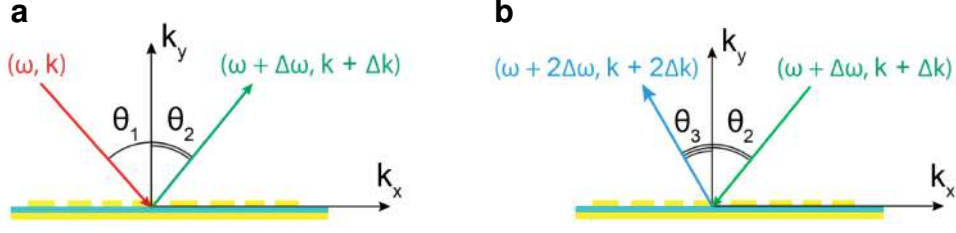


Figure 39: Nonreciprocal effect achieved through a time-varying metasurface [9]. **a)** Reflection angle achieved through a time-varying metasurface. **b)** Reflection angle in the time-reversed case.

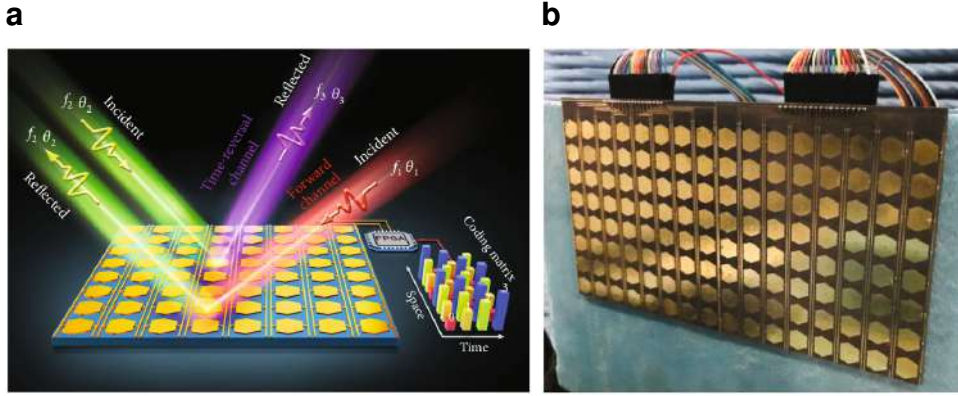


Figure 40: Nonreciprocal effect achieved through a STC-DM [29]. **a)** Conceptual representation of the nonreciprocal reflection phenomenon based on the STC-DM. **b)** Prototype of the 2-bit programmable metasurface.

In particular, a 2-bit STC-DM has recently been designed to achieve programmable nonreciprocal effects [29], enabling nonreciprocal reflection effects along with frequency conversion. In this particular application, a STC-DM composed of N columns of programmable elements was considered, where the phase of the reflection coefficient was modulated using integrated PIN diodes. The metasurface was illuminated by a plane wave impinging at an angle θ_i , with a time-harmonic dependence of the form $\exp(j2\pi f_c t)$. The reflection coefficient of the n -th element is temporally modulated and can be represented as a periodic function of time:

$$\Gamma_n(t) = \sum_{l=1}^L \Gamma_n^{(l)} W^{(l)}(t),$$

where $W^{(l)}(t)$ is the rectangular window defined in Equation (40). Thus, the phase response of each element can be characterized by a set of encoding

sequences of length L . With these premises, the field reflected by the meta-surface, in the far-field region, can be expressed as [29]:

$$\mathbf{f}(\theta, \phi, t) = \mathbf{E}(\theta, \phi) \sum_{n=1}^N \Gamma_n(t) e^{j \frac{2\pi}{\lambda_c} (n-1) d (\sin \theta + \sin \theta_i)}, \quad (60)$$

where $\mathbf{E}(\theta, \phi)$ is the element radiation pattern, assumed identical for all elements, and d is the inter-element spacing. Considering the Fourier series expansion of the periodic function $\Gamma_n(t)$, the reflected field from the meta-surface in question at the harmonic frequency $f_c + \nu f_0$ is given by:

$$\mathbf{F}^{(\nu)}(\theta, \phi) = \mathbf{E}(\theta, \phi) \sum_{n=1}^N a_n^{(\nu)} e^{j 2\pi (n-1) d \left(\frac{\sin \theta}{\lambda_r} + \frac{\sin \theta_i}{\lambda_c} \right)}, \quad (61)$$

where $\lambda_r = c/(f_c + \nu f_0)$ is the wavelength at the ν -th harmonic order, and $a_n^{(\nu)}$ are the Fourier coefficients of $\Gamma_n(t)$, already defined in Equation (44):

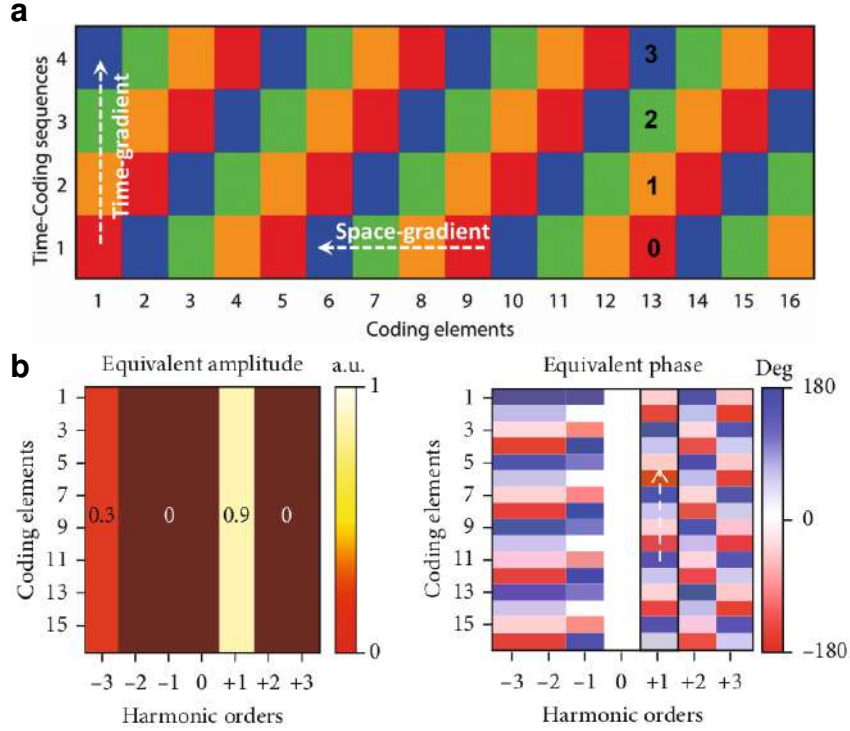


Figure 41: a) 2-D STC matrix. b) Equivalent amplitudes and phases [29].

Figure 40a schematically shows the nonreciprocal effect in a system based on a STC-DM. By properly designing the STC sequence, a plane wave oscillating at frequency f_1 and incident at angle θ_1 will be reflected at an angle θ_2 with frequency f_2 . In the time-reversed scenario, the incident wave at angle θ_2 and frequency f_2 will be reflected at an angle $\theta_3 \neq \theta_1$ and frequency $f_3 \neq f_1$. This effect represents a phenomenon that overcomes Lorentz reciprocity.

To achieve this nonreciprocal behavior, a STC-DM has been designed, consisting of 16 programmable columns and a temporal coding sequence of length $L = 4$, as represented by the STC matrix in Figure 41a. Each constituent element is subject to a temporal gradient phase of $\pi/2$, and the temporal coding sequences are progressively delayed by a time interval of $T_0/4$, such that each pair of adjacent elements exhibits a phase difference of $\pi/2$ in every temporal interval. The distributions of equivalent amplitudes and phases, determined using Equation (44), are shown in Figure 41a, from which it can be observed that the proposed STC leads to an anomalous reflection, where the EM energy is primarily concentrated in the first positive harmonic ($\nu = 1$).

Referring to the case of direct incidence shown in Figure 40a, for a plane wave incident at frequency f_c and wavenumber $k = 2\pi f_c/c$, the main reflected component is located at the first positive harmonic frequency $f_c + f_0$ and wavenumber $k + \Delta k = 2\pi(f_c + f_0)/c$. The relationship between the incident angle θ_1 and the reflection angle θ_2 is [9]:

$$(k + \Delta k) \sin \theta_2 = k \sin \theta_1 + \frac{\partial \psi}{\partial x}. \quad (62)$$

Referring to the time-reversed case shown in Figure 40a, and assuming that the incident wave has a frequency $f_c + f_0$ and wavenumber $k + \Delta k = 2\pi(f_c + f_0)/c$, the main reflected component is located at a frequency $f_c + 2f_0$ and a wavenumber $k + 2\Delta k = 2\pi(f_c + 2f_0)/c$. In this case, the relationship between the incident angle θ_2 and the reflection angle θ_3 is given by [9]:

$$(k + 2\Delta k) \sin \theta_3 = (k + \Delta k) \sin \theta_2 - \frac{\partial \psi}{\partial x}. \quad (63)$$

From Equations (62) and (63), it can be found that the reflection angles for the direct case, θ_2 , and the time-reversed case, θ_3 , satisfy the following relationships [29]:

$$\sin \theta_2 = \frac{k \sin \theta_1 + \frac{\partial \psi}{\partial x}}{k + \Delta k} = \frac{\sin \theta_1 - \frac{\lambda_c}{4d}}{1 + \frac{f_0}{f_c}}, \quad (64a)$$

$$\sin \theta_3 = \frac{k}{k + 2\Delta k} \sin \theta_1 = \frac{\sin \theta_1}{1 + \frac{2f_0}{f_c}}. \quad (64b)$$

The reflected wave in the time-reversed case does not propagate along the same direction as the incident wave in the direct case and also exhibits a frequency shift of $2f_0$, thus demonstrating the breaking of Lorentz reciprocity in both the spatial and frequency domains.

To quantify the difference in the tilt angle between the reflected wave in the time-reversed case and the incident wave in the direct case, a deviation factor has been defined [29]:

$$\Delta_{1,3} = |\sin \theta_3 - \sin \theta_1| = \frac{\sin \theta_1}{1 + \frac{f_c}{2f_0}}, \quad (65)$$

mal incidence and a harmonic time dependence of the form $\exp(j2\pi f_c t)$, the reflected wave $E^{(r)}$, in the frequency domain, can be represented as follows:

$$\begin{aligned} E^{(r)}(f) &= \Gamma(f) * E^{(i)}(f) \\ &= \delta(f - f_c) * \sum_{\nu=-\infty}^{+\infty} a^{(\nu)} \delta(f - \nu f_0) \\ &= \sum_{\nu=-\infty}^{+\infty} a^{(\nu)} \delta(f - f_c - \nu f_0), \end{aligned} \quad (66)$$

where the $*$ symbol denotes the convolution operation, $\delta(f)$ represents the Dirac delta function, and $a^{(\nu)}$ is the Fourier coefficient of the periodic function $\Gamma(t)$ corresponding to the ν -th harmonic. From Equation (66), it follows that the reflected wave is a linear superposition of EM waves at different harmonic frequencies $f_c + \nu f_0$. However, in certain light-matter interaction scenarios, additional nonlinear effects may occur, where components at harmonics different from the fundamental appear.

Figure 42b shows the geometry of a TDC digital metasurface, where each constituent element is composed of two rectangular patches connected via a varactor diode. Several experimental demonstrations based on a TDC digital metasurface, implemented by integrating varactor diodes, have been proposed to demonstrate the operating principle of a programmable nonlinear effect [25]. Figure 42c shows the spectral distributions corresponding to a phase modulation obtained through 1-bit and 2-bit temporal coding sequences. Using the 1-bit time-coding sequence, the incident power is transferred to the odd harmonics of the reflected wave with a symmetric spectral distribution. The 2-bit time-coding sequence, on the other hand, primarily redirects the incident power to the first harmonic with an asymmetric spectral distribution [25].

3.4.5.1 Nonlinear Programmable Effects

From Equation (44), it follows that one of the main limitations of STC-DM arises from the intrinsic coupling between the amplitudes and phases of the coefficients $a_{nm}^{(\nu)}$ at the various harmonics. Therefore, a recent study based on TDC digital metasurfaces proposed a method to independently control the amplitude and phase values at different frequencies using time-varying signals [83]. Figure 43a shows a conceptual representation of a TDC metasurface, in which the phase of the reflection coefficient is periodically modulated in time through a square wave oscillating between two phase values, ϕ_1 and ϕ_2 , corresponding to two bias voltages, V_1 and V_2 .

To decouple the amplitudes and phases at the various harmonics, a time delay t_0 was introduced in the reflection coefficient $\Gamma(t)$, leading to an additional phase shift of $-2\pi f_0 t_0$ in the frequency domain, while leaving the am-

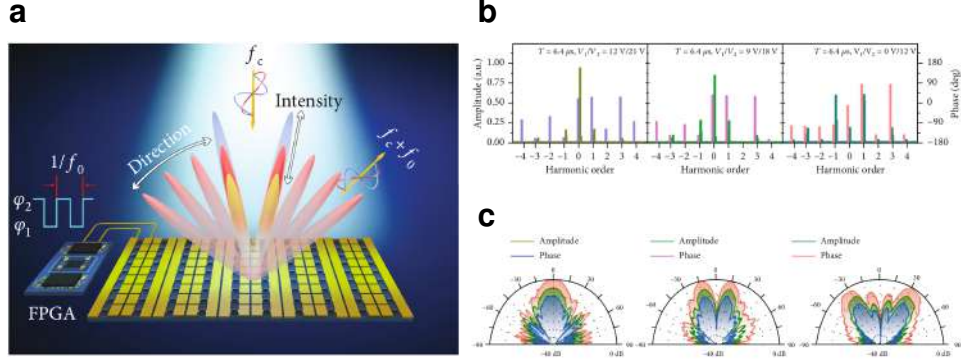


Figure 43: **a)** Conceptual representation of a TDC digital metasurface for independently controlling the amplitudes and phases at the various harmonics. **b)** Measured equivalent amplitudes and phases at different harmonics, corresponding to different combinations of the bias voltages V_1 and V_2 . **c)** Scattering patterns measured at the first harmonic for three different spatial codings [83].

plitudes corresponding to the ν -th harmonic unchanged [83]. The reflected spectral distribution can be expressed as:

$$E^{(r)}(f) = 2\pi A e^{j\Phi_2} \cos \Phi_1 \cdot E^{(i)}(f) + \sum_{\nu=-\infty}^{+\infty} \frac{4A}{2\nu-1} e^{j[\Phi_2 - (2\nu-1)2\pi f_0 t_0]} \sin \Phi_1 \cdot E^{(i)}(f - (2\nu-1)f_0), \quad (67)$$

where $\Phi_1 = (\phi_1 - \phi_2)/2$, $\Phi_2 = (\phi_1 + \phi_2)/2$, A is a constant, and $\nu \in \mathbb{Z}$.

From Equation (67), it can be observed that the spectral components of the reflected waves appear only at the central frequency ($\nu = 0$) and at odd harmonics, due to the properties of the Fourier transform of the square wave [83]. Furthermore, the amplitudes of the reflected waves at the central frequency and at the odd-order harmonics can be controlled by adjusting the phase difference $\phi_1 - \phi_2$, and the time delay t_0 provides an additional degree of freedom to control the phases at the odd harmonics while keeping the amplitudes unchanged. This enables independent manipulation of the amplitudes and phases at the harmonic frequencies.

To experimentally verify the described nonlinear phenomenon, a TDC digital metasurface prototype controlled via an FPGA was designed and characterized in Ref. [83]. Figure 43b shows the measured amplitudes and phases at various harmonics for three different combinations of the voltages V_1 and V_2 applied to the metasurface. Subsequently, time delays of 0 and $T_0/2$ were introduced to different columns of the metasurface, implementing a spatial coding consisting of “0” and “1” elements in order to perform beam shaping. Figure 43c shows three sets of radiation patterns measured at the first positive harmonic, obtained using three different spatial coding sequences. From the results presented in Ref. [83], it is observed that different combina-

tions of the bias voltages V_1 and V_2 allow the control of the radiation power intensity, while preserving the beam shapes.

3.4.6 Polarization Conversion

In the applications presented so far, digital metasurfaces can only function in a single polarization context. In Ref. [84], an anisotropic digital metasurface was proposed to achieve programmable polarization conversions at the central frequency and harmonic frequencies. Based on the theory of nonlinear effects discussed in Section 3.4.5, a general theory was proposed to control the polarization of the reflected wave.

In Ref. [84], the polarization conversion problem was expressed in terms of a matrix $\Gamma(t)$, introduced to characterize the effects of anisotropic reflection:

$$\Gamma(t) = \begin{pmatrix} \Gamma_{xx}(t) & \Gamma_{xy}(t) \\ \Gamma_{yx}(t) & \Gamma_{yy}(t) \end{pmatrix}. \quad (68)$$

Since Ke *et al.* [84] consider an anisotropic digital metasurface, we have $\Gamma_{xx} \neq \Gamma_{yy}$ and the off-diagonal terms are nearly zero. Therefore, the anisotropic reflection effect can be expressed by the following equation:

$$\begin{aligned} \mathbf{E}^{(r)}(t) &= E_x^{(r)}(t) \hat{\mathbf{x}} + E_y^{(r)}(t) \hat{\mathbf{y}} \\ &= E_x^{(i)}(t) \Gamma_{xx}(t) \hat{\mathbf{x}} + E_y^{(i)}(t) \Gamma_{yy}(t) \hat{\mathbf{y}}, \end{aligned} \quad (69)$$

where $E^{(r)}(t)$ and $E^{(i)}(t)$ represent the components of the reflected and incident waves, respectively. Considering a monochromatic incident wave, expressed as

$$\mathbf{E}^{(i)}(t) = (A_{0x} e^{j\varphi_{0x}} \hat{\mathbf{x}} + A_{0y} e^{j\varphi_{0y}} \hat{\mathbf{y}}) e^{j2\pi f_c t},$$

Equation (69) can be rewritten as:

$$\mathbf{E}^{(r)}(t) = \left[|A_{0x} \Gamma_{xx}(t)| e^{j(\varphi_{0x} + \varphi_{xx}(t))} \hat{\mathbf{x}} + |A_{0y} \Gamma_{yy}(t)| e^{j(\varphi_{0y} + \varphi_{yy}(t))} \hat{\mathbf{y}} \right] e^{j2\pi f_c t}, \quad (70)$$

From Equation (70), it follows that one can act on the polarization of the incident wave, at the central frequency, by appropriately programming $\Gamma_{xx}(t)$ and $\Gamma_{yy}(t)$.

3.4.6.1 Programmable Polarization Conversion

As shown in Figure 44a, the metasurface is illuminated with a linearly polarized wave. When the phase difference $\Delta\varphi = \varphi_{xx}(t) - \varphi_{yy}(t) = \pi/2$ and $|A_{0x} \Gamma_{xx}(t)| = |A_{0y} \Gamma_{yy}(t)|$, the metasurface reflects a wave with right-handed circular polarization. Additionally, a left-handed circularly polarized reflected wave can also be obtained by setting $\Delta\varphi = -\pi/2$. Otherwise, an elliptically polarized reflected wave can be achieved. However, due to limitations in modulating the reflection coefficient amplitudes, it is likely that

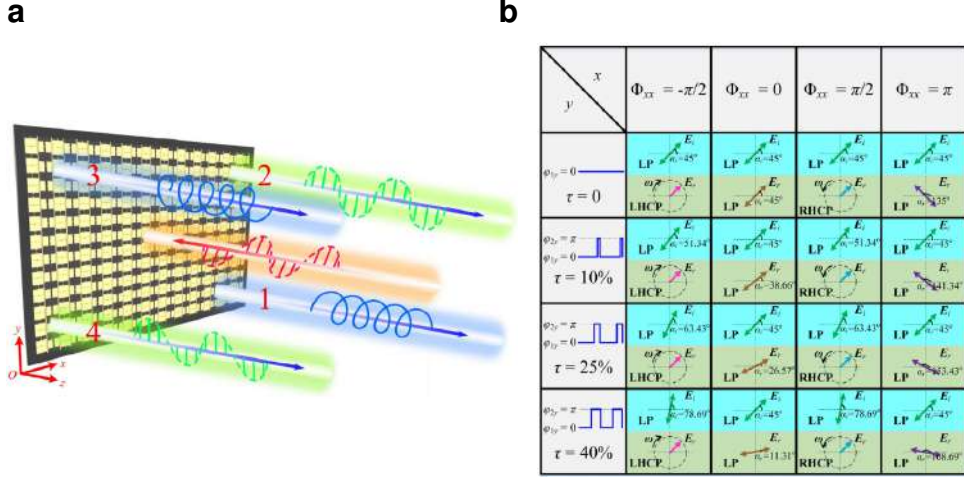


Figure 44: a) Conceptual representation of polarization conversion through a TDC digital metasurface. b) Polarization conversion diagrams associated with different combinations of phase responses along x and y [84].

there is no substantial difference between $|\Gamma_{xx}(t)|$ and $|\Gamma_{yy}(t)|$. Therefore, the condition $|A_{0x}\Gamma_{xx}(t)| = |A_{0y}\Gamma_{yy}(t)|$ is not necessarily satisfied when considering an incident wave with arbitrary polarization.

To overcome this issue, Ref. [84] designed the reflection coefficient as a square wave with a tunable duty cycle D_c , which is adjustable along the polarization directions. By increasing D_c , $|E_y^{(r)}(f_c)|$ is attenuated while $\varphi_y^{(r)}(f_c)$ remains unchanged. Therefore, by adjusting the phase response φ_{xx} so that $\varphi_x^{(r)} = \varphi_y^{(r)}(f_c)$, a linearly polarized wave can be obtained whose electric field is oriented along an arbitrary direction, defined by the following polarization angle:

$$\alpha_{\text{pol}} = \arctan \left(\left| \frac{E_y^{(r)}(f_c)}{A_{0x}} \right| \right).$$

Figure 44b shows the polarization conversion diagrams associated with different combinations of phase responses along x and y , corresponding to different values of D_c .

Subsequently, the reflection coefficient was designed as a square wave with $D_c = 50\%$ to achieve polarization conversion effects at the harmonic frequencies of the reflected field. Considering a reflection coefficient polarized along y and introducing a time delay of $T_0/4$, according to Equation (67), the reflected wave is LHCP at the first negative harmonic, while a RHCP wave is observed at the first positive harmonic. Moreover, it was observed that by varying the introduced time delay, different nonlinear polarization conversion effects can be achieved. By also introducing a phase shift $\varphi_0 = \pi/2$ in the phase response polarized along y , LHCP reflected waves were obtained at the first harmonic frequency. Similarly, if a phase shift $\varphi_0 = \pi/2$ is added to the phase response polarized along x , RHCP reflected waves are observed.

In general, by introducing a time delay t_0 in the phase response polarized along y and a phase shift φ_0 in the phase response polarized along x , the following occurs [84]:

$$\Delta\varphi^{(r)}(f \pm f_0) = \varphi_x^{(r)}(f \pm f_0) - \varphi_y^{(r)}(f \pm f_0) = \varphi_0 \mp f_0 t_0, \quad (71)$$

Therefore, using Equation (71), it is possible to easily derive the values of t_0 and φ_0 through which the nonlinear polarization effect can be engineered.

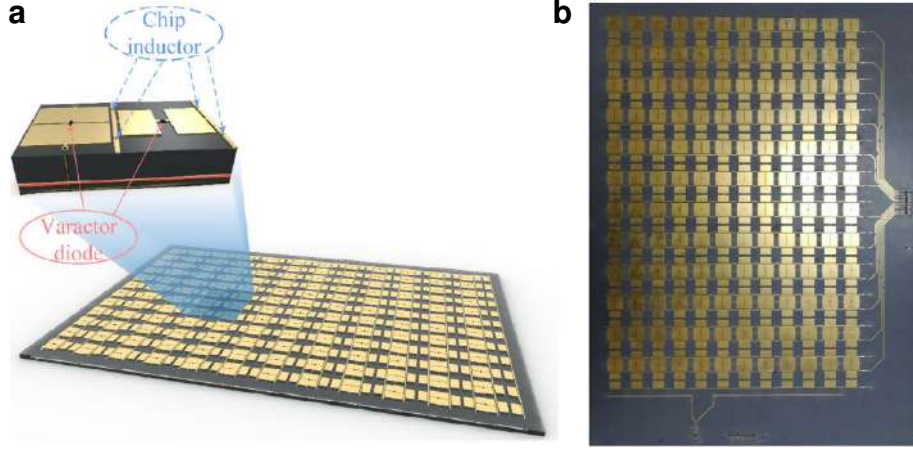


Figure 45: a) Conceptual illustration of the TDC anisotropic digital metasurface. b) Prototype of the digital metasurface designed in Ref. [84].

In Ref. [84], to experimentally verify the results that were obtained through theoretical analysis, a TDC anisotropic digital metasurface was designed and thoroughly characterized, as shown in Figure 45. Each constitutive element of the metasurface was obtained by integrating two varactor diodes between two rectangular metallic patches. As the applied bias voltage changes, the capacitance of the varactor diodes changes, allowing the independent design of phase responses along different polarization directions. Moreover, the pairs of rectangular patches enable the polarization isolation condition (off-diagonal elements of the matrix defined in Equation (68) being nearly zero) and eliminate the crosstalk component between the phase responses associated with different polarization directions. In this way, independent control of the phase responses along the two polarization directions can be achieved.

3.4.7 Polarization Multiplexing

Typical metasurface-based configurations exhibit static polarization behavior when the geometric parameters are fixed, making dynamic polarization control an area of interest. In particular, polarization manipulation can be achieved by exploiting nonlinear effects, while frequency conversion is implemented using time-varying platforms.

Programmable metasurfaces integrated with active elements can be employed to simultaneously control both polarization and frequency. For instance, it has been demonstrated that polarization can be manipulated using anisotropic metasurfaces integrated with varactor diodes, which are appropriately controlled through bias voltages [85]. Moreover, it has been shown that the polarization plane of an incident wave can be rotated using a metasurface with dynamically tunable properties [86]. However, these approaches offer limited control, as they operate in a linear and time-invariant regime. This constraint prevents the simultaneous realization of frequency and polarization conversion.

Figure 46 conceptually illustrates the architecture of a polarization multiplexing scheme implemented using an anisotropic STC-DM [87]. Each element of the metasurface is composed of two partitions that independently control the x - and y -polarized components of the EM waves.

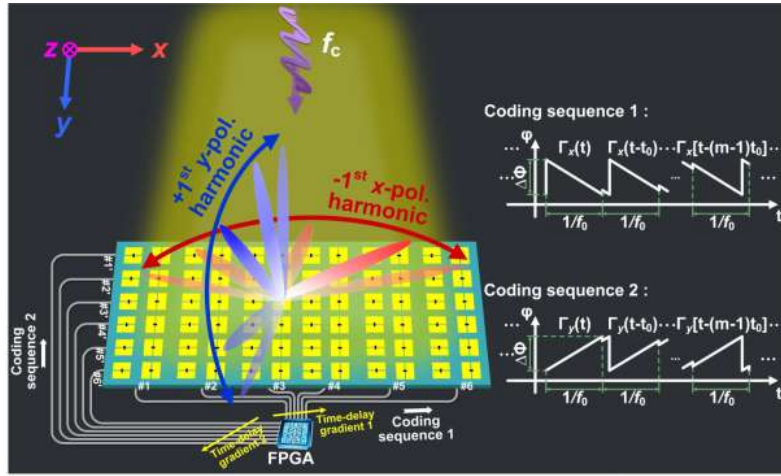


Figure 46: Architecture of the polarization multiplexing scheme based on an anisotropic STC-DM [87]. Coding sequences 1 and 2 are time-varying waveforms that drive the varactor diodes.

A linearly polarized plane wave, whose polarization direction is not aligned with either the x or y axis, is used as the incident wave to illuminate the metasurface perpendicularly. Due to its anisotropy, the metasurface decomposes the incident electric field into two orthogonal components, thereby producing x - and y -polarized fields. As a result, the metasurface simultaneously generates two independent reflected waves. Consequently, by dynamically adjusting the reflection coefficients corresponding to the two polarization directions in real time, it is possible to independently control the amplitudes and phases of the x - and y -polarized reflected waves.

The dual-polarization reflection property can be described using a Jones matrix $\mathcal{H}(t)$, that adheres to the structure of the matrix in Equation (68) and can be expressed as

$$\mathcal{H}(t) = \begin{pmatrix} |\Gamma_{xx}(t)|e^{j\Phi_{xx}(t)} & 0 \\ 0 & |\Gamma_{yy}(t)|e^{j\Phi_{yy}(t)} \end{pmatrix}, \quad (72)$$

where the off-diagonal elements are considered null to model the cross-polarization isolation property of the metasurface, and the elements along the main diagonal are expressed using phasor notation.

3.4.7.1 Frequency-Polarization Multiplexing

From Equation (72) and the general theory presented in Section 3.4.6, it is evident that the simultaneous and independent control of time-varying polarization coefficients is achieved by dynamically adjusting the reflection coefficients corresponding to the two polarization directions, $\Gamma_{xx}(t)$ and $\Gamma_{yy}(t)$, thereby enabling the realization of a polarization conversion scheme.

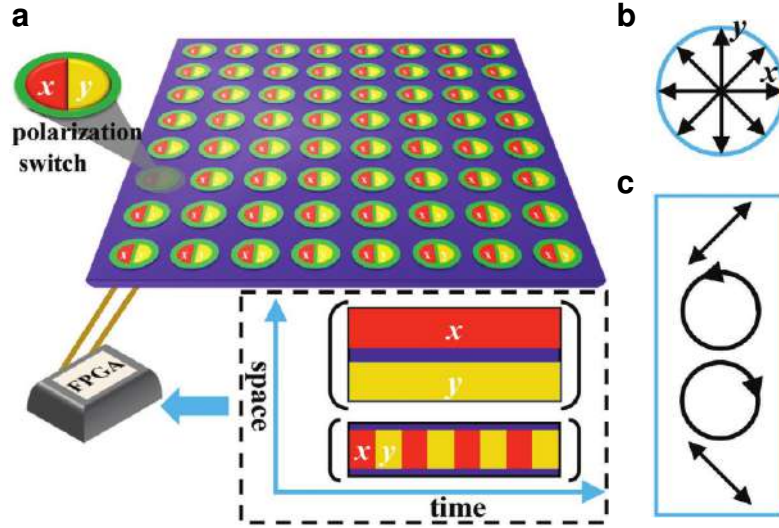


Figure 47: Conceptual illustration showing the time-division multiplexing in a metasurface [88]. **a)** Metasurface integrating polarization switches controlled via an FPGA. **b)** Linear polarization rotation at the central frequency. **c)** Polarization states obtained at the harmonic frequencies.

As already discussed, an anisotropic metasurface allows the control of polarization at harmonic frequencies by adjusting the phases of two orthogonally polarized waves. However, this paradigm is limited by several factors. For instance, varactor diodes are used to continuously correct the amplitude and phase responses along the orthogonal polarization directions, but they require high bias voltages, which limit the switching speeds. Furthermore, to achieve optimal results with this approach, the incident polarization should

be tilted by 45° with respect to the array axes, which limits the potential application fields.

To overcome these restrictions, a time-division multiplexing metasurface composed of $N \times M$ elements integrated with PIN diodes has been proposed [88], as shown in Figure 47a. Unlike traditional schemes based on phase or amplitude modulation, this approach employs the unit cell as a polarization switch that enables the transmission of waves polarized along the x or y direction, depending on the state (“ON” or “OFF”) of the diodes. Those diodes can be reconfigured by controlling the bias voltages supplied through the FPGA hardware. This mechanism can operate with arbitrarily polarized incident waves and makes use of PIN diodes, which require low bias voltages and exhibit a wide operational bandwidth. In particular, it has been shown in Ref. [88] that, by consistently designing the time-switching process, it is possible to arbitrarily rotate the linear polarization at the central frequency, as illustrated in Figure 47b, and to obtain different polarization states at the harmonic frequencies.

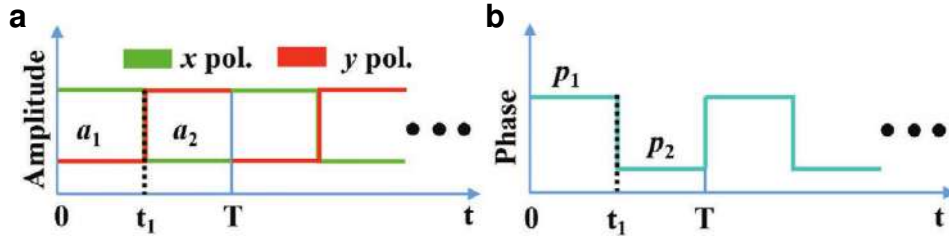


Figure 48: Periodic modulation of the transmission function $\mathcal{T}(t)$ [88]. **a)** Amplitude behavior of the transmission function. **b)** Phase behavior of the transmission function.

As schematized in Figure 48, for each unit cell, the transmission function is assumed to be periodic with period T , and the modulation frequency is defined as $f_0 = 1/T$. Within each period, the x -polarization is transmitted with phase p_1 in the interval $0 \leq t < t_1$, while the y -polarization is transmitted with phase p_2 in the interval $t_1 \leq t < T$. Therefore, the transmission function can be defined within one period as follows:

$$\mathcal{T}(t) = \begin{cases} a_1 e^{jp_1}, & 0 \leq t < t_1, \\ a_2 e^{jp_2}, & t_1 \leq t < T, \end{cases} \quad (73)$$

where a_1 and a_2 denote the amplitude factors associated with the respective polarizations. As a consequence of the temporal modulation, the transmitted wave contains spectral components not only at the central frequency f_c , but also at the harmonic frequencies $f_c \pm \nu f_0$. Considering modulation frequencies such that $f_0 \ll f_c$, the amplitudes of the x - and y -polarized components

can be explicitly expressed by expanding the transmission function defined in Equation (73) into a Fourier series of coefficients:

$$c^{(\nu)} = \begin{cases} a_1 e^{jp_1} R_x + a_2 e^{jp_2} (1 - R_x), & \nu = 0, \\ (a_1 e^{jp_1} - a_2 e^{jp_2}) R_x e^{-j\pi\nu f_0 t_1}, & \nu \neq 0. \end{cases} \quad (74)$$

Assuming $a_1 = a_x$ and $a_2 = 0$ for the x -polarization, and $a_1 = 0$ and $a_2 = a_y$ for the y -polarization, the complex amplitudes of the two polarizations at the harmonic frequencies can be obtained by particularizing Equation (74) as follows:

$$A_x^{(\nu)} = \begin{cases} a_x e^{jp_1} R_x, & \nu = 0, \\ a_x e^{jp_1} R_x e^{-j\pi\nu f_0 t_1}, & \nu \neq 0, \end{cases} \quad (75a)$$

$$A_y^{(\nu)} = \begin{cases} a_y e^{jp_2} (1 - R_x), & \nu = 0, \\ -a_y e^{jp_2} R_x e^{-j\pi\nu f_0 t_1}, & \nu \neq 0, \end{cases} \quad (75b)$$

where $R_x = t_1/T$ is the duty cycle of the transmission function. Thus, at each harmonic frequency, the polarization angle $\theta^{(\nu)}$ can be expressed as:

$$\tan \theta^{(\nu)} = \frac{A_y^{(\nu)}}{A_x^{(\nu)}} = \begin{cases} \frac{a_y}{a_x} e^{j(p_2-p_1)} \left(\frac{1-R_x}{R_x} \right), & \nu = 0, \\ -\frac{a_y}{a_x} e^{j(p_2-p_1)}, & \nu \neq 0. \end{cases} \quad (76)$$

From Equation (76), it can be observed that the polarization state obtained at the central frequency ($\nu = 0$) and at the harmonic frequencies ($\nu \neq 0$) can be modified by adjusting the phase difference $\Delta p = p_2 - p_1$, the amplitude factors a_x , a_y , and the duty cycle R_x .

In other words, by varying the phase difference Δp , an additional degree of freedom is introduced in the design process of the polarization states at the harmonic frequencies. However, the PIN diode-based architecture imposes limitations on the control that can be exercised over these parameters. To address this issue, a more general modulation of the transmission function can be considered:

$$\mathcal{T}(t) = \begin{cases} a_1 e^{j\varphi(t)}, & 0 \leq t < T/2, \\ a_2 e^{j\varphi(t)}, & T/2 \leq t < T, \end{cases} \quad (77)$$

where $\varphi(t)$ is a time-varying phase. Based on this more general formulation, the Fourier coefficients of the transmission function can be expressed as:

$$c^{(\nu)} = \frac{1}{2} C^{(\frac{\nu}{2})} (a_1 + a_2 e^{-j\pi\nu}) \quad (78)$$

where $C^{(\nu)}$ are the Fourier coefficients of the signal $\exp(j\varphi(t))$.

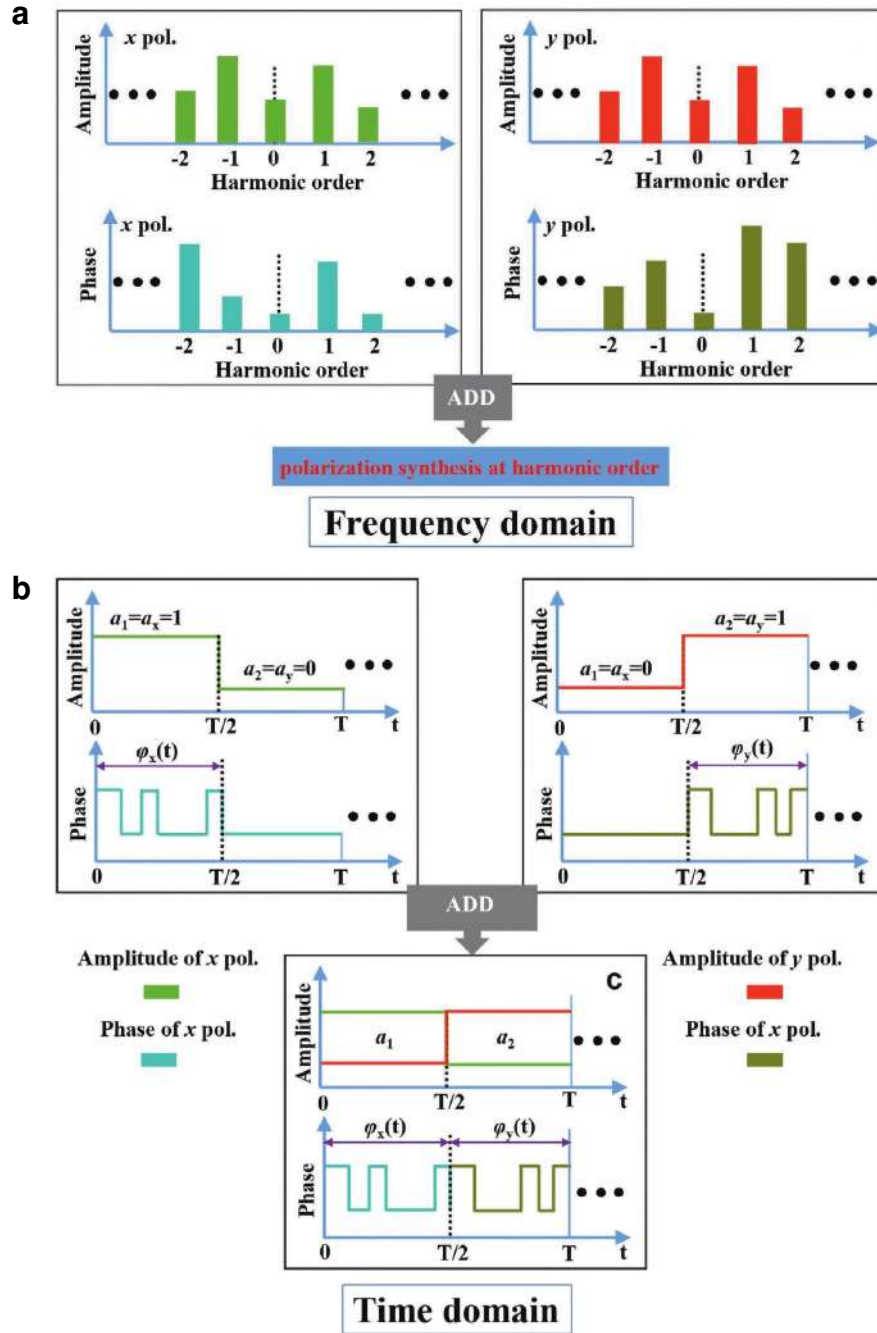


Figure 49: Schematic representation of the synthesis process [88]. **a)** Periodic modulation of the transmission function for both polarizations and temporal overlap of the modulated polarizations. **b)** Corresponding amplitude and phase distributions of the polarizations at the harmonic frequencies.

Considering Equation (78) and specializing it for even harmonic orders ($v = 2k$), with the assumption that for the x -polarization, $a_1 = a_x$, $a_2 = 0$, and $\varphi(t) = \varphi_x(t)$, and for the y -polarization, $a_1 = 0$, $a_2 = a_y$, and $\varphi(t) = \varphi_y(t)$, the corresponding harmonic amplitude values can be derived as:

$$A_x^{(2k)} = \frac{1}{2} C_x^{(v)} a_x, \quad (79a)$$

$$A_y^{(2k)} = \frac{1}{2} C_y^{(v)} a_y, \quad (79b)$$

Since $C_x^{(v)}$ and $C_y^{(v)}$ can be independently controlled via the phases $\varphi_x(t)$ and $\varphi_y(t)$, the complex amplitudes $A_x^{(2k)}$ and $A_y^{(2k)}$ can also be synthesized independently at the even harmonic orders. Based on this, a synthesis technique was proposed [88], schematically shown in Figure 49, which proceeds as follows:

1. The time-varying phases $\varphi_x(t)$ and $\varphi_y(t)$ are independently designed to achieve specific amplitude and phase distributions for the two polarizations at the even harmonic frequencies;
2. These signals are time-interlaced using polarization switches, ensuring that the spectral distributions for the two polarizations are effectively overlapped.

This enables independent and concurrent manipulation of polarization states at even harmonic orders.

For further considerations on the temporal delays and their role as an additional degree of freedom, the reader is referred to Ref. [88].

3.4.7.2 Space-Frequency-Polarization Multiplexing

Equation (72) can be used to describe the reflection function of the unit cell in a frequency-polarization multiplexing scheme. A space-frequency-polarization multiplexing scheme can be implemented along both polarization directions by exploiting the harmonic beamforming properties of STCDMs.

To achieve harmonic beamforming, it is essential to design the phase profile for the harmonics. As shown in Figure 50c, an arbitrary harmonic phase profile can be obtained by constructing the corresponding time-delay functions for the reflection coefficients of the unit cells. In a general case, a STCDM can be divided into N rows and M columns to independently control the properties of waves polarized along the x and y directions. By introducing time-delay functions for the x and y directions, denoted as $\tau_1(x)$ and $\tau_2(y)$, respectively, between the control signals of different rows and columns (as shown in Figure 46), the reflection function can be expressed as:

$$\mathcal{H}(x, y, t) = \begin{pmatrix} \Gamma_x(t - \tau_1(x)) & 0 \\ 0 & \Gamma_y(t - \tau_2(y)) \end{pmatrix}, \quad 0 \leq t \leq T, \quad (80)$$

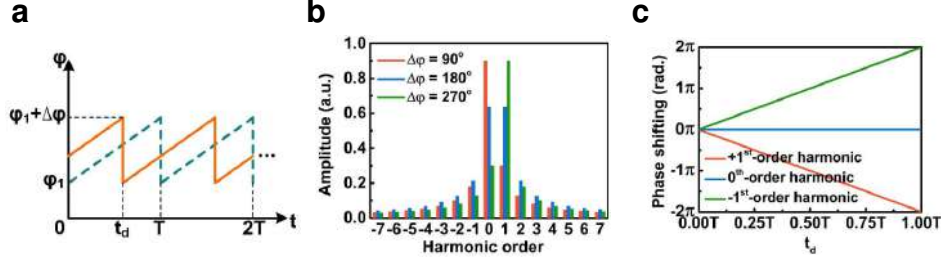


Figure 50: **a)** Time-varying phases of the original and time-delayed reflection coefficients. **b)** Amplitude distribution at harmonic frequencies. **c)** Phase delay trend at the central frequency and the first harmonic frequency as the time delay t_d varies [88].

where $\Gamma_x(t - \tau_1(x))$ and $\Gamma_y(t - \tau_2(y))$ satisfy the following functional expression [87]:

$$\Gamma(t - t_d) = \begin{cases} Ae^{j[\varphi_0 + \frac{\Delta\varphi}{T}(t + T - t_d)]}, & 0 \leq t \leq t_d, \\ Ae^{j[\varphi_0 + \frac{\Delta\varphi}{T}(t - t_d)]}, & t_d < t \leq T, \end{cases}$$

where t_d represents a general time delay, A is the amplitude of the reflection coefficient, and φ_0 and $\Delta\varphi$ are the initial phase and phase difference, respectively. The harmonic phase profiles, produced by the time-delay functions $\tau_1(x)$ and $\tau_2(y)$, steer the x -polarized wave at the first negative harmonic frequency and the y -polarized wave at the first positive harmonic frequency along the directions defined by the angles

$$\theta_x = \arcsin\left(\frac{cf_0}{f_c - f_0} \frac{d\tau_1}{dx}\right), \quad (81a)$$

$$\theta_y = \arcsin\left(-\frac{cf_0}{f_c + f_0} \frac{d\tau_2}{dy}\right), \quad (81b)$$

where c is the speed of light in vacuum and $f_0 = 1/T$ is the switching frequency.

3.4.8 Integrated Sensing and Communication

Recent developments in wireless technology underscore the transformative potential of programmable metasurfaces in shaping intelligent wireless environments. Among these, STC-DMs have emerged as a compelling solution for seamlessly integrating sensing and communication within a unified hardware platform. By employing sophisticated STC strategies, STC-DMs dynamically control the propagation of the fundamental frequency wave to ensure robust wireless communication, while simultaneously synthesizing harmonics for spatial sensing.

In Ref. [89], Chen *et al.* propose an innovative method for Integrated Sensing and Communication (ISAC) that leverages the unique capabilities of STC-

DMs. The core idea relies on the metasurface's ability to manipulate incoming EM waves such that the fundamental frequency component is directed toward the intended communication target, while spatially distributed harmonics are generated for sensing. Figure 51 presents a conceptual overview of the system.

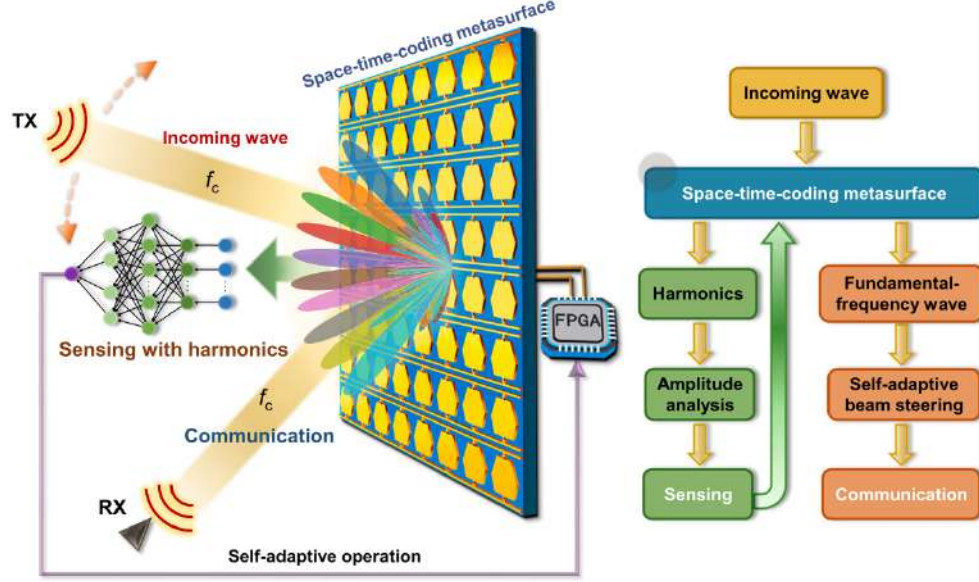


Figure 51: Conceptual illustration of the proposed STC-DM-based ISAC scheme [89].

In this setup, the fundamental frequency wave is utilized for communication, while the spatial-spectral properties of the harmonics are analyzed to precisely estimate the Direction of Arrival (DOA) of the incoming signal. Using the DOA information, the STC-DM's coding pattern is adjusted to ensure that the direction of the beam of the fundamental frequency wave is consistently steered towards the user's location.

In particular, considering a 2-bit coding STC-DM composed of N columns, under a plane-wave illumination $\exp(j2\pi f_c t)$ at an incidence angle θ_i , the far-field scattering pattern at the central frequency is described by

$$f_S(\theta) = E(\theta) \sum_{n=1}^N \Gamma_n e^{jk_c d(n-1)(\sin \theta + \sin \theta_i)}, \quad (82)$$

where $E(\theta)$ denotes the element-level scattering pattern, $k_c = 2\pi f_c / c$ is the wavenumber corresponding to the central frequency f_c , c is the speed of light in free space, and d is the spacing between adjacent meta-atoms. By expanding the time-varying reflection coefficient $\Gamma_n(t)$ into its Fourier series, with $a_n^{(\nu)}$ representing the general coefficient, the far-field scattering pattern at ν -th harmonic order becomes:

$$f_T^{(\nu)}(\theta) = E(\theta) \sum_{n=1}^N a_n^{(\nu)} e^{jd(n-1)(k^{(\nu)} \sin \theta + k_c \sin \theta_i)}, \quad (83)$$

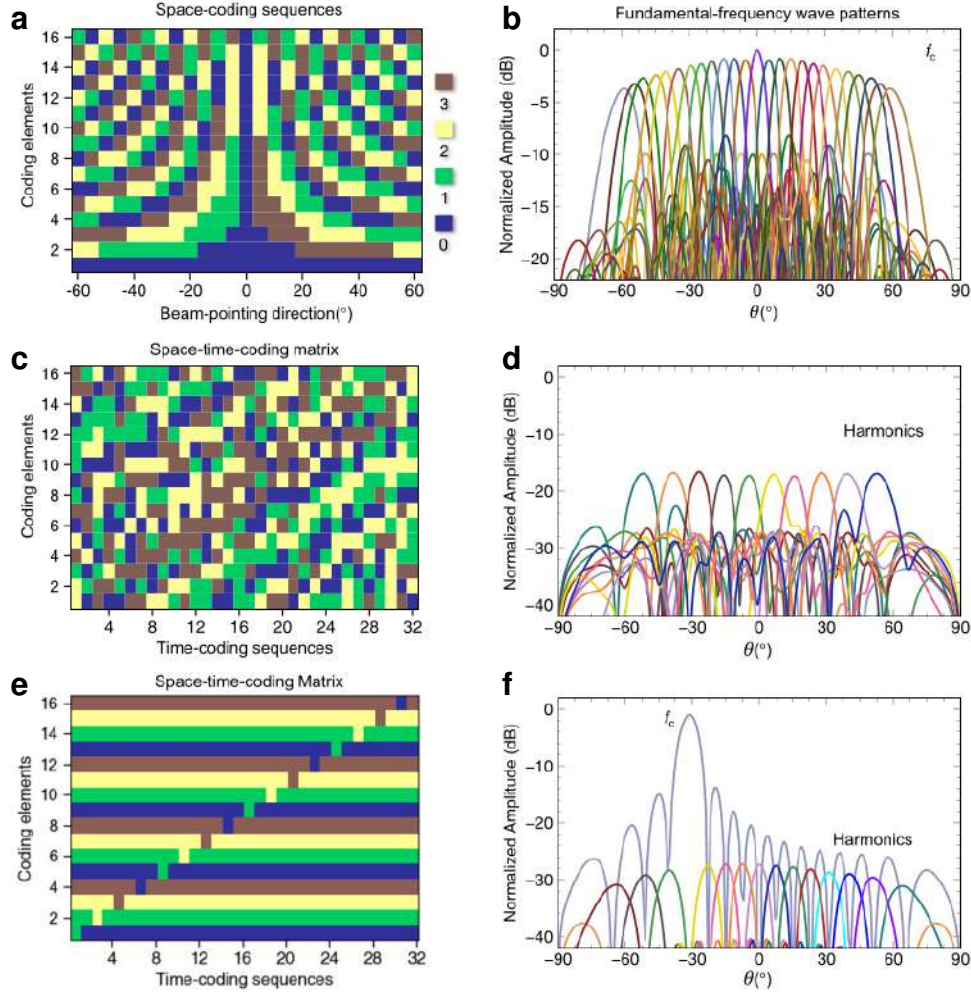


Figure 52: STC matrices and corresponding far-field scattering pattern of the proposed STC-DM [89]. The blue, green, yellow, and brown pixels represent the “0,” “1,” “2,” and “3” digits of the 2-bit coding, respectively. **a,b)** Space-coding sequences for $N = 16$ and the corresponding theoretical far-field scattering pattern for the fundamental frequency wave at $\theta_i = 0^\circ$, respectively. **c,d)** Optimized STC matrix for controlling the harmonics when $N = 16$ and $L = 32$, and corresponding far-field scattering patterns at $\theta_i = 0^\circ$, respectively. **e,f)** Optimized STC matrix for simultaneously controlling the fundamental frequency wave and for generating multiple harmonics, and corresponding far-field scattering patterns, respectively.

where $k^{(\nu)} = 2\pi(f_c + \nu f_0)/c$ is the wavenumber corresponding to the harmonic frequency $f_c + \nu f_0$.

These spatial-spectral responses are carefully designed through optimized coding matrices, as illustrated in Figure 52, which shows a set of space-coding sequences and the resulting far-field scattering patterns for communication and sensing purposes.

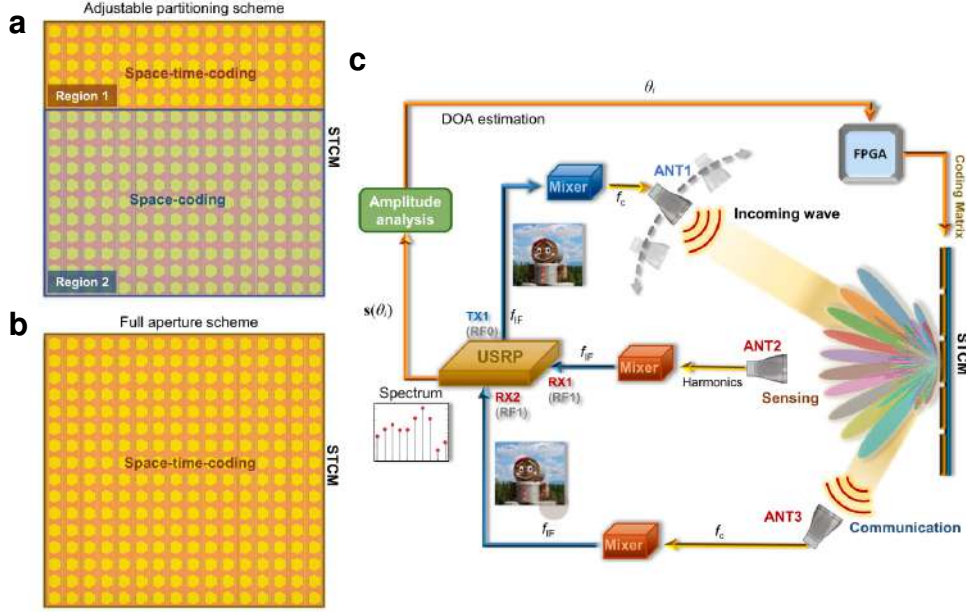


Figure 53: STC-DM-based ISAC system architecture [89]. **a)** Adjustable partitioning scheme of the STC-DM, which is divided into two regions for sensing and communication. The size of the two regions can be adjusted based on the required needs. **b)** Full-aperture scheme of the STC-DM without partitioning, which can utilize the entire STC-DM aperture by using STC matrices. **c)** Architecture of the ISAC system, in which a universal software radio peripheral is used for signal modulation and demodulation, and the STC-DM establishes a reliable communication link by sensing the direction of the incoming wave in real-time and updating the coding matrices.

To enhance ISAC performance, two complementary coding strategies are introduced: *adjustable partitioning* and *full-aperture modulation*, allowing STC-DMs to manipulate both the communication and sensing functions without interference, effectively eliminating the need for additional sensors or separate control links. This not only streamlines system design but also leads to significant reductions in hardware costs and power consumption compared to conventional systems like phased array antennas.

In the partitioned scheme (Figure 53a), the metasurface is divided into two independently coded regions: Region 1 applies STC for sensing via harmonic generation, while Region 2 uses space-only coding for beamforming the communication signal. The size of each region can be adjusted to balance

power between the two functions, minimizing interference and optimizing performance.

Conversely, the full-aperture scheme (Figure 53b) applies STC uniformly across the entire aperture, allowing simultaneous control of the fundamental and harmonic wavefronts without physical partitioning. This method maximizes aperture usage while maintaining functional decoupling through STC matrix design.

Finally, Figure 53c provides details on the hardware implementation. A universal software-defined radio (NI USRP-2943R) performs signal generation and reception. The modulated signal, transmitted at frequency f_c via antenna ANT1, impinges on the STC-DM. The spatio-temporally modulated harmonics are captured by ANT2 and analyzed to estimate the DOA. Based on this estimate, the STC matrix is updated in real time to redirect the beam toward ANT3, ensuring adaptive beam steering without mechanical movement or external sensors.

This architecture enables reliable, direction-aware wireless communication while simultaneously performing environmental sensing. The elimination of separate sensor arrays or mechanical beam-steering components leads to lower hardware complexity, reduced power consumption, and greater adaptability—offering a compelling alternative to conventional phased array systems.

3.4.9 Non-Uniform Modulation

In the simplest STC-DM configuration, a uniform modulation frequency is applied across all meta-atoms [28]. This enables a straightforward Fourier-based modeling approach, where the metasurface is represented as an array of equivalent sources with complex excitation currents at the generated harmonic orders. However, adopting *non-uniform* modulation—where each meta-atom operates at a distinct modulation frequency—introduces additional degrees of freedom for enhanced wavefront control.

Among such strategies, *asynchronous* [90, 91] and *co-prime* modulation [92] schemes have emerged as effective solutions to achieve broadband manipulation, such as scattering reduction.

3.4.9.1 Asynchronous STC-DMs

Wang *et al.* [90] introduced an Asynchronous Space-Time-Coding Digital Metasurface (*ASTC-DM*), where each element operates with an independent local clock. This design enables the encoding of distinct temporal phases across the metasurface, thereby eliminating the need for a global synchronization mechanism. The asynchronous configuration allows for the synthesis of complex spectral modulations in the reflected wavefront and introduces a novel degree of freedom in space-time EM design.

In the proposed ASTC-DM, which consists of $N \times M$ meta-atoms, the modulation period varies significantly across the surface due to position-dependent modulation frequencies. The time-varying reflection coefficient for the (n, m) -th meta-atom is given by [90]

$$\Gamma_{nm}(t) = \sum_{v=-\infty}^{+\infty} a_{nm}^{(v)} e^{j2\pi v(f_0 + \Delta f_{nm})t}, \quad (84)$$

where $a_{nm}^{(v)}$ are the Fourier coefficients of $\Gamma_{nm}(t)$, while f_0 and Δf_{nm} represent the reference modulation frequency and the local frequency offset of the (n, m) -th meta-atom, respectively. As a result, the actual modulation period for each meta-atom becomes $1/(f_0 + \Delta f_{nm})$.

Under normal incidence of a monochromatic wave at frequency f_c , the far-field scattering pattern of the ASTC-DM is expressed as [90]

$$\mathbf{f}(\theta, \phi, t) = \sum_{v=-\infty}^{+\infty} \sum_{n=1}^N \sum_{m=1}^M \mathbf{E}_{nm}(\theta, \phi) a_{nm}^{(v)} e^{j(\psi_{nm} + \emptyset_{nm}^{(v)})}, \quad (85)$$

where ψ_{nm} and $\emptyset_{nm}^{(v)}$ are defined as

$$\begin{cases} \psi_{nm} = \frac{2\pi f_c}{c} d \sin \theta [(n-1) \cos \phi + (m-1) \sin \phi], \\ \emptyset_{nm}^{(v)} = 2\pi v \Delta f_{nm} \left(t - \frac{R'}{c} \right). \end{cases} \quad (86)$$

Here, d is the spacing between adjacent meta-atoms in both x and y directions, c is the speed of light in vacuum, and R' denotes the distance between the center of the metasurface and the observation point. In Equation (86), ψ_{nm} depends solely on the spatial location of the meta-atom, while $\emptyset_{nm}^{(v)}$ accounts for the temporal phase shift due to the modulation frequency offsets, enabling dynamic harmonic wavefront control under a fixed coding sequence. The reader is referred to Ref. [90] for a more detailed theoretical treatment and additional discussion.

3.4.9.2 Co-Prime Modulation

A more advanced non-uniform scheme was introduced in Ref. [92], where co-prime integers are assigned as modulation factors to individual meta-atoms, resulting in aperiodic temporal modulation over the aperture. As conceptually illustrated in Figure 54, this aperiodicity disrupts the global coherence of the scattered field and promotes joint spatial-spectral dispersion.

The authors demonstrated that by carefully choosing the co-prime set for the (n, m) -th meta-atom and ensuring that these numbers are relatively prime, the resulting modulation sequence achieves an extremely low time-domain periodicity—formally, the Least Common Multiple (LCM) of the co-prime set becomes very large. Consequently, the temporal coherence among

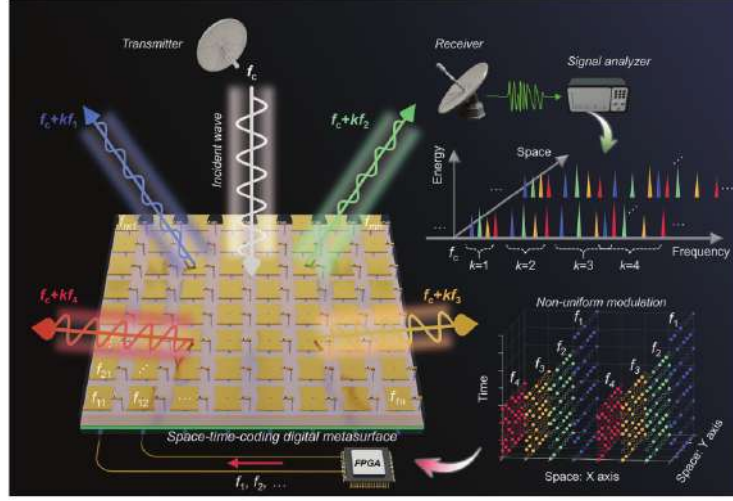


Figure 54: Conceptual illustration of non-uniform temporal modulation in a STC-DM [92].

the reflected harmonic components is drastically reduced, leading to a *broad-band suppression of coherent scattering*. In this framework, the time-varying reflection coefficient for the (n, m) -th element is modeled as

$$\Gamma_{nm}(t) = \sum_{l=1}^L \Gamma_{nm}^{(l)} G_{nm}^{(l)}(t), \quad (87)$$

where the rectangular window function $G_{nm}^{(l)}$ can be expressed as

$$G_{nm}^{(l)}(t) = \begin{cases} 1, & (l-1)\tau_{nm} \leq t < l\tau_{nm}, \\ 0, & \text{otherwise,} \end{cases} \quad (88)$$

where $\tau_{nm} = T_{nm}/L$ is the local modulation period.

The far-field scattering pattern is then expressed via a Fourier expansion as

$$\mathbf{f}(\theta, \phi, t) = E(\theta, \phi) \sum_{f \in \mathbb{F}} e^{j2\pi ft} \sum_{n=1}^N \sum_{m=1}^M b_{nm}(f) e^{j\frac{2\pi}{\lambda_c} \sin \theta [(n-1) \cos \phi + (m-1) \sin \phi]}, \quad (89)$$

where $\mathbb{F} = \{f \mid f = f_c + \nu f_{nm}, \nu \in \mathbb{Z}\}$ is the set of harmonic frequencies generated by the periodic time-varying reflection coefficient $\Gamma_{nm}(t)$ on each coding element under monochromatic incidence $\exp(j2\pi f_c t)$, and $b_{nm}(f)$ denotes a modified equivalent reflection coefficient

$$b_{nm}(f) = \begin{cases} a_{nm}\left(\frac{f-f_c}{f_{nm}}\right), & \frac{f-f_c}{f_{nm}} \in \mathbb{Z}, \\ 0, & \frac{f-f_c}{f_{nm}} \notin \mathbb{Z}, \end{cases} \quad (90)$$

where $a_{nm}(\nu)$ are the complex-valued equivalent currents defined by Equation (44).

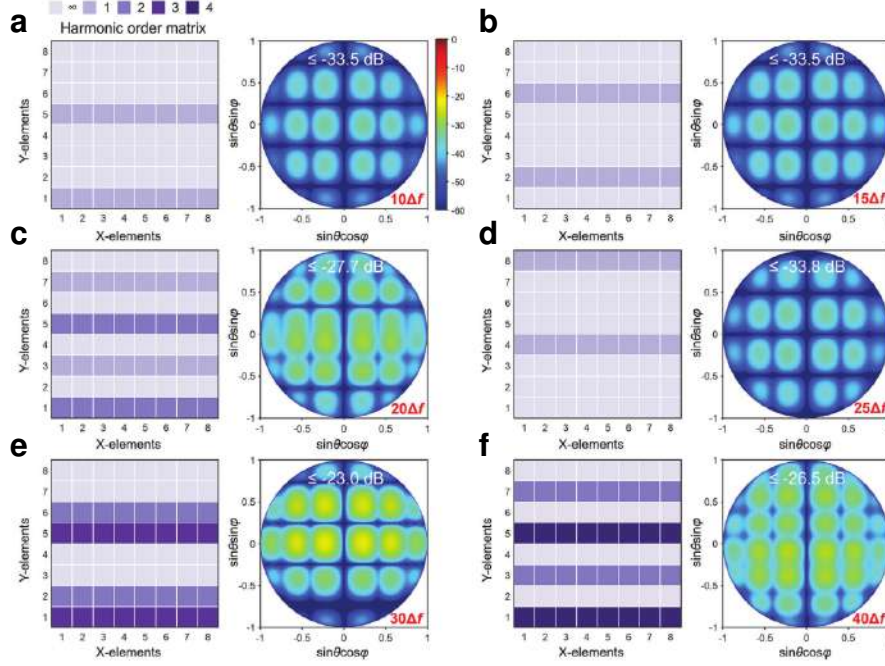


Figure 55: Non-uniform modulation with four frequencies [92]. Harmonic order matrices and the corresponding 2-D far-field scattering patterns produced by a non-uniform STC-DM at different harmonic frequencies.

This structure leads to a dense harmonic distribution that resembles a noise-like spectral profile and suppresses coherent scattering across a broad frequency band. For a more comprehensive discussion about the implication of Equation (90), the reader is referred to Ref. [92].

Numerical and experimental results confirmed that co-prime modulation can achieve ≈ 33.5 dB of scattering reduction when compared to a metallic reference plate of equal dimensions, as shown in Figure 55. Moreover, it provides ≈ 11 dB of improvement over conventional uniformly modulated STC-DMs [28], as shown in Figure 56. These gains are attributed to the enhanced spectral spreading and the disruption of deterministic interference patterns.

Finally, co-prime modulation is compatible with current digital metasurface hardware, as it only requires programmable local clocks. This makes it a promising candidate for stealth-oriented and low-scattering EM systems.

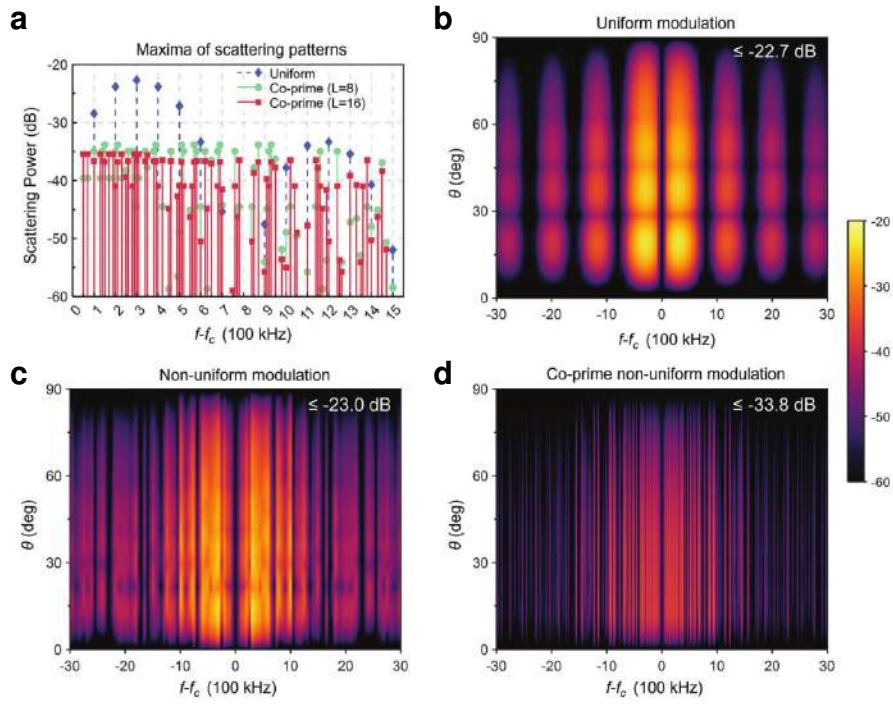


Figure 56: Maximum scattering powers for different modulation cases [92]. **a)** Maxima of scattering patterns for uniform and co-prime modulation. **b–d)** Angular-frequency spread diagrams of a STC-DM with identical coding sequences but different modulation strategies (uniform, non-uniform, and co-prime non-uniform, respectively).

Part III

SYNTHESIS AND DIAGNOSTICS FOR SPACE-TIME-CODING DIGITAL METASURFACES

This part of the Thesis investigates the synthesis and diagnostics of space-time-coding digital metasurfaces, highlighting advanced techniques in both fields.

The design methodology begins with a generalized analytical framework for the multi-frequency synthesis. In addition, a machine-learning-based approach to synthesis is explored, showcasing its potential to enhance design processes.

The diagnostic techniques for these metasurfaces are then presented. This includes advanced methods such as sparse regularization and inverse modeling, as well as the application of deep learning for data-driven diagnostics. A significant portion is dedicated to space-time modulation-based diagnostics, where both uniform and non-uniform coding schemes are examined in detail.

SYNTHESIS OF SPACE-TIME-CODING DIGITAL METASURFACES

4.1 BACKGROUND AND GENERALITIES

As discussed in [Chapter 3](#), STC-DMs have emerged as powerful and versatile architectures for the practical realization of space-time-varying media. These platforms leverage digital coding by coupling spatial and temporal modulations, enabling advanced control over the spatial and spectral characteristics of EM waves.

The synthesis of STC-DMs fundamentally involves selecting appropriate digital coding sequences that define the response of each meta-atom. These sequences are typically periodic in time, defined by specific modulation frequencies, and are often represented as a STC matrix with two spatial and one temporal dimension. From a computational perspective, this design process remains largely unresolved. In relatively simple scenarios, it can be approached through Brute-Force (BF) numerical optimization methods [28].

This chapter explores design strategies that significantly enhance the functionality of STC-DMs. It begins with a summary of recent advances in this area and highlights the computational complexity inherent to the general design problem. To address these challenges, we introduce a generalized autoencoder architecture based on Deep Neural Networks (DNNs). This novel approach enables the simultaneous and independent synthesis of harmonic scattering patterns, providing fine control over harmonic tapering and phasing, and thereby improving spectral efficiency.

4.2 GENERALIZATION OF THE ANALYTICAL SYNTHESIS PROCEDURE

In [Section 3.4.1.3](#), the analytical design strategy is put forward to achieve the independent and simultaneous syntheses of prescribed scattering patterns at given harmonic frequencies, but has attained only partial control on the equivalent complex-valued amplitudes described by Equation (44).

This section presents a possible generalization of the aforementioned synthesis procedure, aimed at enabling greater control over the equivalent amplitudes. In particular, to decouple the syntheses at the various harmonic frequencies and achieve control over both amplitude and phase modulation, the temporal coding sequence can be decomposed into Q subsequences, each of length R , repeated periodically with a period P . Consequently, the total length of the interleaved coding sequence becomes $L = PQR$.

The elements that compose the subsequences are determined by imposing the following condition:

$$\begin{aligned}\Gamma_{pQR+qR+r+1} &= \Omega_{pq}\Gamma_{qR+r+1}, & p &= 0, \dots, P-1, \\ & & q &= 0, \dots, Q-1, \\ & & r &= 0, \dots, R-1.\end{aligned}\tag{91}$$

By applying this condition to Equation (44), the complex-valued equivalent amplitudes for (n, m) -th meta-atom are obtained as:

$$a_{nm}^{(v)} = c^{(v)} \sum_{q=0}^{Q-1} \Lambda_q^{(v)} \Theta_q^{(v)} e^{-j\frac{2\pi vq}{PQ}},\tag{92}$$

where

$$c^{(v)} = \frac{1}{L} \text{sinc}\left(\frac{\pi v}{L}\right) e^{-j\frac{\pi v}{L}},\tag{93a}$$

$$\Lambda_q^{(v)} = \sum_{p=0}^{P-1} \Omega_{pq} e^{-j\frac{2\pi vp}{P}},\tag{93b}$$

$$\Theta_q^{(v)} = \sum_{r=0}^{R-1} \Gamma_{qR+r+1} e^{-j\frac{2\pi vr}{L}}.\tag{93c}$$

By formulating the problem in this way, in order to decouple the synthesis at the various harmonic frequencies, it is necessary to find coefficients Ω_{pq} such that $\Lambda_q^{(v)} = \chi^{(v)} \tilde{\delta}_{qv}$ for $v = 0, \dots, Q-1$, where $\chi^{(v)}$ are non-zero constants.

Let S denote the number of accessible states for a given digital encoding. In the particular case where the periodicity of the coding sequence is an integer multiple of the number of states, i.e., $P = \kappa S$, with $\kappa \in \mathbb{N} \setminus \{0\}$, Equation (93b) can be written as

$$\begin{aligned}\Lambda_q^{(v)} &= \sum_{p=0}^{S-1} \left(\Omega_{\kappa p, q} + \Omega_{\kappa p+1, q} e^{-j\frac{2\pi v}{P}} + \Omega_{\kappa p+2, q} e^{-j\frac{4\pi v}{P}} + \dots \right. \\ &\quad \left. \dots + \Omega_{\kappa p+\kappa-1, q} e^{-j\frac{2(\kappa-1)\pi v}{P}} \right) e^{-j\frac{2\pi pv}{S}},\end{aligned}$$

or, equivalently, in the more compact form:

$$\Lambda_q^{(v)} = \sum_{p=0}^{S-1} \left(\sum_{s=0}^{\kappa-1} \Omega_{\kappa p+s, q} e^{-j\frac{2\pi vs}{P}} \right) e^{-j\frac{2\pi pv}{S}}.\tag{94}$$

At this point, to simplify the problem, one can impose the factorization $\Omega_{\kappa p+s, q} = \gamma_{qs} \tilde{\Omega}_{\kappa p+s, q}$, where the auxiliary coefficients are defined as

$$\tilde{\Omega}_{\kappa p+s, q} = e^{j\frac{2\pi pq}{S}}, \quad s = 0, \dots, \kappa-1.$$

Substituting this into Equation (94) yields:

$$\Lambda_q^{(v)} = \tilde{\Lambda}_q^{(v)} \sum_{s=0}^{\kappa-1} \gamma_{qs} e^{-j\frac{2\pi vs}{P}},\tag{95}$$

where $\gamma_{q0} = 1 \forall q$, expressing a self-consistency condition. By introducing the auxiliary variable $\xi = q - \nu$, the term $\tilde{\Lambda}_q^{(\nu)}$ is explicitly given by

$$\begin{aligned}\tilde{\Lambda}_q^{(\nu)} &= \sum_{p=0}^{S-1} e^{j\frac{2\pi p\xi}{S}}, \\ &= \frac{1 - e^{j2\pi\xi}}{1 - e^{j\frac{2\pi\xi}{S}}} = \frac{S \operatorname{sinc}(\pi\xi)}{\operatorname{sinc}\left(\frac{\pi\xi}{S}\right)} e^{j\frac{(S-1)\pi\xi}{S}},\end{aligned}\tag{96}$$

which is directly related to the Dirichlet kernel [93]. In this way, to determine the coefficients γ_{qs} needed to decouple the problem, it is possible to set up a BF optimization of Equation (95) by exploring a space of S^K combinations, rather than S^P .

4.2.1 Feasible Domain at the Central Frequency

Here we analyze the characteristics of the coefficient $a_{nm}^{(0)}$ in the simplified case where $P = Q = 1$, focusing on the impact of different values of R . Figure 57 illustrates the feasible domain of the complex-valued equivalent amplitudes at the central frequency for $R = 2, 4, 8$. In all cases, the domain lies within a square in the complex plane and contains $(R + 1)^2$ unique values.

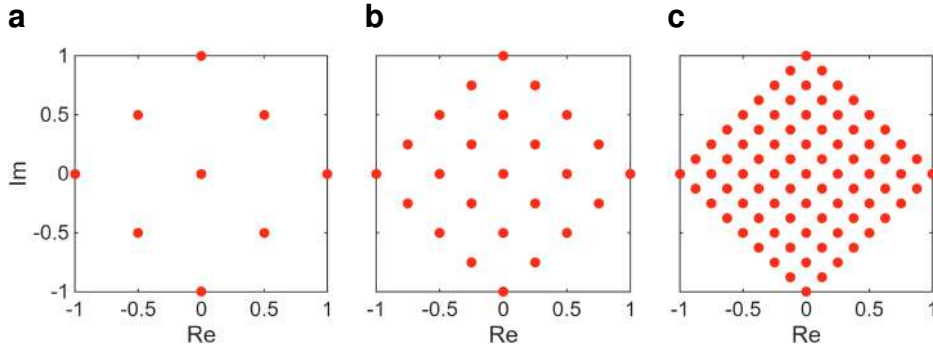


Figure 57: Feasible domains at the central frequency for different values of R , represented in the complex plane. **a)** $R = 2$, **b)** $R = 4$, **c)** $R = 8$.

As the length of the basic time sequence increases (i.e., with increasing R), the number of equivalent phase states grows accordingly, resulting in improved resolution of the achievable amplitudes for specific angular directions. In particular, the resolution along orthogonal directions is $2/R$, while along the diagonal directions it is $\sqrt{2}/R$ (when R is even). Importantly, increasing R does not affect the maximum achievable magnitude of the equivalent amplitudes.

As shown in Figure 57a–c, increasing R leads to a visible densification and expansion of the feasible domain at the fundamental harmonic. This

behavior stems from the definition of the coefficient $\Theta_q^{(\nu)}$ in Equation (93c), which implies that for a given R , the feasible domain tends to further expand with increasing harmonic order. Nevertheless, since the trends observed at the central frequency are representative of the general behavior, higher-order harmonic domains are omitted here to avoid redundancy.

Additionally, it is important to consider practical constraints associated with the selection of the sequence length L . Assuming a fixed modulation period T_0 , a longer time sequence implies a shorter duration per coding state, equal to T_0/L , and thus a higher state-switching rate. In practice, however, this rate cannot be arbitrarily high due to hardware limitations. Conversely, if the coding state duration is fixed, increasing L leads to a higher modulation frequency f_0 , which may degrade the frequency separation between the fundamental and higher harmonics. Consequently, the choice of L must balance resolution requirements with system-level limitations and performance trade-offs.

4.2.2 Optimization Criterion

In order to synthesize specific scattering patterns of the STC-DM at various harmonic frequencies, it is required to determine the required STC matrix. To achieve this, we can apply the Minimum Distance (MD) criterion, aiming to minimize the distance in the complex plane between the optimized complex excitation vector and the ideal solution corresponding to the optimal excitation coefficients. Under this criterion, the problem used to determine the optimal coding sequence $\hat{\mathbf{G}}_{nm}$ for the generic (n, m) -th meta-atom, can be described as

$$\hat{\mathbf{G}}_{nm} = \underset{\mathbf{G}_{nm}}{\operatorname{argmin}} \left\| \hat{\mathbf{A}}_{nm} - \mathbf{A}_{nm} \right\|_1, \quad (97)$$

where $\|\cdot\|_1$ denotes the L_1 -norm, $\mathbf{G}_{nm} = (\Gamma_{nm}^{(1)}, \dots, \Gamma_{nm}^{(L)})^T$ is the time coding sequence pertaining to the (n, m) -th meta-atom, subject to the feasibility constraint that all its components must belong to the alphabet $\{1, j, -1, -j\}$, $\hat{\mathbf{A}}_{nm} = (\hat{a}_{nm}^{(0)}, \dots, \hat{a}_{nm}^{(Q-1)})^T$ denotes the optimal excitation coefficients for Q consecutive harmonic orders of interest, while $\mathbf{A}_{nm} = (a_{nm}^{(0)}, \dots, a_{nm}^{(Q-1)})^T$ denotes the feasible excitation coefficients.

4.2.3 Example of Synthesis

As a proof of concept, we consider a 2-bit ($S = 4$) coding STC-DM with $N = 16$ meta-atoms, a spacing $d = \lambda_c/2$, and a temporal coding sequence composed by $Q = 5$ subsequences, each of length $R = 6$, repeated with a

periodicity $P = 8$, for a total length $L = 240$. To determine the coefficients $\hat{\gamma}_{qs}$ such that $\Lambda_q^{(v)} \propto \tilde{\delta}_{qv}$, the following minimization problem was solved:

$$\begin{aligned} \hat{\gamma}_{qs} = \underset{\gamma_{qs}}{\operatorname{argmin}} \frac{|\Lambda_q^{(v \neq q)}|}{|\Lambda_q^{(q)}|}, \quad v = 0, \dots, Q-1, \\ \text{s. t. } \gamma_{qs} \in \{1, j, -1, -j\} \forall s. \end{aligned} \quad (98)$$

Figure 58 shows the coefficients $\Lambda_q^{(v)}$ obtained via a BF optimization for this configuration. As can be observed, the magnitude of these coefficients is not equal for all the harmonic orders. In particular, in correspondence of $q = v \in \{1, 3\}$, we have $|\Lambda_q^{(v)}| = \sqrt{2 + \sqrt{2}} \cdot S$. While the magnitude $|\Lambda_q^{(v)}|$ directly constrains the maximum spectral efficiency for the corresponding harmonic synthesis, the values shown for $q = v \in \{1, 3\}$, despite falling below the ideal maximum of $2S$, are nevertheless very close to this boundary. Crucially, this result represents the best solution for the chosen parameter set, having been obtained through an exhaustive search of all γ_{qs} combinations.

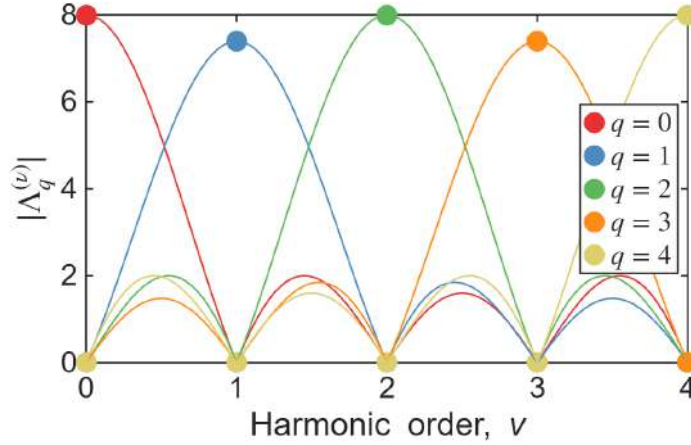


Figure 58: Magnitude of the coefficients $\Lambda_q^{(v)}$ achieved via the BF optimization, when $P = 8$, $Q = 5$, $R = 6$, $S = 4$. Continuous curves are guides to the eye only.

In this example, the targets are $Q = 5$ representative 1-D scattering patterns with nontrivial magnitude and phase distributions for harmonic orders $v = 0, 1, 2, 3, 4$, namely:

1. Golay–Rudin–Shapiro diffuse-scattering pattern, with a steering angle of 35° [15];
2. Three-beam pattern with peaks at $-30^\circ, 15^\circ, 50^\circ$ [94];
3. Difference pattern with cosine tapering and a steering angle of 30° ;
4. Binomial pattern [94];
5. Dolph–Chebyshev pattern with a Sidelobe Level (SLL) of -20 dB [94];

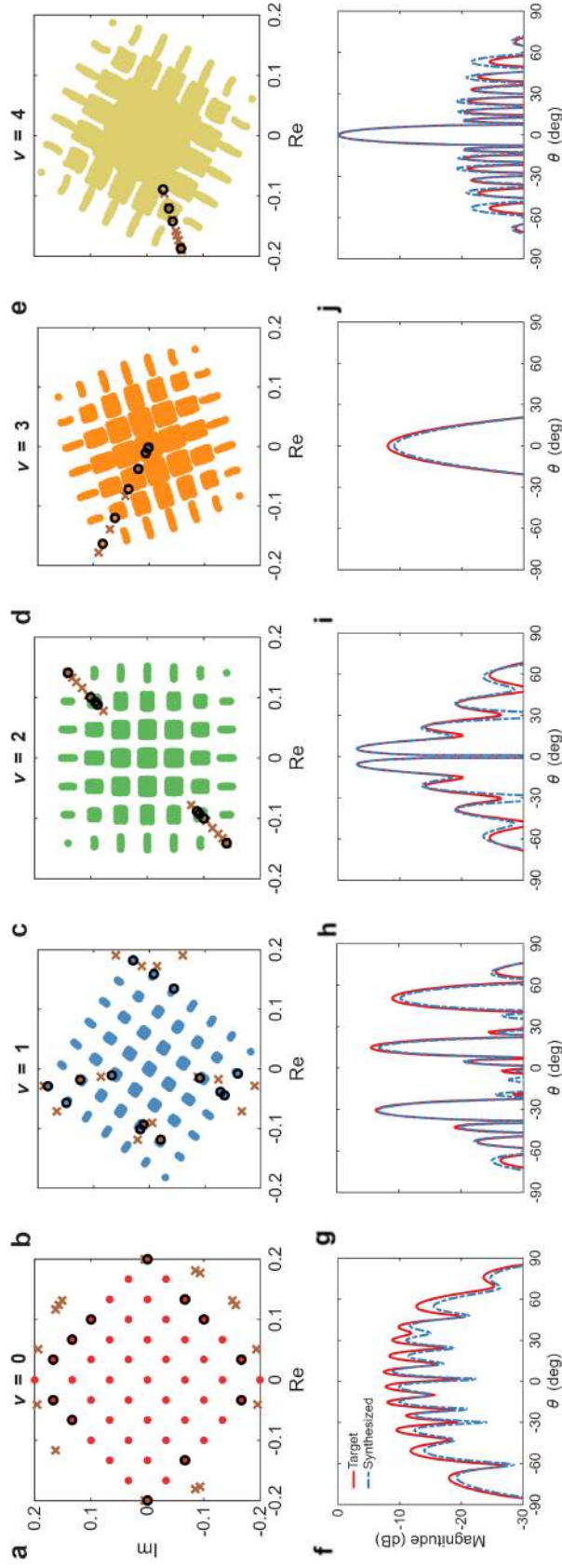


Figure 59: Example of synthesis of the scattering patterns for a 2-bit coding STC-DM composed by 16 meta-atoms with period $d = \lambda_c/2$, when $P = 8, Q = 5, R = 6$. **a–e)** Feasible domains for the targeted harmonic orders ($\nu = 0, 1, 2, 3, 4$). The filled markers indicate feasible equivalent amplitudes, the empty markers are the optimal solutions found through the MD criterion, while the crosses indicate the ideal equivalent amplitudes. **f–j)** Corresponding scattering patterns with intensity normalized to the maximum value at the harmonic order $\nu = 4$.

This example of joint synthesis is not related to any specific application; rather, it was selected to illustrate how this method can shape scattering patterns across different harmonics.

In order to improve the agreement between the synthesized and desired coefficients, a constant phase shift of $\exp(j\varphi^{(v)})$ was applied to the target coefficients for each harmonic synthesis. This phase shift is a degree of freedom that can be exploited without affecting the magnitude of the scattering patterns. In particular, the harmonic phases $\varphi^{(v)}$ correspond to the phases at which the points in the feasible domain exhibit the maximum magnitude for each harmonic order. Figure 59 illustrates the synthesized results for the harmonic frequencies of interest. Specifically, Figure 59a–e display the feasible domains for the targeted harmonic orders. The filled markers represent the feasible equivalent amplitudes, calculated using Equation (44) for all combinations of the coding states. The empty markers denote the optimal solutions found through the MD criterion, while the crosses indicate the ideal equivalent amplitudes. Figure 59f–j show the synthesized scattering patterns, compared with the targeted ones.

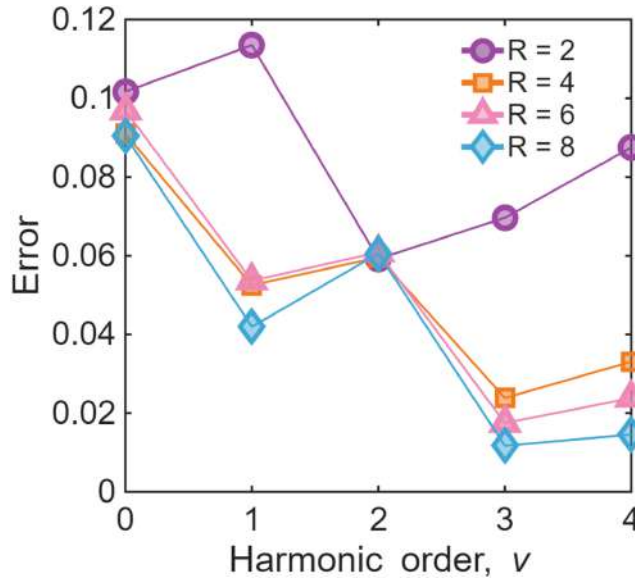


Figure 60: Variation of the error metric $\mathcal{E}^{(v)}$ as a function of the subsequence length R , for the harmonic orders of interest $\nu = 0, 1, 2, 3, 4$. Continuous curves are guides to the eye only.

To further assess the effectiveness of the synthesis approach, we introduce an error metric that quantifies the deviation between the target and synthesized scattering patterns across different harmonic orders. Specifically, the metric is defined as the mean absolute error between the magnitude of the target scattering pattern $\mathcal{M}^{(v)}$ and the magnitude of the synthesized pattern $\hat{\mathcal{M}}^{(v)}$ for the harmonic orders $\nu = 0, 1, 2, 3, 4$:

$$\mathcal{E}^{(v)} = \frac{1}{N_\theta} \left\| \mathcal{M}^{(v)} - \hat{\mathcal{M}}^{(v)} \right\|_1, \quad (99)$$

where N_θ denotes the number of samples of θ used to compute the scattering patterns, which are derived from Equation (46) for $\phi = 0$.

Referring to the synthesis shown in Figure 59f–j, Figure 60 illustrates the variation of the error metric as a function of the subsequence length R . The plot provides insights into how the error evolves with different values of R , offering a performance benchmark for the synthesis process and demonstrating the trade-offs between synthesis accuracy and coding sequence length.

4.2.4 Spectral Efficiency

The syntheses generated across the first Q harmonic orders replicate periodically due to the inherent periodicity of the digital-filter terms $\tilde{\Lambda}_q^{(\nu)}$ defined in Equation (96). However, the amplitudes of these syntheses decay according to the relation given in Equation (92). Therefore, depending on the value of L , this decay may occur at a rather slow rate, which can result in reduced spectral efficiency.

One approach to effectively control the spectral decay of these higher-order terms involves intentionally shaping the temporal coding by applying a digital low-pass filter. To provide a concrete example, with reference to Figure 59, we implemented a 16th-order Finite-Impulse-Response (FIR) low-pass filter with a normalized cut-off frequency $\nu_c = (2Q - 1)/(2L)$. The filtering operation is performed by numerically convolving the original reflection-coefficient sequence $\{\Gamma_{nm}^{(l)}\}_{l=1,\dots,L}$ with the filter's coefficient sequence. The resulting, extended sequence is now of length $L + 16$, and its coefficients generally no longer maintain the identical amplitude or the 2-bit quantized phases of the initial sequence.

As a meaningful metric for assessing the spectral efficiency, we evaluate the normalized intensities of the complex-valued equivalent amplitudes, as given in Equation (92), averaged across all coding elements:

$$I^{(\nu)} = \frac{\sum_{n=1}^N \sum_{m=1}^M |a_{nm}^{(\nu)}|^2}{\sum_{n=1}^N \sum_{m=1}^M \sum_{k=-\infty}^{+\infty} |a_{nm}^{(k)}|^2}, \quad (100)$$

which essentially provides a measure of the average *fraction of power* distributed across the various harmonic orders. Figure 61 compares the results in the absence and presence of ideal filtering, from which the effects on the higher-order terms are evident. Specifically, when filtering is not employed, $\approx 4\%$ of the total power is allocated to each of the five controlled harmonic orders ($\nu = 0, 1, 2, 3, 4$). These controlled harmonic orders, highlighted by the green-shaded region, thus account for a cumulative power fraction of $\approx 21\%$. Conversely, when the ideal filtering is applied, the individual power fractions delivered to the controlled harmonic orders rise significantly, now spanning from $\approx 6\%$ up to $\approx 18\%$, yielding a much higher total power concentration of $\approx 63\%$.

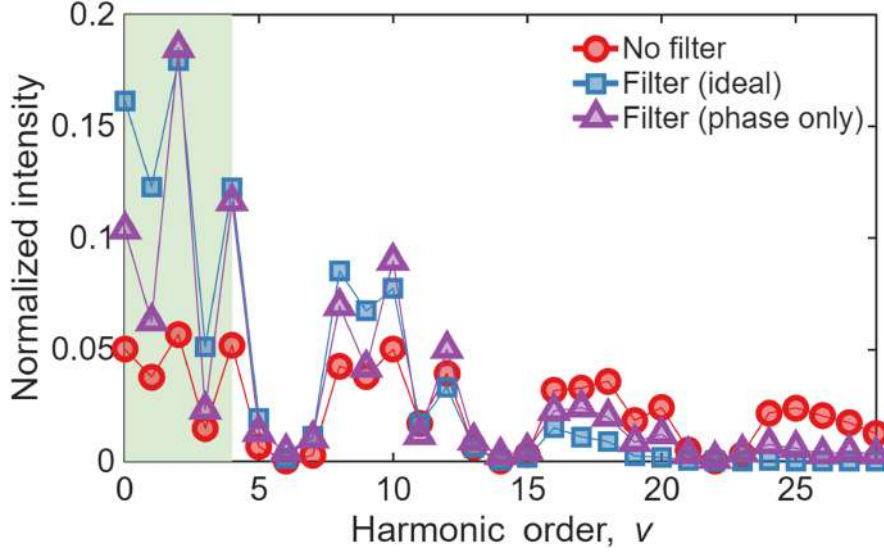


Figure 61: Filtering of the higher-order harmonic terms. The normalized intensities $I^{(\nu)}$ are plotted as a function of the harmonic order ν , for the configuration and parameters as in Figure 59. Red circles: Results in the absence of low-pass filtering. Blue squares: Effects of ideal low-pass FIR filtering. Purple triangles: Effects of approximate low-pass FIR filtering, with 2-bit quantization of phase only. Continuous curves are guides-to-the-eye only; the green-shaded region indicates the spectral range of direct interest for the synthesis.

The primary obstacle, however, lies in the fact that the resulting amplitude and phase distributions are incompatible with our digital, phase-only platform. Consequently, investigating simplified or approximate filtering methods becomes necessary. Specifically, Figure 61 also illustrates the outcome achieved when the filter is approximated using only 2-bit quantized phases. It is evident that even this phase-only quantized version, which is compatible with our hardware, offers significant advantages over the original, unfiltered results, with power fractions ranging from $\approx 3\%$ to $\approx 19\%$, for a total of $\approx 49\%$.

Overall, there appears to be promising potential for improvements, particularly in achieving better power distribution across the controlled harmonic orders, which could be accomplished by optimizing the filtering process or the underlying modulation waveform. Nevertheless, applying any form of filtering inevitably increases the overall length of the code sequence, presenting a trade-off between performance gains and sequence length. Additionally, it is essential to optimize filtering without compromising the alignment between target and synthesized coefficients, as the code sequence must comply with the 2-bit coding scheme of the STC-DM prototype, introducing another layer of complexity to the design process.

4.2.5 Experimental Details

MODELING The numerical implementation of the theoretical analysis was carried out using MATLAB[®], with the far-field scattering pattern of the basic element approximated as $E(\theta, \phi) \approx \cos \theta$.

4.2.6 Summary and Outlook

To summarize, the analysis has outlined the primary intrinsic limitations of the synthesis technique based on interleaved time coding sequences, which directly impact the viability of this approach:

- *Constraints on the sequence length:* For a fixed total modulation period, extending the time sequence necessitates a reduction in the duration allocated to each coded state. This demands a faster switching rate, which may exceed the physical limitations of the hardware system. Conversely, if the duration of each state is kept constant, increasing the sequence length implies a higher modulation frequency. This, in turn, leads to a deterioration in the spectral separation between the fundamental harmonic and the higher-order harmonics.
- *Reduced spectral efficiency:* Due to the intrinsic periodicity of the digital coefficients described in Equation (96), the synthesized responses generated at the first Q harmonic orders replicate with progressively decreasing amplitudes, following the spectral envelope detailed in Equation (92).

Therefore, while the results shown in Figure 59 may initially appear promising, the interconnected technical constraints discussed above place serious restrictions on the practical effectiveness of the proposed synthesis methodology. Consequently, these challenges discourage further exploration of this approach under the current conditions.

In light of these considerations, it is essential to explore alternative strategies to address these challenges and improve the overall effectiveness of the synthesis approach.

4.3 MACHINE-LEARNING-ENABLED SYNTHESIS

The independent and simultaneous multi-frequency synthesis of scattering patterns is a key challenge for realizing the full potential of STC-DMs in wireless communication scenarios [30], enabling efficient multiplexing strategies. However, BF optimization methods can become computationally prohibitive when dealing with complex patterns and a large number of frequencies. Semi-analytical approaches only partially address this issue by decoupling the synthesis across different frequencies, but they are typically limited to phase-only designs and result in suboptimal spectral efficiencies [78].

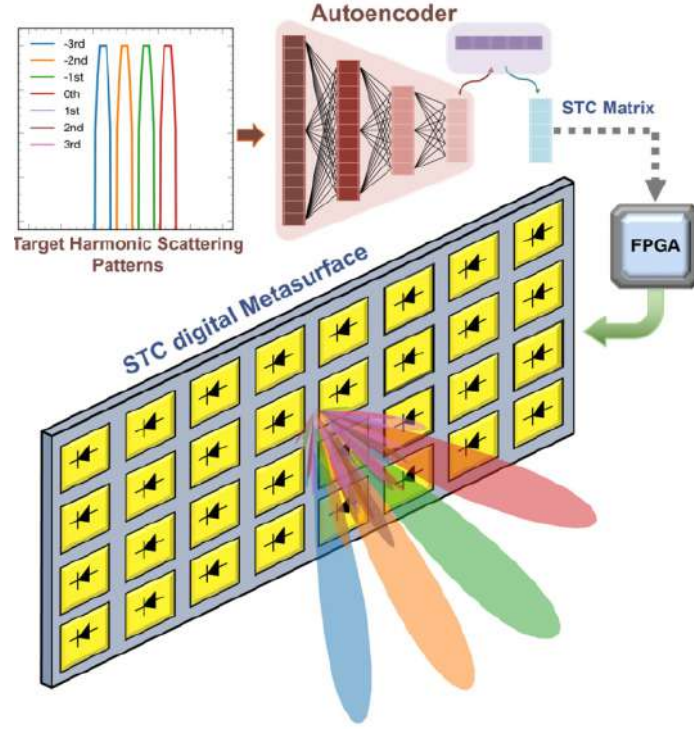


Figure 62: Conceptual illustration of the physics-driven vector-quantized intelligent autoencoder [95].

In recent years, Machine Learning (ML) approaches have been explored as a more efficient solution. For instance, in Ref. [95], a physics-driven vector-quantized intelligent autoencoder was proposed for harmonic beam steering. As illustrated in Figure 62, this system consists of an encoder, a vector-quantizer layer, and a physics-driven decoder. The system takes a set of desired harmonic scattering patterns as input and, after training, generates the corresponding optimized STC matrix. This approach was successfully applied to harmonic beam steering for up to five harmonic orders.

Although significant, previous research has predominantly targeted harmonic beam steering, offering limited capabilities in beam shaping. As a result, precise and computationally efficient control of the radiation pattern remains a major challenge in the field.

In our approach, we adopt a distinct strategy—still grounded in ML—that focuses on synthesizing the equivalent current (in both magnitude and phase) at harmonic frequencies, rather than directly targeting the scattering patterns. This method utilizes DNNs to enable the generation of a diverse set of scattering patterns by simultaneously and independently applying traditional array synthesis techniques.

To address the inherent ill-posedness of the inverse problem, which often leads to slow and inefficient training of the DNN, we employ a *tandem architecture*. This architecture has already proven successful in various domains,

including nanophotonics [96], plasmonics [97], cloaking [98], scattering reduction [99], and antenna design [100].

To showcase the potential of our approach, we provide several representative examples of multi-frequency synthesis that involve both phase and amplitude control. These designs encompass Dolph–Chebyshev, Fourier, multi-beam, diffuse scattering, and OAM configurations. The results are experimentally validated using a 2-bit coding STC-DM prototype operating in the microwave range (X-band), showing strong agreement between theoretical predictions and measurements.

In summary, our findings substantially enhance the design capabilities of STC-DMs, paving the way for advanced applications in areas such as wireless communications, sensing, and imaging. The results below are also documented in Ref. [101].

4.3.1 Proposed Approach

Figure 63 provides a conceptual illustration of the proposed ML-based reflective STC-DM. In this configuration, each meta-atom (represented as a golden square) has a reflection phase that can be toggled between multiple quantized levels using embedded diodes. This modulation scheme successfully translates physical states into digital signals, with both spatial and temporal control handled dynamically by an FPGA. The management is directed by an STC matrix, which comprises two spatial axes and one temporal axis of digital values.

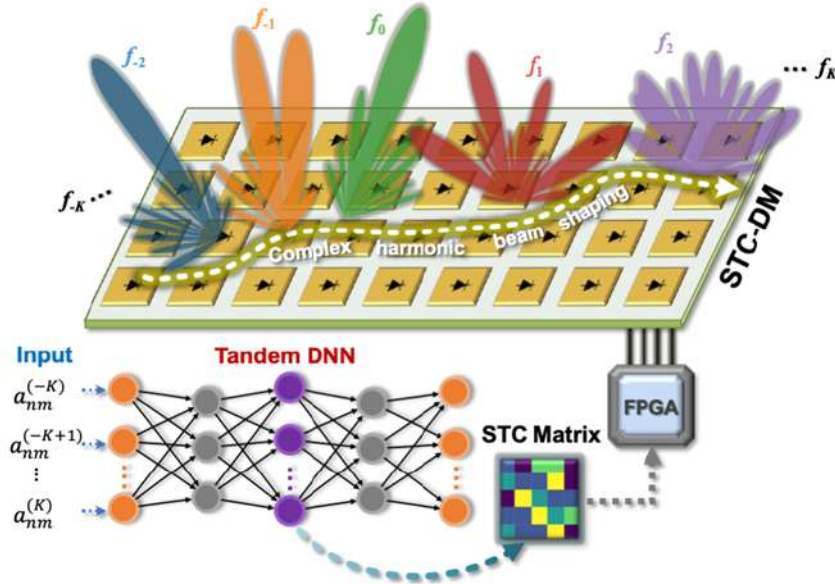


Figure 63: Conceptual illustration of an STC-DM showing its multi-frequency beam shaping and steering capabilities, powered by a tandem DNN.

Although conceptually similar to previous studies [28, 78], the current approach employs ML to achieve a more ambitious goal: The independent and simultaneous synthesis across multiple frequencies, offering improved capabilities in beam shaping and steering, as well as enhanced spectral efficiency.

From a modeling standpoint, we consider a STC-DM consisting of an $N \times M$ periodic array of meta-atoms, with a lattice period d , lying on the x - y plane. The structure is illuminated by a linearly polarized, normally incident, time-harmonic plane wave of the form $\exp(j2\pi f_c t)$, with the electric field oriented along the x -axis.

Each meta-atom experiences a temporally periodic modulation of the reflection coefficient, with a period T_0 , which is assumed to be much longer than the EM timescale. This modulation is described by Equation (38). For simplicity, we initially assume unit-magnitude reflection, i.e., $\Gamma_{nm}^{(l)} = \exp(j\varphi_{nm}^{(l)})$, where the phase response is uniformly quantized into S discrete states within the interval $[-180^\circ, 180^\circ]$. Nonetheless, we will later investigate the impact of non-ideal factors, such as reflection losses and non-uniform phase quantization. Under these assumptions, the harmonic scattering pattern is given by Equation (46).

Remarkably, despite the inherently unit-magnitude and phase-quantized nature of $\Gamma_{nm}^{(l)}$, the resulting equivalent currents typically exhibit a significantly richer and broader coverage in both magnitude and phase, as discussed in Ref. [28]. This characteristic effectively broadens the parameter space available for synthesis. However, Equation (44) defines a complex and nonlinear relationship between these equivalent currents and the actual design variables, namely, the quantized phase values of the reflection coefficients in the temporal sequence described by Equation (38). This intrinsic entanglement across different harmonic frequencies hinders the direct application of conventional semi-analytical array design methods that rely on independent control of phase and magnitude, such as the Dolph–Chebyshev or Fourier method [102].

4.3.2 Machine-Learning-Based Synthesis

The tandem architecture comprises two sub-blocks: a forward DNN and an inverse DNN, as shown in Figure 64.

For now, we assume a symmetric set of $Q = 2K + 1$ consecutive harmonic orders of interest, i.e., $\nu = -K, -K + 1, \dots, K - 1, K$. The synthesis is performed for a generic meta-atom, so we omit the dependence on indices (n, m) to avoid notational redundancy. Essentially, for a given meta-atom, the system takes the target harmonic equivalent currents $a^{(\nu)}$ as input and, through intermediate layers and a quantizer, outputs the L reflection phases $\varphi^{(l)}$ of the coding sequence in Equation (38). More specifically, the input is a real-valued vector $\mathbf{a} \in \mathbb{R}^{2Q}$, where the first half corresponds to $\Re\{a^{(\nu)}\}$ and the second half to $\Im\{a^{(\nu)}\}$. The intermediate-layer output

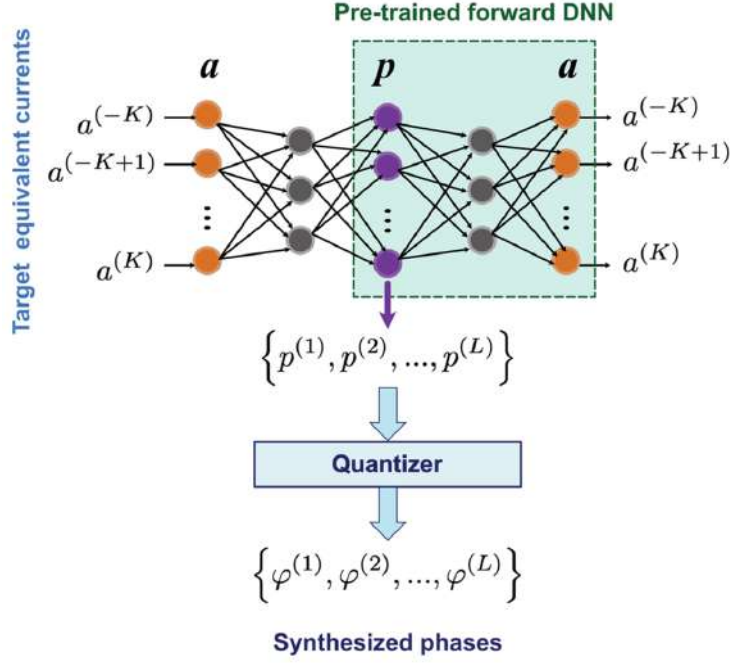


Figure 64: Schematic illustration of the tandem DNN architecture.

is a real-valued vector $\mathbf{p} \in \mathbb{R}^L$ comprising continuous normalized phases $p^{(l)} = \hat{\varphi}^{(l)}/\pi \in [-1, 1]$, which are subsequently quantized within the B -bit alphabet of cardinality $S = 2^B$, yielding the desired discrete phases $\varphi^{(l)}$.

First, the forward DNN is pre-trained on a dataset constituted by $D = 85 \cdot 10^3$ sequences of unit-magnitude reflection coefficients $\Gamma^{(l)} = \exp(j\varphi^{(l)})$, $l = 1, \dots, L$, where $\hat{\varphi}^{(l)}$ are random phases uniformly distributed in $[-\pi, \pi]$, and the corresponding equivalent currents $a^{(v)}$ are derived from the analytical model in Equation (44). At this stage, the forward mapping $\mathbf{a} = \mathbb{O}\mathbf{p}$ is well-defined and can be accurately captured by the forward DNN. However, the inherent ill-posed nature of the inverse problem makes it impossible to directly invert by merely swapping the input and output vectors. To overcome this limitation, our proposed tandem scheme seeks a nonlinear operator $\mathbb{O}'$ such that $\mathbb{O}(\mathbb{O}'\mathbf{a}) = \mathbf{a}$, essentially aiming for an identity relationship between the input and the output. This strategy is known to mitigate the one-to-many mapping issues that occur in the training of DNNs for inverse problems [96]. To achieve this, we fix the forward DNN with the pre-trained configuration and train the inverse DNN on a dataset of the same cardinality D as in the previous stage. Additionally, we introduce a power constraint:

$$P_r \leq \sum_{v=-K}^K |a^{(v)}|^2 \leq 1. \quad (101)$$

Here, the lower bound P_r represents the desired fraction of power to be allocated within the spectral range of interest, while the upper bound arises from power conservation. The choice of the lower bound strikes a balance

between optimizing spectral efficiency and ensuring successful performance during the training phase. In our subsequent syntheses, as a reasonable trade-off, we assume $P_r = 0.8$, aiming at concentrating up to 80% of the scattered power within the targeted spectral range. For training the inverse DNN, we apply a rejection scheme, generating the training set similarly to the forward case, but discarding sequences that do not satisfy the power constraint in Equation (101). The loss function used during the training process is the half Mean Squared Error (MSE) [103], which can be defined as

$$\mathcal{L} = \frac{1}{4QD} \|\mathbf{X} - \mathbf{T}\|_F^2, \quad (102)$$

where $\|\cdot\|_F$ denotes the Frobenius norm, $\mathbf{X} \in \mathbb{R}^{2Q \times D}$ contains the predicted harmonic equivalent amplitudes (i. e., the network outputs), and $\mathbf{T} \in \mathbb{R}^{2Q \times D}$ holds the corresponding target values.

It is worth noting that our forward model can be described analytically, as presented in Equation (44). This opens the possibility of adopting a hybrid approach, where only the inverse model is implemented using a DNN. However, in our implementation, we opt for a full DNN setup to streamline the training process. This approach allows for the direct use of built-in metrics and routines, simplifying the overall implementation.

Moreover, while more sophisticated tandem DNN architectures exist [98, 99, 104, 105], here we opt for a conventional fully connected implementation [96] because it adequately demonstrates the effectiveness and advantages of our approach in synthesizing STC-DMs with a simple architecture. Indeed, there is potential for enhancements in the tandem DNN architecture that could provide more advanced control features (see Section 4.3.4.2).

4.3.3 Results and Discussion

4.3.3.1 Tandem DNN Training

Figure 65 illustrates the convergence of the inverse-DNN training process. Figure 65a shows the MSE for a scenario with $Q = 5$ harmonic frequencies ($\nu = -2, -1, 0, 1, 2$) and sequences of length $L = 24$.

The plot displays the error metrics for the training, validation, and test datasets, with the phase quantization effects explicitly excluded. The errors for the three datasets are hardly distinguishable and reach values as low as $\approx 3 \cdot 10^{-3}$. Figure 65b illustrates the correlation between the optimal error and the temporal sequence length, for various numbers of target harmonic frequencies ($Q = 3, 5, 7$). Intuitively, handling a larger number of harmonic frequencies typically requires longer sequences. However, beyond a certain threshold, the error may increase, likely due to overfitting. Further increases in the number of harmonic frequencies ($Q > 7$) may demand more complex DNN architectures, with additional hidden layers and neurons.

The corresponding results for the forward DNN are presented in Figure 66, where trends appear qualitatively similar but with significantly

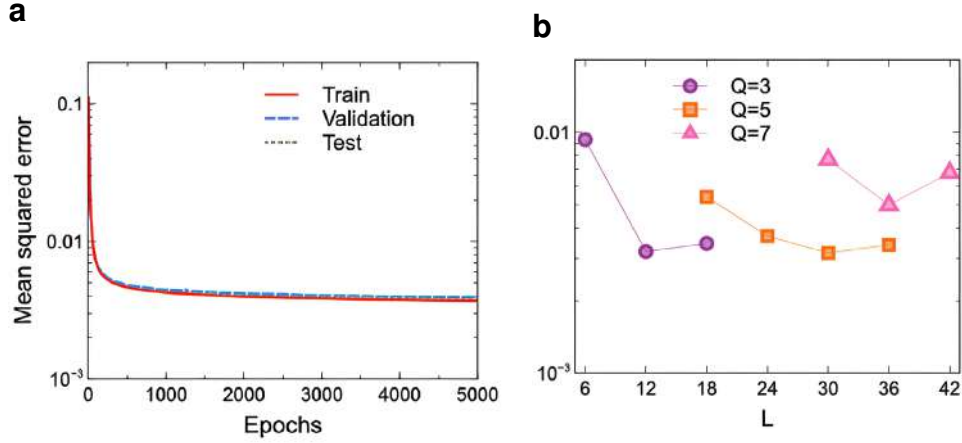


Figure 65: Convergence for inverse DNN training. **a)** MSE for a scenario with $Q = 5$ harmonic frequencies ($\nu = -2, -1, 0, 1, 2$) and sequences of length $L = 24$. **b)** Optimal error as a function of sequence length L for various numbers of target harmonic frequencies. Continuous curves are guides to the eye only.

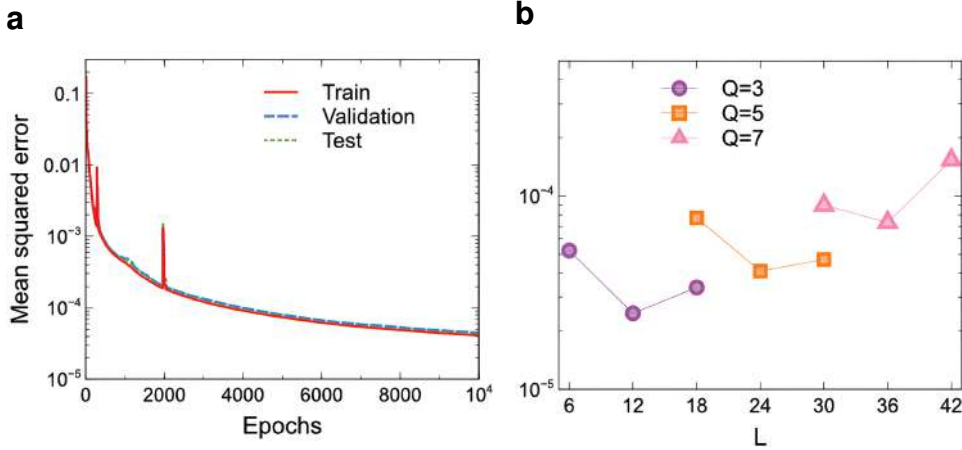


Figure 66: Convergence for forward DNN training. **a)** MSE for a scenario with $Q = 5$ harmonic frequencies ($\nu = -2, -1, 0, 1, 2$) and sequences of length $L = 24$. **b)** Optimal error as a function of sequence length L for various numbers of target harmonic frequencies. Continuous curves are guides to the eye only.

smaller errors, reflecting the inherently lower complexity of the forward problem.

In our subsequent examples of application, we will focus on a scenario featuring $Q = 5$ harmonic frequencies, choosing a coding sequence length of $L = 24$ as a reasonable trade-off between accuracy and complexity, and a 2-bit coding ($S = 4$) in view of its demonstrated practical viability in several STC-DM prototypes [28].

4.3.3.2 Example of Syntheses

Figure 67 shows two illustrative cases of multi-frequency synthesis for the complex-valued equivalent currents $a^{(\nu)}$ of a generic meta-atom, targeting $Q = 5$ consecutive harmonic frequencies ($\nu = -2, -1, 0, 1, 2$).

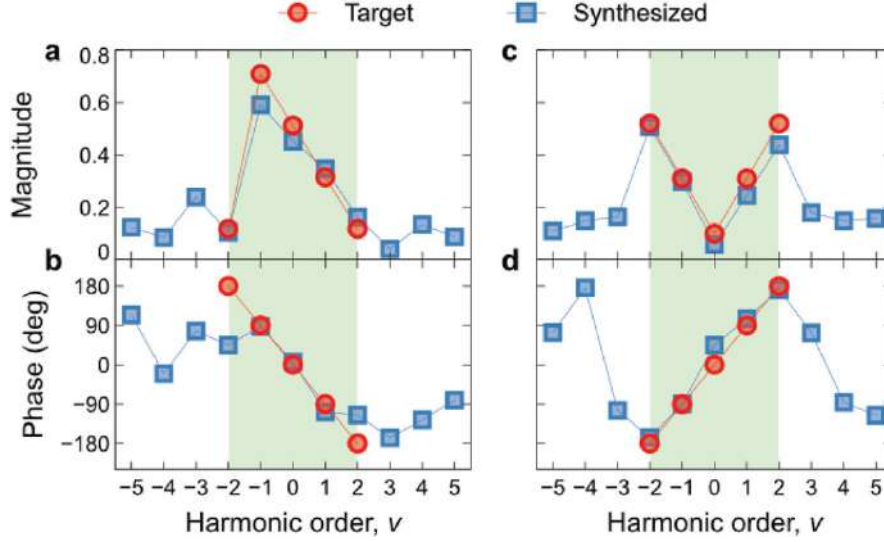


Figure 67: Examples of joint syntheses for the complex-valued equivalent currents $a^{(\nu)}$ in a generic meta-atom across different harmonic frequencies, assuming as a target $Q = 5$ consecutive harmonic frequencies ($\nu = -2, -1, 0, 1, 2$), and a 2-bit coding sequence of length $L = 24$. **a,b)** Comparison between target and synthesized magnitudes and phases, respectively, for an example featuring piecewise asymmetric linear behavior for the magnitude, and linear decreasing behavior for the phase. **c,d)** Same as above, but for an example featuring piecewise symmetric linear behavior for the magnitude, and linear increasing behavior for the phase. The green-shaded regions indicate the targeted spectral range. The total power fraction concentrated within the target region is 71% and 74% in the two examples. Continuous curves are guides to the eye only.

These examples utilize a 2-bit coding sequence of length $L = 24$ and ideal phase jumps of 90° , and focus on synthesizing piecewise linear magnitudes (exhibiting either asymmetric or symmetric profiles) together with linear phase shifts (either increasing or decreasing). The comparison between the targeted and synthesized values, both in magnitude and phase, demonstrates a good agreement within the desired spectral domain (highlighted by a green-shaded region). Additionally, the results for higher harmonic orders (ν up to ± 5), which are not directly controlled, are also shown. These harmonics exhibit relatively small magnitudes, ensuring that the majority of the power (71% Figure 67a and 74% in Figure 67b) remains confined within the targeted spectral range.

Once the problem is decoupled, we proceed to apply these results to the multi-frequency synthesis of harmonic scattering patterns, treating each harmonic frequency independently. We then derive the desired (magnitude and phase) distributions of the equivalent currents for each meta-atom. This approach allows the use of conventional semi-analytical array design techniques [102], eliminating the need for computationally expensive BF optimization methods.

To illustrate the capabilities of our approach, we consider an STC-DM with 16×16 meta-atoms, with period $d = \lambda_c/2$ and the same parameters as previous examples. We target $Q = 5$ representative 2-D scattering patterns with nontrivial magnitude and phase distributions for harmonic orders $\nu = -2, -1, 0, 1, 2$, namely:

1. Tseng–Cheng-type optimal pattern with a SLL of -15 dB [102];
2. OAM beam with topological charge $\ell = 1$ (i. e., constant magnitude and spiral-type phase) [106];
3. Triangular-shaped sector beam synthesized via the Fourier method [102];
4. Golay–Rudin–Shapiro diffuse-scattering pattern (with constant magnitude and binary phase) [15];
5. Cross-shaped sector beam synthesized via the Fourier method [102].

This example of joint synthesis is not related to any specific application. Rather, it is selected to demonstrate the robustness and versatility of our approach in synthesizing diverse scattering patterns, including those requiring either magnitude-phase or phase-only equivalent current distributions.

The semi-analytically obtained magnitude and phase distributions are shown in Figure 68, along with the corresponding scattering patterns. Figure 69 illustrates the synthesized results for the five harmonic frequencies of interest. Specifically, Figure 69a–e display the synthesized magnitude distributions of the complex-valued $a_{nm}^{(\nu)}$, while Figure 69f–j show the corresponding phase distributions, and Figure 69k–o the resulting scattering patterns.

The comparison with Figure 68 highlights the good agreement between the targeted and synthesized results, demonstrating the potential of the approach. Remarkably, three of these scattering patterns, which require non-uniform magnitude distributions, were previously impossible to synthesize with the former semi-analytical approach [78], and have not been demonstrated with the previous ML-based strategy [95]. Therefore, they represent the first examples of joint magnitude-phase multi-frequency syntheses with STC-DMs. The scattering patterns at higher harmonic orders ($|\nu| > 2$) not shown for brevity, consistently exhibit levels below ≈ -8 dB, thereby confirming the increased spectral efficiency.

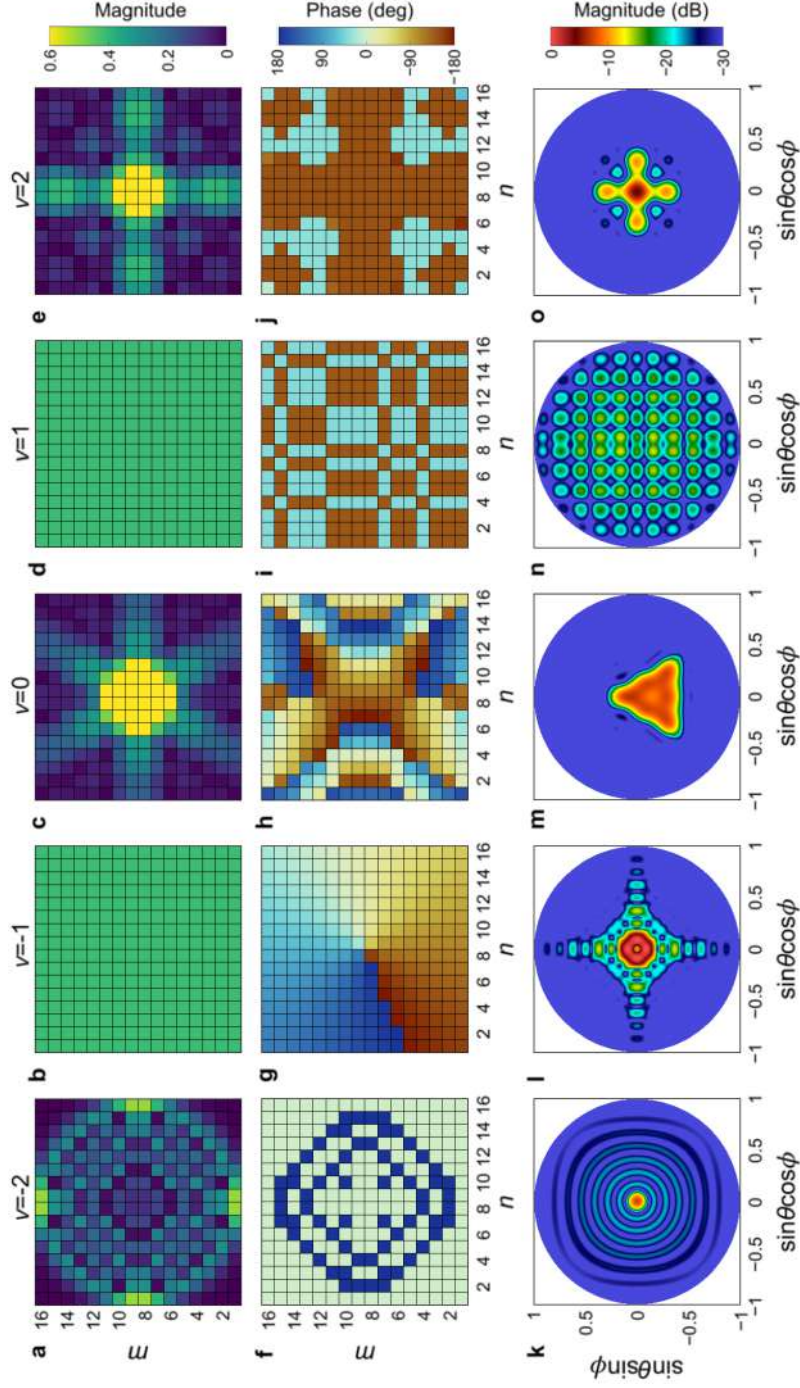


Figure 68: a–e) Target magnitude distributions for the complex-valued equivalent amplitudes $a_{nm}^{(\nu)}$ at harmonic orders $\nu = -2, -1, 0, 1, 2$, respectively. f–j) Corresponding phase distributions. k–o) Corresponding scattering patterns, with intensity normalized to the maximum value at harmonic order $\nu = -1$.

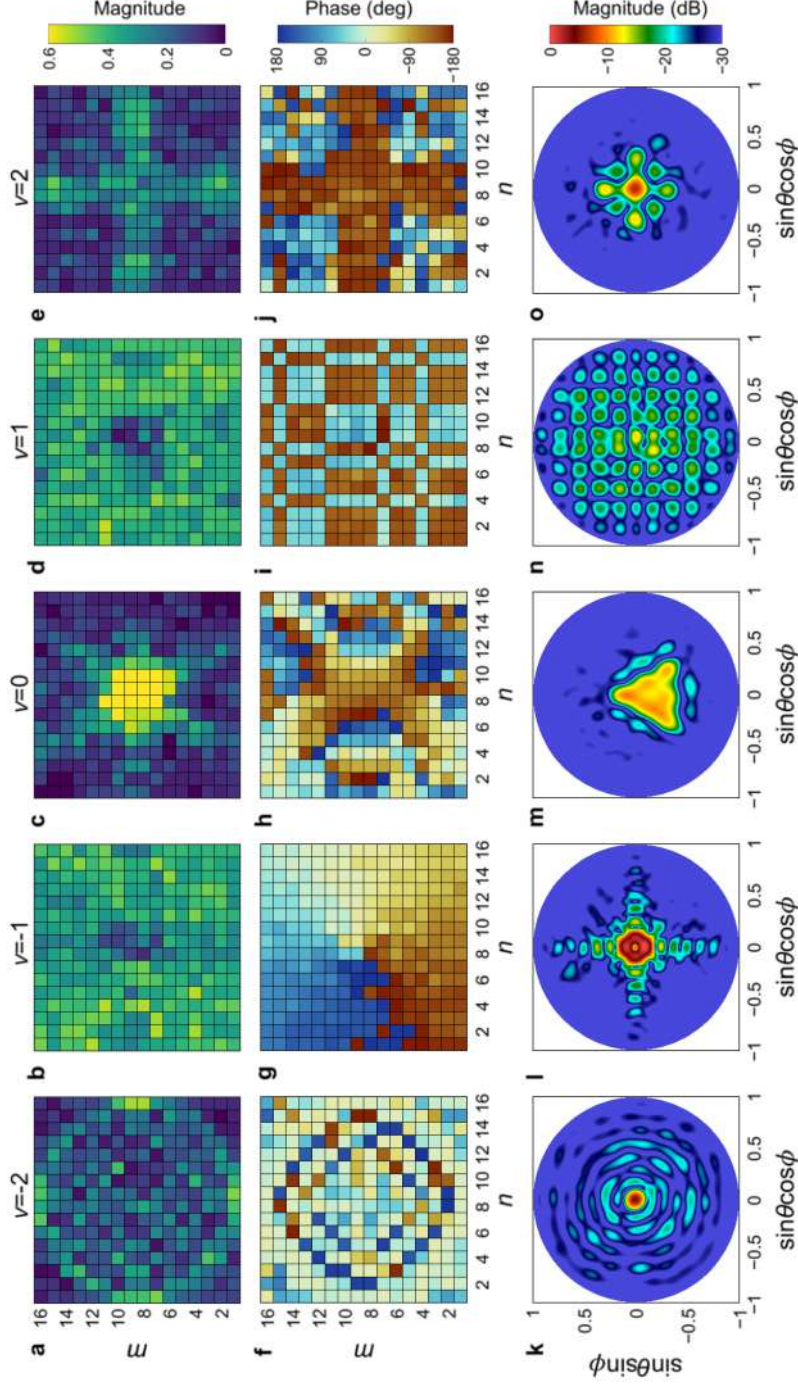


Figure 69: Numerical example of joint syntheses at $Q = 5$ harmonic frequencies, for a STC-DM featuring 16×16 meta-atoms with period $d = \lambda_c/2$, 2-bit coding sequence of length $L = 24$, and ideal 90° phase jumps. **a–e)** Synthesized magnitude distributions for the complex-valued equivalent currents $a_{nm}^{(\nu)}$ at harmonic orders $\nu = -2, -1, 0, 1, 2$, respectively. **f–j)** Corresponding phase distributions. **k–o)** Corresponding scattering patterns, with intensity normalized to the maximum value at the harmonic order $\nu = -1$.

4.3.3.3 Experimental Validation

For our experimental validation, we utilize a 2-bit, X-band STC-DM prototype. This prototype features a layout of 16 columns and 8 connected meta-atoms, as schematized in Figure 71a. Each column shares common control voltages, simplifying the electronics but restricting the beam-shaping and steering to one plane ($\phi = 0$) only. The meta-atom, illustrated in Figure 71b, consists of a hexagonal metal patch on a grounded dielectric substrate, connected via two PIN diodes to biasing lines, with a total unit cell size of $14 \times 14 \text{ mm}^2$. A photograph of the fabricated prototype, measuring $224 \times 140 \text{ mm}^2$, is shown in Figure 71c.

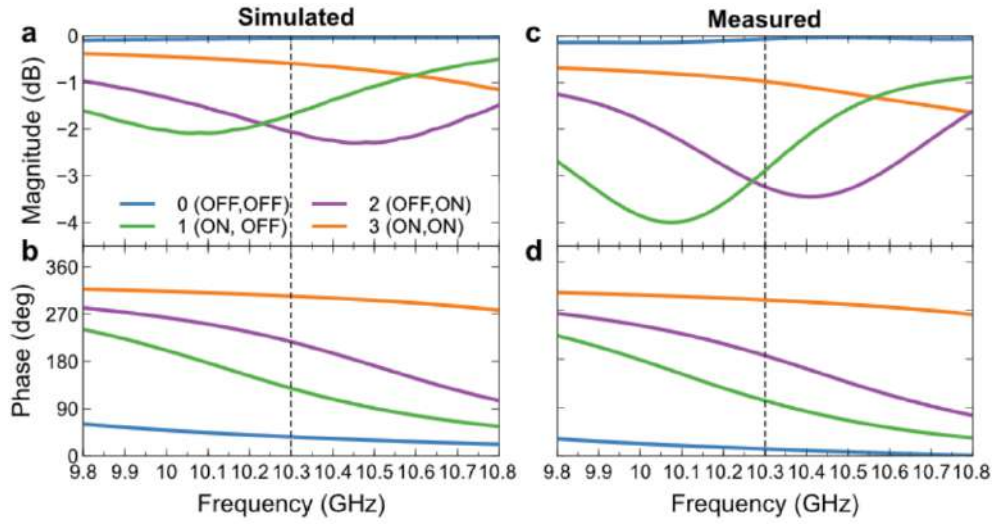


Figure 70: a,b) Simulated magnitude and phase reflection responses, respectively, as a function of frequency, pertaining to the meta-atom in Figure 71b, for the four possible combinations of the PIN diodes' bias states. c,d) Corresponding measured responses. The vertical dashed line indicates the design frequency.

As illustrated in Figure 70, the meta-atom is numerically designed so that, at the operational frequency of 10.3 GHz, the four possible ON/OFF combinations of the two PIN diodes yield reflection phases differing by $\approx 90^\circ$ steps, with reflection magnitudes above $\approx -2 \text{ dB}$. However, from experimental characterization, we found slight discrepancies, with the actual reflection phases of 12° , 102° , 186° , 289° , and corresponding magnitudes of -0.06 , -2.86 , -3.25 , -0.98 dB , due to unmodeled imperfections, approximations, and fabrication tolerances.

The measurement setup, visualized as a photograph in Figure 71d, comprises an anechoic chamber housing a turntable hosting the prototype, along with a feeding antenna connected to a signal generator, and a receiving antenna connected to a spectrum analyzer. Accordingly, we implement our joint syntheses across $Q = 5$ harmonic frequencies ($\nu = -2, -1, 0, 1, 2$). Each of the 16 columns, consisting of 8 connected meta-atoms, is controlled with

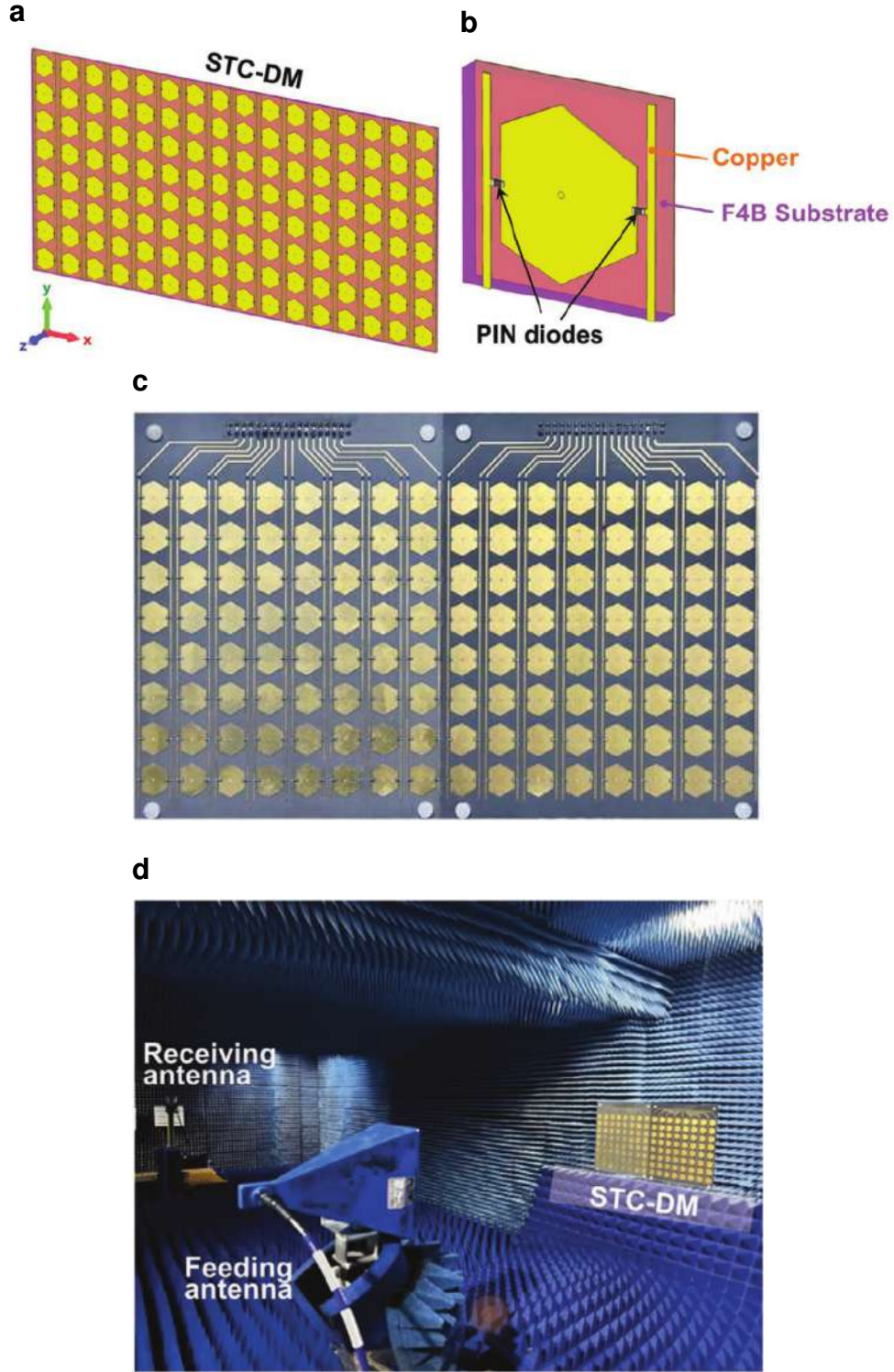


Figure 71: Microwave prototype and measurement setup. **a,b)** Schematics of the STC-DM and meta-atom, respectively. **c,d)** Photographs of the STC-DM prototype and measurement setup, respectively.

a temporal coding sequence of length $L = 24$, having a period $T_0 = 1 \mu\text{s}$ (modulation frequency $f_0 = 1 \text{ MHz}$, corresponding to a diode switching rate of 24 MHz). In this experimental scenario, considering the restrictions imposed by the prototype, we carry out 1-D syntheses, synthesized in a semi-analytical fashion via conventional antenna array design methods:

1. Dolph–Chebyshev pattern with a SLL of -15 dB and a steering angle of -10° [102];
2. Difference pattern with cosine tapering and a steering angle of -30° ;
3. Uniform aperture pattern with a steering angle of 20° ;
4. Four-beam pattern with peaks at $\pm 50^\circ$ and $\pm 15^\circ$ [94];
5. Golay–Rudin–Shapiro diffuse-scattering pattern, with a steering angle of 35° [15];

Figure 72 compares the target and synthesized equivalent current distributions, demonstrating once again a satisfactory agreement. Figure 73 presents a comparison between measured and simulated scattering patterns, considering both ideal and imperfect phase responses. In general, there is good agreement, especially when phase imperfections are considered. Minor discrepancies, such as slightly wider beams and elevated sidelobes in the measured results, can be ascribed to inherent non-idealities and modeling approximations. At the center frequency ($\nu = 0$), moderately larger deviations are observable. Specifically, the effects around the main lobe are mainly due to blockage from the feeding antenna, whereas the speckle at larger angles is attributable to the interference between the STC-DM reflection and the backward radiation from the feeding antenna. Simulated and measured higher-order scattering patterns are shown in Figure 74, consistently exhibiting levels $\approx 10 \text{ dB}$ below the maximum.

These results validate the proposed approach through experimentation, confirming its practical viability.

4.3.4 Additional Results

To further validate the flexibility and robustness of the proposed synthesis framework, this section presents two additional sets of results. First, we demonstrate that the methodology can be effectively extended to non-consecutive harmonic orders, highlighting its versatility beyond previously shown configurations. Then, we explore an alternative implementation of the tandem DNN, which replaces the learned forward model with an analytical counterpart and integrates a more expressive inverse network. These results reinforce the generality of the proposed approach and offer insights into its adaptability to different design choices and constraints.

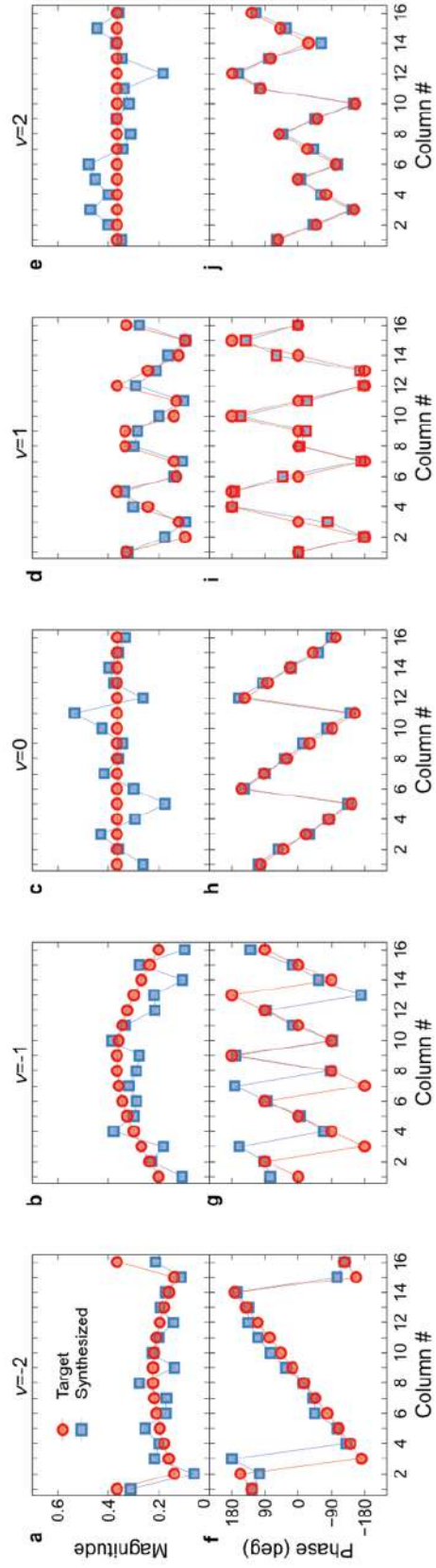


Figure 72: a–e) Comparison between target and synthesized magnitude distributions for the complex-valued equivalent currents $a_{nm}^{(v)}$, as a function of the column number, at harmonic orders $v = -2, -1, 0, 1, 2$, respectively. **f–j)** Comparison for corresponding phase distributions. Continuous curves are guides to the eye only.

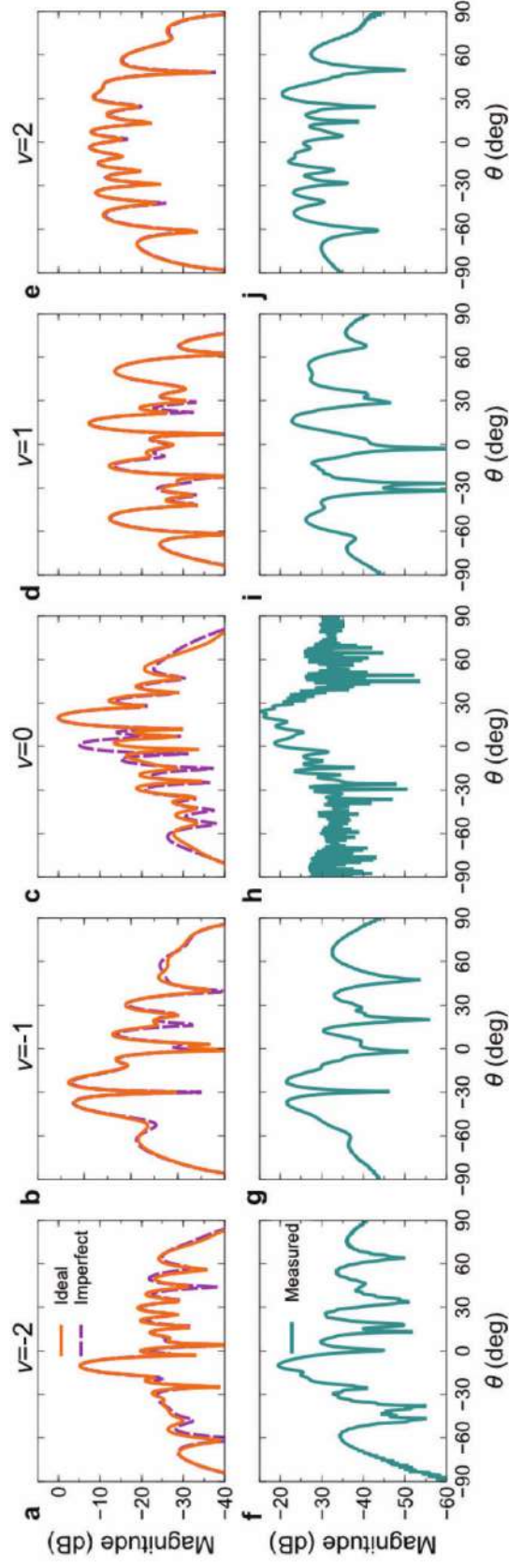


Figure 73: Experimental validation of joint syntheses at $Q = 5$ harmonic frequencies, for a STC-DM featuring 16×8 meta-atoms with period $d = 0.48\lambda_c$ and a 2-bit coding sequence of length $L = 24$. **a–e)** Numerically computed scattering patterns at harmonic orders $\nu = -2, -1, 0, 1, 2$, respectively. Orange-solid and purple-dashed curves pertain to the ideal (90° -step) and imperfect phase responses, respectively. **f–j)** Corresponding measured results at frequencies 10.298, 10.299, 10.3, 10.301, 10.302 GHz, respectively.

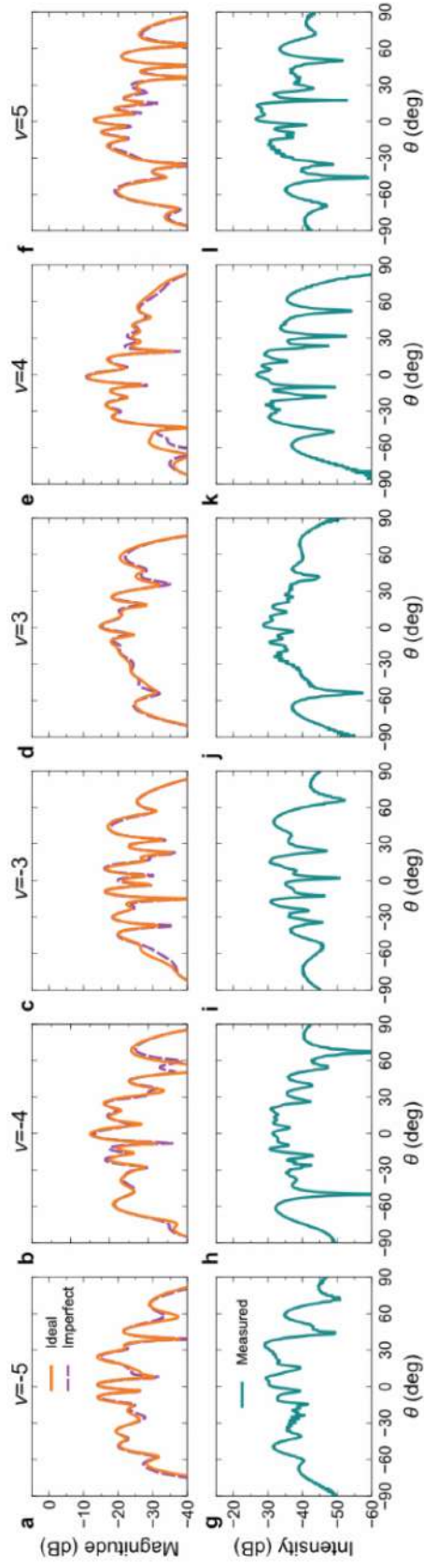


Figure 74: a–f) Numerically computed scattering patterns at harmonic orders $\nu = -5, -4, -3, 3, 4, 5$, respectively. Orange-solid and purple-dashed curves pertain to ideal (90° -step) and imperfect phase responses, respectively. **g–l)** Corresponding measured results at frequencies 10.295, 10.296, 10.297, 10.303, 10.304, 10.305 GHz, respectively.

4.3.4.1 Results for Non-Consecutive Harmonic Orders

While the examples presented so far focus on targeting consecutive harmonic frequencies, our approach can be generalized to accommodate more diverse configurations. For instance, Figure 76 illustrates numerical results for the same target patterns as in Figure 73, but now assuming non-consecutive harmonic orders of interest, specifically $\nu = -2, -1, 0, 1, 3$. The results demonstrate performance comparable to the consecutive case. Additionally, Figure 77 confirms that the scattering patterns at other harmonic orders, including the interior harmonic order $\nu = 2$, maintain levels ≈ 10 dB below the maximum. Once again, this indicates increased flexibility in the design, by comparison with the previous semi-analytical approach [78]. Nevertheless, there are limitations due to the inherent sinc-type decay of the equivalent currents in Equation (44). Specifically, focusing too much power on higher harmonic orders may cause the synthesis to become ill-conditioned, hindering the convergence of the tandem DNN.

4.3.4.2 Alternative Implementation of the Tandem DNN

Here, we present results obtained using an alternative tandem DNN scheme, which implements the forward model analytically and employs a more complex inverse DNN with both convolutional and fully connected layers (see Section 4.3.5 for more implementative details).

Figure 75 illustrates an alternative tandem DNN scheme. Unlike the scheme in Figure 64, this approach directly implements the forward model based on Equation (44), rather than relying on a secondary DNN to simulate this operation. In this way, the overall training process is simplified. Moreover, this strategy ensures perfect fidelity to the theoretical model, thereby avoiding the approximation errors typical of a data-trained DNN and allowing for a direct focus on the error introduced by the inverse DNN.

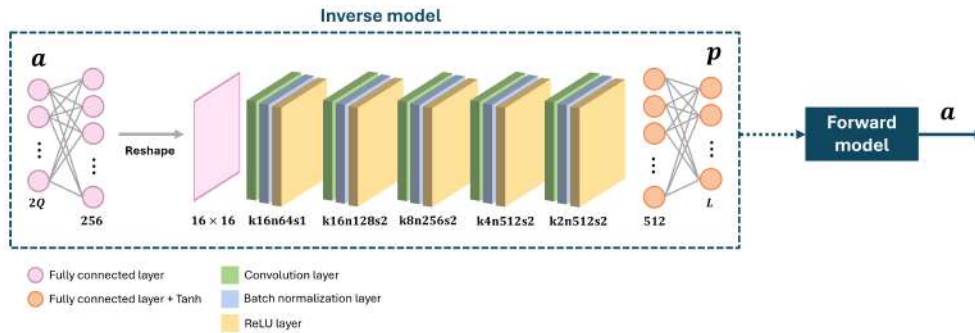


Figure 75: Schematic illustration of the alternative tandem DNN architecture.

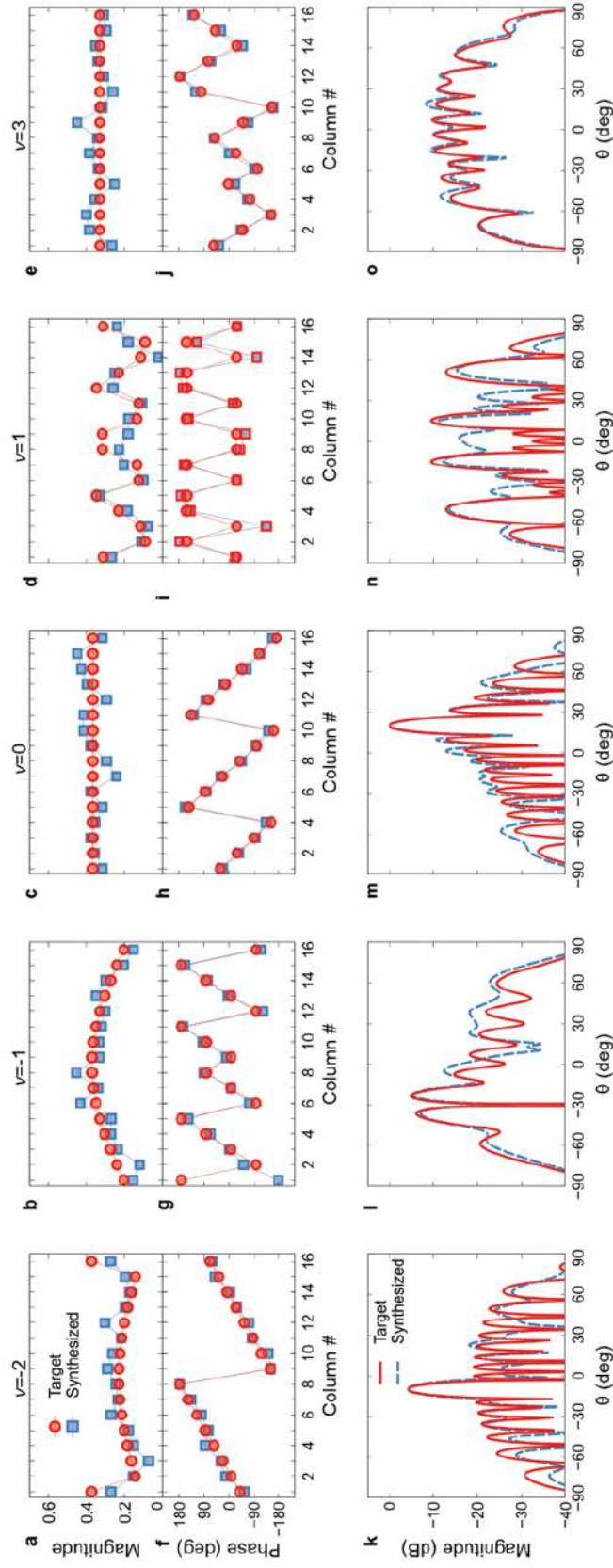


Figure 76: a–e) Comparison between target and synthesized magnitude distributions for the complex-valued equivalent currents $a_{mm}^{(v)}$, as a function of the column number, at harmonic orders $v = -2, -1, 0, 1, 3$, respectively. Continuous curves are guides to the eye only. **f–j)** Comparison for corresponding phase distributions. **k–o)** Comparison for the resulting scattering patterns.

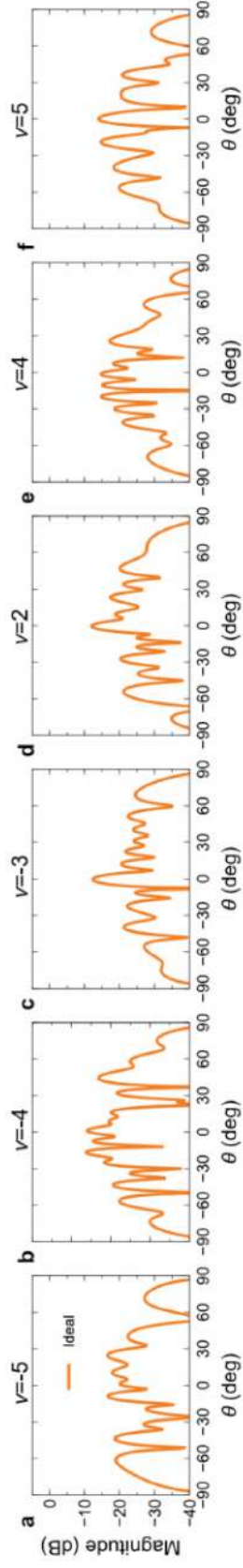


Figure 77: a–f) Numerically computed scattering patterns at harmonic orders $v = -5, -4, -3, -2, -1, 0$, respectively, assuming ideal (90° -step) phase responses.

For the intermediate layers, we use the conventional notation “ $\hat{k}\hat{n}\hat{s}$,” where the convolutional layer has a kernel of size $\hat{k} \times \hat{k}$, $\hat{n}$ channels in the output feature map, and a stride of $\hat{s}$. The data structures for the complex amplitudes $\mathbf{a}$ and the reflection coefficient phases $\mathbf{p}$, along with the generation of the training dataset, the training metric, and phase quantization, are identical to those discussed previously. For notational compactness, we refer to the original tandem DNN scheme (Figure 64) as T-DNN1, and the alternative scheme (Figure 75) as T-DNN2.

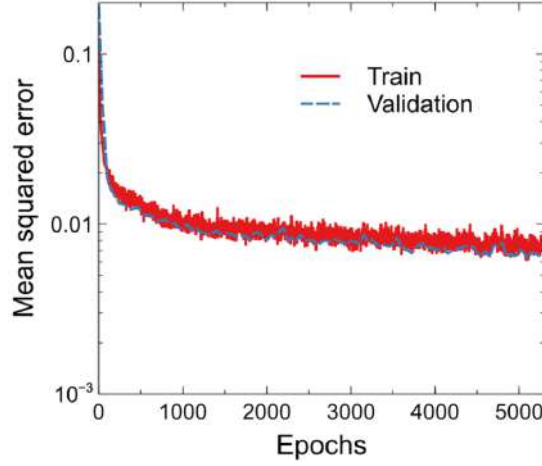


Figure 78: Convergence for T-DNN2 scheme. MSE for a scenario with $Q = 5$ harmonic frequencies ($\nu = -2, -1, 0, 1, 2$) and sequences of length $L = 24$.

Figure 78 illustrates the convergence behavior for T-DNN2 under a typical parameter configuration. Overall, the results are comparable to those observed for T-DNN1 (Figure 65), although T-DNN1 exhibits a smoother error behavior. On the other hand, T-DNN2 does not require the training of the forward model.

Using the same parameter configuration, Figure 79 compares the results of multi-frequency synthesis of complex-valued equivalent currents $a^{(\nu)}$ for a generic meta-atom using both tandem DNN schemes. For a meaningful comparison, the results are presented both without phase quantization in Figure 79a,b, and with a 2-bit phase quantization in Figure 79c,d. As observed, the results are generally comparable, especially in the presence of phase quantization. In general, T-DNN2 tends to offer slightly better spectral efficiency, characterized by lower amplitudes at harmonic orders $|\nu| > 2$.

For the same 1-D synthesis examples shown in Figure 73, Figure 80 compares T-DNN1 and T-DNN2 in terms of equivalent amplitude distributions and scattering patterns for the harmonic orders of interest $\nu = -2, -1, 0, 1, 2$, without phase quantization. Figure 81 provides the comparison for a 2-bit phase quantization under the same scenario. These results confirm that the two tandem DNN schemes produce comparable syntheses, particularly when phase quantization is applied.

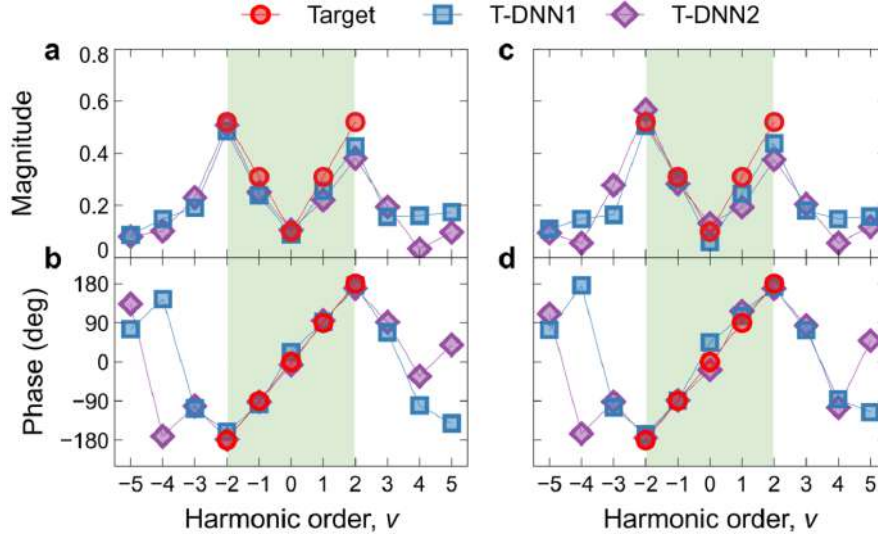


Figure 79: Comparison of joint syntheses, via T-DNN1 and T-DNN2, for the complex-valued equivalent currents $a^{(v)}$ for a generic meta-atom across different harmonic orders assuming as a target $Q = 5$ consecutive harmonic orders ($v = -2, -1, 0, 1, 2$). **a,b**) No phase quantization. **c,d**) 2-bit phase quantization.

Finally, while the main goal was to demonstrate the feasibility and potential of our synthesis method with a simple tandem DNN architecture, further improvements are possible. For example, employing tensor or convolutional layers could enhance the modeling efficiency [104, 107] and increase the number of controllable harmonics. Additionally, constrained implementations [105] may improve the control over spectral efficiency and other system-level metrics. For practical deployment, especially on edge computing devices like FPGAs, a simple, real-time DNN-based encoding scheme is crucial. To accomplish this, the DNN architecture should be kept as simple as possible while still fulfilling the accuracy requirements. Thus, the fully connected architecture in Figure 64 may represent a reasonable compromise.

4.3.5 Experimental Details

MODELING The numerical implementation of the analytic modeling in Equations (44)–(46) was carried out using MATLAB[®] [108]. The design and modeling of the 2-bit meta-atom in the X-band microwave prototype were performed by means of full-wave numerical simulations relying on CST Microwave Studio[®] [109]. In these simulations, a normally incident plane-wave excitation was assumed, port-type terminations along the z -direction were employed, and periodic boundary conditions in the x - y plane were implemented. The PIN diodes were modeled as equivalent circuits, namely an RL circuit for the “ON” state (with $R = 2 \, \Omega$, $L = 200 \, \text{pH}$), and an RLC circuit for the “OFF” state (with $R = 3.3 \, \Omega$, $L = 200 \, \text{pH}$, $C = 52 \, \text{fF}$).

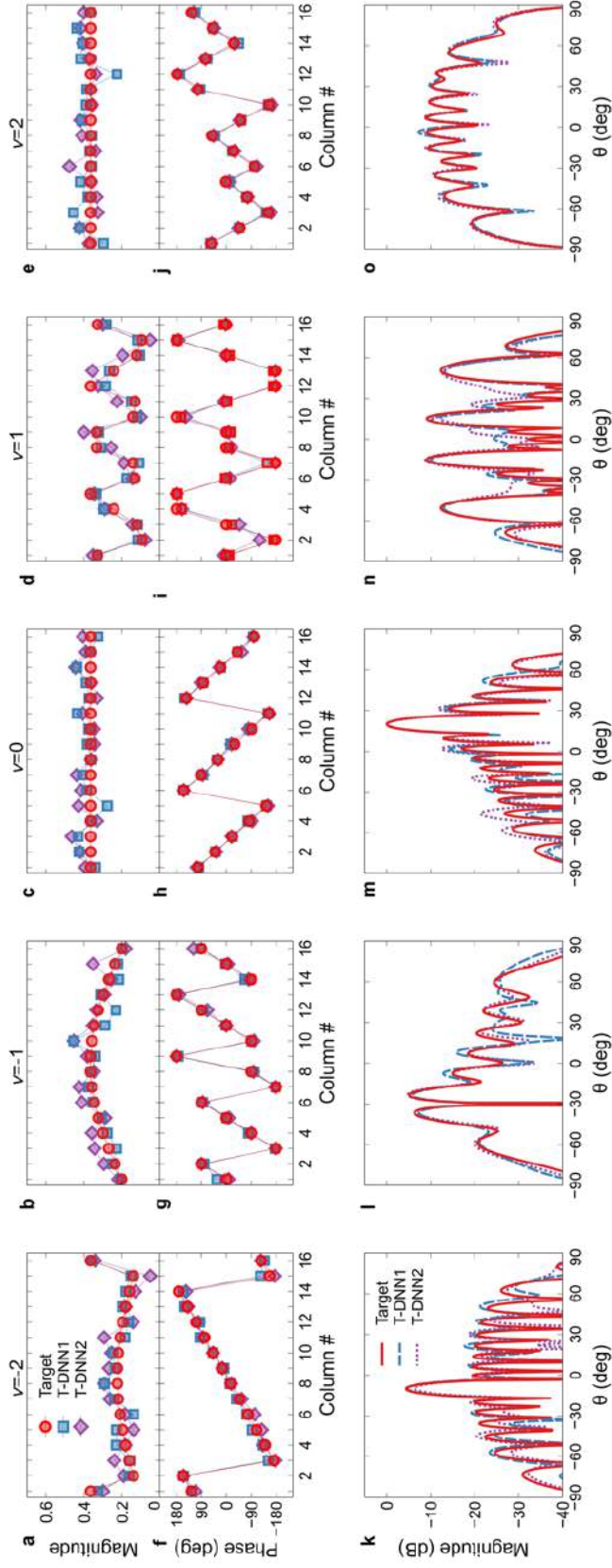


Figure 80: a–e) Comparison between target and synthesized, via T-DNN1 and T-DNN2, magnitude distributions for the complex-valued equivalent currents $a_{lm}^{(v)}$, as a function of the column number, at harmonic orders $v = -2, -1, 0, 1, 2$, respectively, assuming no phase quantization. Continuous curves are guides to the eye only. **f–j)** Comparison for corresponding phase distributions. **k–o)** Comparison for resulting scattering patterns.

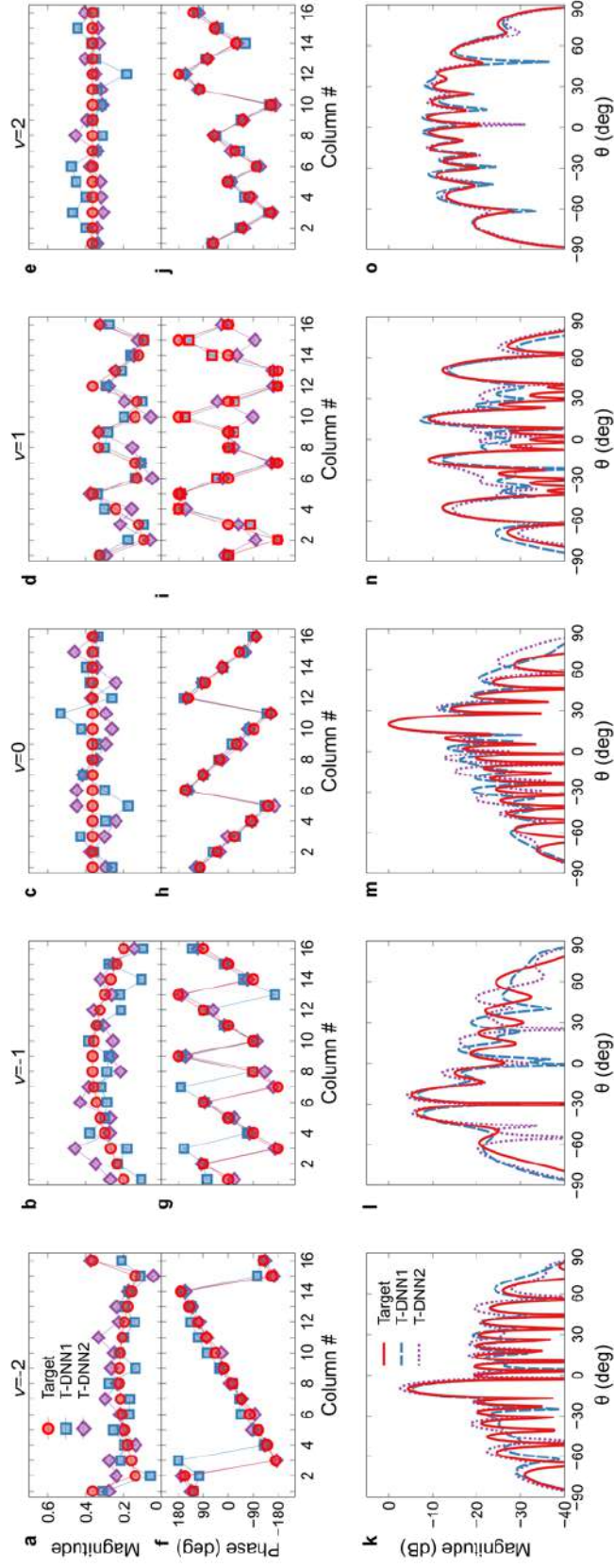


Figure 81: a–e) Comparison between target and synthesized, via T-DNN1 and T-DNN2, magnitude distributions for the complex-valued equivalent currents $a_{nm}^{(v)}$, as a function of the column number, at harmonic orders $v = -2, -1, 0, 1, 2$, respectively, assuming a 2-bit phase quantization. Continuous curves are guides to the eye only. **f–j)** Comparison for corresponding phase distributions. **k–o)** Comparison for resulting scattering patterns.

TANDEM DNN IMPLEMENTATION The fully connected tandem DNN was implemented using the `fitnet` library in MATLAB[®] [110]. Both the forward and inverse DNNs were composed of two hidden layers. Specifically, the first hidden layer of the forward DNN comprised $10L$ neurons, while the second hidden layer contained $20Q$ neurons. Radial basis functions were employed as activation functions for both DNNs, while the output layers utilized the Elliot sigmoid function [111]. Regarding the training algorithm, the resilient back-propagation method [112] was used, as it is particularly well-suited for sigmoid-type activation functions. For both forward and inverse DNNs, the datasets were divided as follows: 70% for training, 15% for validation, 15% for testing. The training process typically required 3 hours on a personal computer equipped with an Intel Core i7-8550U CPU (@ 1.80–2.00 GHz) and 8 GB of RAM. In the final phase quantization step, a nearest-neighbor search was performed, utilizing the degree of freedom in the reference phase to minimize errors.

ALTERNATIVE TANDEM DNN IMPLEMENTATION The alternative tandem DNN scheme was implemented and tested using the functionalities provided by the `dlnetwork` library in MATLAB[®] [113]. In this scheme, the inverse model adopts a more complex architecture, as summarized in Table 1. Each hidden layer comprises a convolutional layer, followed by a batch normalization layer and a Rectified Linear Unit (ReLU) as activation function. The layers are listed in the table in the same order in which they appear in the network.

Table 1: Architecture of the alternative inverse model using a CNN.

Layer	Description
Input	$2Q$ features
Fully connected	256 neurons
Reshape	16×16 feature map
Hidden ($5 \times$)	Conv [†] , BatchNorm [‡] , ReLU activation
Fully connected	512 neurons, tanh activation
Output	L features, tanh activation

[†] Convolutional layer

[‡] Batch normalization layer

For the training algorithm, we employ the ADAM algorithm [114] with the following parameters: learning rate $\alpha = 0.001$, gradient decay factor $\beta_1 = 0.9$, squared gradient decay factor $\beta_2 = 0.999$, denominator offset $\epsilon = 10^{-8}$. The dataset is split into two partitions: 80% for training and 20% for validation.

PROTOTYPE REALIZATION The fabrication of the STC-DM prototype in Figure 71c was carried out by means of standard printed-circuit-board technology, utilizing a 1.5 mm-thick F4B substrate with a dielectric constant of $\epsilon_r = 2.65$ and a loss tangent of $\tan \delta = 0.001$. Each meta-atom integrates two embedded PIN diodes (MACOM MADP-000907-14020x) connected to the biasing lines. To implement dynamic biasing, an FPGA hardware control board (ALTERA Cyclone IV) was employed. This board was pre-programmed with a computer code designed to generate control signals according to the desired STC matrices.

EXPERIMENTS The experimental setup shown in Figure 71d comprised a linearly polarized X-band horn antenna, employed as a transmitter, and a signal generator (Keysight E8257D) providing the carrier microwave signal. The transmitter and STC-DM prototype were coaxially mounted on a rotary support at a distance of 0.85 m, enabling a 360° rotation in the horizontal plane with a precision of 0.1° . To measure the harmonic scattering patterns, an identical horn antenna was utilized as a receiver and connected to a spectrum analyzer (Keysight E4447A). The characterization of the meta-atom's reflection response was performed using a vector network analyzer (Keysight N5230C).

4.3.6 Summary and Outlook

We have put forward a novel ML-based method for the multi-frequency synthesis of STC-DMs, which utilizes a tandem DNN architecture to map desired equivalent-current distributions at different harmonic frequencies directly onto the coding sequences of a generic meta-atom. This effectively decouples the synthesis problem, enabling the use of conventional semi-analytical array design techniques for harmonic beam shaping and steering, with precise control over magnitude and phase distributions.

Within the emerging framework of STC-DMs, our approach stands out as a remarkable integration of ML-based and semi-analytical techniques. It enables the synthesis of complex scattering patterns that were previously unattainable with existing design methodologies while maintaining a computational complexity that is essentially independent of the STC-DM electrical size. This advantage essentially arises from synthesizing the equivalent currents across harmonic frequencies for individual meta-atoms, rendering the computational complexity of the tandem DNN independent of the size of the STC-DM. Scattering pattern synthesis is then effectively carried out using traditional array design techniques. Although our examples focus on semi-analytical approaches, numerical methods [115] remain computationally feasible thanks to the decoupling strategy and could be employed for each individual frequency component. This stands in contrast to previous ML-based methods that directly target scattering pattern synthesis [95], which results in a computational complexity that scales unfavorably with

the STC-DM size. Another key advantage is that our DNN training depends solely on the targeted harmonic frequencies and coding-sequence length and hence it remains valid even if the desired scattering pattern or STC-DM size changes, eliminating the need for retraining.

The promising results of our approach have been experimentally validated using a 2-bit, X-band STC-DM prototype, paving the way for significant advancements in applications such as smart radio environments for upcoming wireless networks [116, 117], as well as in sensing and imaging technologies [118, 119]. Within this framework, the proposed ML-based approach can be seamlessly integrated with system-level, artificial-intelligence-driven strategies tailored for smart radio environments [120, 121]. Another promising direction is the investigation of rapid switching mechanisms, such as those leveraging graphene or vanadium dioxide, to enable operations at higher frequencies, such as millimeter-wave and terahertz bands [122, 123], where the use of PIN diodes is not viable.

DIAGNOSTICS OF SPACE-TIME-CODING DIGITAL METASURFACES

5.1 BACKGROUND AND GENERALITIES

The diagnostics of metasurfaces, whether configured as reflectarrays or RISs, plays a critical role in ensuring optimal performance in practical deployment scenarios. As these structures are increasingly integrated into advanced wireless communication systems, maintaining their operational integrity becomes essential. In particular, modern diagnostic techniques have shifted toward data-efficient and model-aware strategies, leveraging methods such as sparse recovery, Compressed Sensing (CS), and DL. These approaches aim to identify faults or anomalies at the level of individual elements within the array by analyzing far-field radiation measurements. By detecting disruptions such as amplitude degradation, phase errors, or complete element failure, these techniques enable real-time monitoring and correction, thus preserving the metasurface's intended EM functionality.

Recent advancements in diagnostics have focused on model-based inversion techniques, CS frameworks, ML approaches, all of which aim to identify local defects using only partial observations—typically in the far-field domain. These methods are particularly well-suited to metasurfaces, where faults are often sparse (i.e., only a few elements are defective at any given time) and measurements are costly or limited in resolution.

5.1.1 *Sparse Regularization and Inverse Modeling*

A commonly used technique for antenna diagnostics is the back-propagation algorithm [124]. This method relies on the Fourier relationship between the tangential components of the electric field distribution on the antenna array surface, $E(x, y)$, and the radiated far field, $F(r, \theta, \phi)$. As is well known, the spatial resolution on the array aperture is practically limited to $\lambda_c/2$. Using fewer unknowns than this limit would result in an undersampling of the aperture field, compromising reconstruction accuracy. Consequently, for an array of dimensions $2N_x \times 2N_y$, the minimum number of unknowns required is $N_u \approx N_x N_y / \lambda_c^2$. Using a larger number of unknowns does not improve the accuracy of the reconstruction; rather, it leads to an oversampling of a band-limited function, which results in an interpolation of the available data.

An alternative technique for identifying faults in large antenna arrays is the Matrix Method Algorithm (MMA) [125, 126]. This approach exploits the linear relationship between the sampled values of the aperture field $\mathbf{x} \in \mathbb{C}^N$ and those of the far-field pattern $\mathbf{y} \in \mathbb{C}^M$, where $k_x = k_0 \sin \theta \cos \phi$,

$k_y = k_0 \sin \theta \sin \phi$, and $k_0 = 2\pi/\lambda_c$. The mapping is described by a system matrix $\mathbf{A}$, such that $\mathbf{y} = \mathbf{A}\mathbf{x}$. However, the matrix $\mathbf{A}$ is typically ill-conditioned, which necessitates the use of regularization techniques to stabilize the inversion process and ensure a meaningful solution [127].

Building on this class of inverse approaches, Ref. [128] presents a model-driven diagnostic technique that leverages prior knowledge of the ideal (i. e., fault-free) radiation pattern of the array. By subtracting the expected far-field response from the measured one, the diagnostic problem is reformulated to highlight only the local discrepancies introduced by faulty elements. This differential formulation transforms the original problem into an equivalent one in which radiation is attributed solely to the region affected by the failure-induced field perturbation. As a result, the method achieves higher accuracy and reduced computational complexity by focusing the analysis on a limited portion of the aperture. In particular, the tangential field distribution $\mathbf{E}(x, y)$ and its corresponding far field pattern $\mathbf{F}(r, \theta, \phi)$ are given by the difference between the distributions of the reference antenna and the antenna under test

$$\mathbf{E}(x, y) = \mathbf{E}^{(r)}(x, y) - \mathbf{E}^{(d)}(x, y), \quad (103a)$$

$$\mathbf{F}(r, \theta, \phi) = \mathbf{F}^{(r)}(r, \theta, \phi) - \mathbf{F}^{(d)}(r, \theta, \phi). \quad (103b)$$

The reconstruction task is then solved by minimizing a sparsity-promoting objective function:

$$\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2 + \alpha_{\text{reg}} \mathcal{R}(\mathbf{x}), \quad (104)$$

where $\mathbf{x}$ represents the spatial distribution of anomalies, $\mathbf{A}$ is a forward operator mapping anomalies to far-field deviations, $\mathbf{y}$ is the difference between the observed and nominal field responses, α_{reg} is a coefficient that controls the relative contribution of the regularization term and $\mathcal{R}(\mathbf{x})$ is a regularization function (e. g., L_1 -norm, or mixed norms) enforcing sparsity or some *a priori* knowledge in the solution.

Compared to classical back-propagation or matrix inversion techniques, this approach significantly reduces the number of required measurements and offers faster convergence in real-time settings, making it highly suitable for large and reconfigurable metasurfaces. The reader is referred to Ref. [128] for further details and numerical examples that validate this model.

5.1.2 Compressed Sensing-Based Diagnostics

In Ref. [129], a diagnostic technique is proposed for millimeter-wave arrays affected by partial or complete blockages caused by external agents (e. g., dust, water, fingers). The impact of these blockages is modeled as attenuation and phase variation on the incident signal. Considering an $N \times M$ re-

flectarray, with an antenna spacing of d , without any temporal modulation, the model focuses on the far-field scattering pattern:

$$F(\theta, \phi) = \sum_{n=1}^N \sum_{m=1}^M w_{nm} e^{j\frac{2\pi}{\lambda_c} d(n-1) \sin \theta \cos \phi} e^{j\frac{2\pi}{\lambda_c} d(m-1) \sin \theta \sin \phi}, \quad (105)$$

where λ_c is the operating wavelength, and w_{nm} is the (n, m) -th complex antenna weight. Equation (105) can be reduced to

$$F(\theta, \phi) = \text{vec}(\mathbf{W})^T \boldsymbol{\Psi}(\theta, \phi), \quad (106)$$

where $\text{vec}(\mathbf{W})$ is the $N \times M$ column vector obtained by stacking the columns of the matrix $\mathbf{W}$ on top of one another, the vector $\boldsymbol{\Psi}(\theta, \phi) = \boldsymbol{\psi}_x(\theta, \phi) \otimes \boldsymbol{\psi}_y(\theta, \phi)$ is the 1-D array response vector, where the n -th component of $\boldsymbol{\psi}_x(\theta, \phi)$ and the m -th component of $\boldsymbol{\psi}_y(\theta, \phi)$ are given by the two exponential terms in Equation (105), respectively, while the operator $\otimes$ denotes the Kronecker product. This formulation enables the reduction of the 2-D array to an equivalent 1-D representation, thereby simplifying the overall problem structure.

In presence of blockages, the far-field scattering pattern of the array in Equation (105) becomes

$$F(\theta, \phi) = \text{vec}(\mathbf{W})^T (\mathbf{b} \circ \boldsymbol{\Psi}(\theta, \phi)), \quad (107)$$

where the operator $\circ$ represents the Hadamard product, and the n -th component of the vector $\mathbf{b} \in \mathbb{C}^{NM}$ is defined by

$$b_n = \begin{cases} \varsigma_n, & \text{if the element is blocked,} \\ 1, & \text{otherwise,} \end{cases} \quad (108)$$

where $\varsigma_n = \rho_n \exp(j\Phi_n)$, with $0 \leq \rho_n \leq 1$ and $0 \leq \Phi_n \leq 2\pi$ are the resulting absorption and scattering coefficients of the n -th antenna. A value of $\rho_n = 0$ represents maximum blockage at the n -th element, and the Φ_n coefficient measures the phase-shift caused by the n -th element. Under this model, b_n is treated as a random variable that takes the value $b_n = \varsigma_n$ with probability P_b when the n -th antenna is blocked, and $b_n = 1$ with probability $1 - P_b$ otherwise. From Equation (107), it is evident that blockages will change the array scattering pattern. For further details and validating numerical examples, the reader is referred to Ref. [129].

5.1.3 Data-Driven Diagnostics via Deep Learning

In Ref. [130], a purely data-driven method is introduced, where a deep CNN is trained to identify faults in antenna arrays directly from far-field radiation patterns. The proposed architecture includes multiple blocks of convolution, batch normalization, and activation layers, followed by fully connected layers to perform fault classification. This DL framework offers a number of advantages:

- It captures *nonlinear relationships* between radiation patterns and faults that are difficult to model analytically;
- It is robust to noise and measurement imperfections;
- Once trained, the model enables *instantaneous inference*, suitable for real-time diagnostics.

This method is particularly promising for RIS applications, where the metasurface is often dynamic and the relationship between control states and observed patterns can be highly complex. The flexibility and scalability of DL make it a compelling solution for large-scale reconfigurable systems.

For a detailed description of the architecture, training methodology, and numerical results, the reader is referred to Ref. [130].

5.2 DIAGNOSTICS VIA ORTHOGONAL CODES

Common methods for diagnosing RISs focus on the signal level [129, 131]. These approaches leverage the statistical properties of transmitted and received signals, along with channel state information, in order to assess the operational status (normal or faulty) of individual elements. This is typically accomplished using CS-based algorithms, which often involve iterative procedures and advanced signal-processing techniques. These methods are commonly integrated into core communication tasks, such as channel estimation [131], resulting in significant computational complexity. Additionally, most of these techniques have been validated primarily through theoretical models or simulations, limiting their experimental validation.

On the other hand, hardware-based diagnostics methods evaluate RIS performance by directly measuring physical parameters, such as far-field [128, 130, 132] or near-field [133] radiation patterns. These methods detect faulty elements by comparing measured data from the tested structure with reference data from a fault-free configuration. However, they require prior measurements of fully functioning structures and highly accurate data collection at specific locations. Furthermore, they typically assume fault sparsity—meaning that only a few elements are defective—which limits their effectiveness in cases of more extensive failures.

To address these limitations, we propose a novel RIS diagnostic method based on STC modulation using orthogonal codes. This approach encodes the reflections of individual elements into distinct, orthogonal channels via time-varying control signals. Elements that fail to correctly modulate the signal leave their corresponding code channels empty, enabling straightforward fault identification. Unlike previous methods, our approach eliminates the need for complex algorithms or iterative measurement processes. It offers several practical benefits, including rapid testing, ease of implementation, minimal computational demand, and robust performance—making it

especially suitable for applications like factory inspections and routine RIS maintenance. The results below are also documented in Ref. [134].

5.2.1 Theory and Modeling

Figure 82 illustrates the core concept behind the proposed RIS diagnostic approach, which leverages spatio-temporal modulation. This modulation is achieved by dynamically controlling the state of reconfigurable elements through an FPGA. As depicted on the left side of the figure, the RIS modulation is represented by a 3-D STC matrix based on orthogonal codes. When the RIS is illuminated by a monochromatic wave, the reflected signal is captured and analyzed to extract the power associated with each code channel. Faulty elements, which fail to modulate correctly, result in significantly lower power in their corresponding channels. By examining the power distribution across these channels, faulty elements can be precisely identified. The next section provides a detailed theoretical model of the RIS diagnostic procedure.

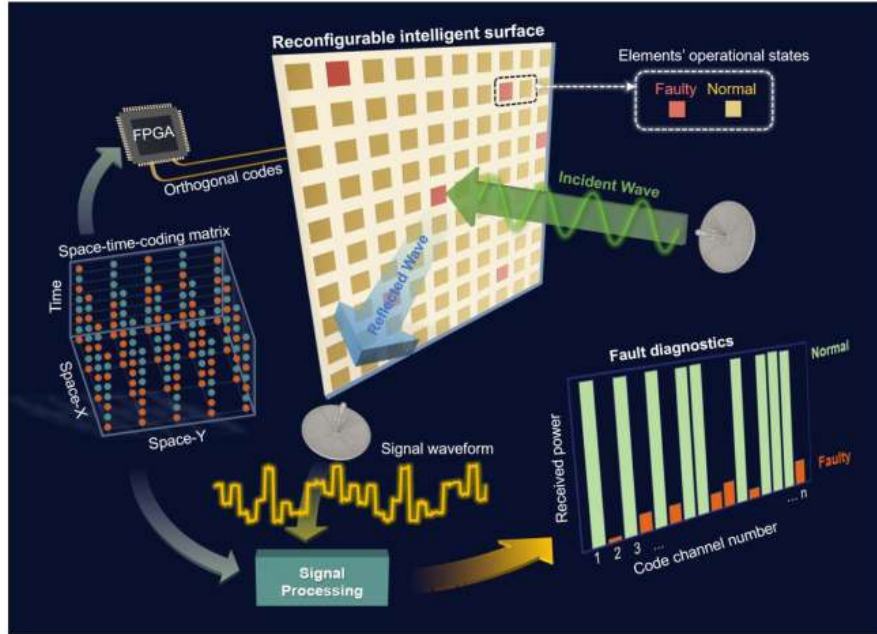


Figure 82: Conceptual illustration of the RIS diagnostics modulation strategy. Each coding element is assigned a unique orthogonal Hadamard sequence, enabling rapid fault detection by analyzing the recovered power associated with each code channel in the received signal.

5.2.1.1 Coding Strategy

As the RIS under test, we consider a 1-bit coding STC-DM comprising N independent coding elements. An independent coding element refers to a

group of meta-atoms that share the same control signal from the FPGA control module—for example, all meta-atoms in the same column in a column-controlled STC-DM, or a single meta-atom in a point-controlled STC-DM. Each coding element is modulated using a distinct Hadamard code, which is commonly employed in Code-Division Multiple Access (CDMA) communication systems and signal processing applications [135]. These codes are obtained from the rows of a Hadamard matrix $\mathbf{H}_L$, where L denotes the code order. Due to their orthogonality, Hadamard codes are particularly well-suited for this diagnostic method. An L -order Hadamard matrix is recursively defined as follows:

$$\mathbf{H}_L = \begin{pmatrix} \mathbf{H}_{L/2} & \mathbf{H}_{L/2} \\ \mathbf{H}_{L/2} & -\mathbf{H}_{L/2} \end{pmatrix}, \quad (109)$$

$$\mathbf{H}_1 = 1.$$

Equation (109) defines an orthogonal matrix whose rows constitute mutually orthogonal Hadamard codes. These codes, composed solely of ± 1 elements, can be directly mapped to binary phase states (0 or π) in complex notation. This discrete phase representation aligns naturally with the capabilities of 1-bit coding STC-DMs, where the 90° -phase-shift functionality enables straightforward implementation of the codes.

However, not all Hadamard codes are suitable for this application. Specifically, the first row of a Hadamard matrix, as shown in Equation (109), consists entirely of $+1$ entries—implying no temporal modulation. Consequently, coding elements assigned this sequence would produce no observable difference between functional and faulty states, aside from a 90° -phase offset, which is impractical to detect in most scenarios. To overcome this limitation, the first row of the Hadamard matrix is omitted, and each n -th coding element is instead assigned the $(n+1)$ -th row. When all coding elements are modulated simultaneously, the minimum required order of the Hadamard matrix is given by:

$$L_{\min} = 2^{\lceil \log_2(N+1) \rceil}, \quad (110)$$

where $\lceil \cdot \rceil$ denotes the ceiling operator. Consequently, as N increases, higher-order Hadamard codes are required, resulting in increased computational resource demands. A practical strategy to mitigate this issue is to partition the RIS into smaller segments and perform the diagnostic procedure on each segment sequentially.

To implement Hadamard code modulation in the time domain, the control sequence $\mathbf{c}_n$ in the n -th element of the STC-DM must satisfy

$$c_n^{(l)} = \begin{cases} 0, & \mathbf{H}_L(n+1, l) = -1, \\ 1, & \mathbf{H}_L(n+1, l) = +1, \end{cases} \quad (111)$$

where $c_n^{(l)}$ denotes the bit symbol of $\mathbf{c}_n$ in the l -th time interval. Under the control sequence $\mathbf{c}_n$, the reflection coefficient of the n -th elements during the l -th time interval is given by

$$\begin{aligned}\Gamma_n^{(l)} &= A_e e^{j(\Phi_0 + c_n^{(l)} \pi)} \\ &= -A_e e^{j\Phi_0} \mathbf{H}_L(n+1, l),\end{aligned}\quad (112)$$

where A_e represents the reflection amplitude, Φ_0 is the reflection phase of state “0,” and a time-harmonic dependence $\exp(j2\pi f_c t)$ is assumed. Given that the symbol duration in the modulation sequence is τ , the time-varying reflection coefficient $\Gamma_n(t)$ of an element controlled by the sequence in Equation (109) can be expressed as:

$$\begin{aligned}\Gamma_n(t) &= \sum_{l=1}^L \Gamma_n^{(l)} W^{(l)}(t) \\ &= -A_e e^{j\Phi_0} \sum_{l=1}^L \mathbf{H}_L(n+1, l) W^{(l)}(t),\end{aligned}\quad (113)$$

where $W^{(l)}(t)$ is the same window function defined in Equation (40).

5.2.1.2 Signal Model

Assuming a monochromatic plane wave at frequency f_c impinging on the RIS from the direction (θ_i, ϕ_i) , the received signal at a receiver located at (θ_o, ϕ_o) is given by

$$\mathbf{r}(t) = e^{j2\pi f_c t} \sum_{n=1}^N \Gamma_n(t) E_n^{(i)} E_n^{(o)} \psi_n(\theta_i, \phi_i) \psi_n(\theta_o, \phi_o) + \mathbf{n}(t), \quad (114)$$

where $E_n^{(i)}$ and $E_n^{(o)}$ represent the angular response of the n -th element in the incident and outgoing directions, respectively, $\mathbf{n}(t)$ is additive Gaussian white noise, and $\psi_n(\theta, \phi)$ is the steering factor of the n -th element, given by

$$\psi_n(\theta, \phi) = e^{j\frac{2\pi}{\lambda_c} (n-1) \sin \theta (d_x \cos \phi + d_y \sin \phi)}, \quad (115)$$

where d_x and d_y denote the dimensions of the n -th coding element along the x and y directions, respectively. Equation (114) can be discretized with a time interval τ as

$$\mathbf{r}^{(l)} = \sum_{n=1}^N \Gamma_n^{(l)} E_n^{(i)} E_n^{(o)} \psi_n(\theta_i, \phi_i) \psi_n(\theta_o, \phi_o) + \mathbf{n}^{(l)}. \quad (116)$$

For a properly functioning element, $\Gamma_n(t)$ exhibits a periodic behavior following an L -order Hadamard code, as defined in Equation (113). In contrast, for a malfunctioning element, the reflection coefficient becomes time-invariant, i.e., $\Gamma_n(t) = A_e \exp(j\Phi_0)$, establishing a direct structural correspondence with $\mathbf{H}_L(1, \cdot)$ —a row consisting solely of +1 entries—through the decomposition

$$\Gamma_n(t) = A_e e^{j\Phi_0} \sum_{l=1}^L \mathbf{H}_L(1, l) W^{(l)}(t). \quad (117)$$

Equation (117) highlights that the reflected signal components from faulty elements remain orthogonal to those of properly functioning ones, owing to the intrinsic orthogonality of $\mathbf{H}_L$. This inherent energy separation guarantees that faulty elements do not interfere with the code channel energy of operational elements, thereby enabling accurate and unambiguous RIS diagnostics.

5.2.1.3 RIS Diagnostics

At the receiving terminal, the signal power of the m -th coding channel, $\hat{P}_m$, is recovered as

$$\hat{P}_m = \left| \frac{1}{L} \sum_{l=1}^L \mathbf{H}_L(m+1, l) \mathbf{r}^{(l)} \right|^2. \quad (118)$$

Substituting Equation (116) into Equation (118) and interchanging the summation order, we obtain

$$\hat{P}_m = \left| \frac{1}{L} \sum_{n=1}^N E_n^{(i)} E_n^{(o)} \psi_n(\theta_i, \phi_i) \psi_n(\theta_o, \phi_o) \sum_{l=1}^L \mathbf{H}_L(m+1, l) \Gamma_n^{(l)} + \mathbf{n}^{(l)} \right|^2. \quad (119)$$

Analyzing Equation (119) and defining the following auxiliary term:

$$\Xi_{nm} = \frac{1}{L} \sum_{l=1}^L \mathbf{H}_L(m+1, l) \Gamma_n^{(l)}, \quad (120)$$

we can distinguish two cases:

- Case 1: The m -th element is normally functioning, i. e., the coding sequence is $\Gamma_m^{(l)} = -A_e \exp(j\Phi_0) \mathbf{H}_L(m+1, l)$. In this case, using Equation (117) and the orthogonality of $\mathbf{H}_L$, we obtain

$$\Xi_{nm} = \begin{cases} 0, & n \neq m, \\ -A_e e^{j\Phi_0}, & n = m, \end{cases} \quad (121)$$

Thus, the recovered code channel power is

$$\hat{P}_m = \left| A_e E_m^{(i)} E_m^{(o)} \psi_m(\theta_i, \phi_i) \psi_m(\theta_o, \phi_o) \right|^2 + P_n, \quad (122)$$

where P_n denotes the noise power.

- Case 2: The m -th element is faulty, i. e., $\Gamma_m^{(l)} = A_e \exp(j\Phi_0) \mathbf{H}_L(1, l)$. In this case, using Equation (117) and the orthogonality of $\mathbf{H}_L$, we obtain

$$\Xi_{nm} = 0, \quad (123)$$

which leads to

$$\hat{P}_m = P_n. \quad (124)$$

By comparing Equations (122) and (124), it follows that in high Signal-to-Noise Ratio (SNR) conditions, an appropriate power threshold P_{thr} can be established to assess the state of the m -th element. Specifically, if $\hat{P}_m \geq P_{\text{thr}}$, the element is classified as normally functioning; otherwise, if $\hat{P}_m < P_{\text{thr}}$, it is identified as faulty. Although the angles influence the absolute intensity $|A_e E_n^{(i)} E_n^{(o)} \psi_n(\theta_i, \phi_i) \psi_n(\theta_o, \phi_o)|^2$, the diagnostic decision is based on the relative difference in intensity between functioning and faulty elements, ensuring robust and consistent fault identification.

In summary, the proposed diagnostic procedure includes the following steps:

1. Modulate each coding element with a distinct Hadamard code as defined in Equations (109)–(113);
2. Illuminate the RIS under test with a monochromatic plane wave and collect the reflected signal;
3. Compute the power associated with each code channel using Equation (118);
4. Determine the state of the m -th element based on the power threshold:

$$\begin{cases} \text{Normal,} & \text{if } \hat{P}_m \geq P_{\text{thr}}, \\ \text{Faulty,} & \text{if } \hat{P}_m < P_{\text{thr}}. \end{cases}$$

This diagnostic approach avoids the sparsity assumptions typically employed in CS-based techniques, making it well-suited for RIS scenarios characterized by a high fault density [128, 129, 131–133].

5.2.2 Results and Discussion

5.2.2.1 Simulation Results

To validate the effectiveness of the proposed diagnostic method, we first perform a numerical simulation using a 16×16 column-controlled metasurface as the RIS under test. The metasurface operates at a frequency of 9.8 GHz and comprises meta-atoms of size $15 \times 15 \text{ mm}^2$, consistent with the configuration of Prototype 1 in Table 2.

Following Equation (110), we set the code order to $L = 32$, with each code symbol lasting $\tau = 1 \text{ } \mu\text{s}$, corresponding to a modulation frequency $f_0 = 62.5 \text{ kHz}$. The digital sampling frequency is $f_s = 8 \text{ MHz}$. We set the incident angle to $\theta_i = 0^\circ$ and position the receiver at $\theta_o = 20^\circ$ relative to the RIS normal. The SNR of the incident signal is set at 30 dB.

Building on the strategy described in the previous section, we implement space-time modulation of the RIS using the STC matrix shown in Figure 83a. This matrix is derived from a 32nd-order Hadamard matrix, with the first

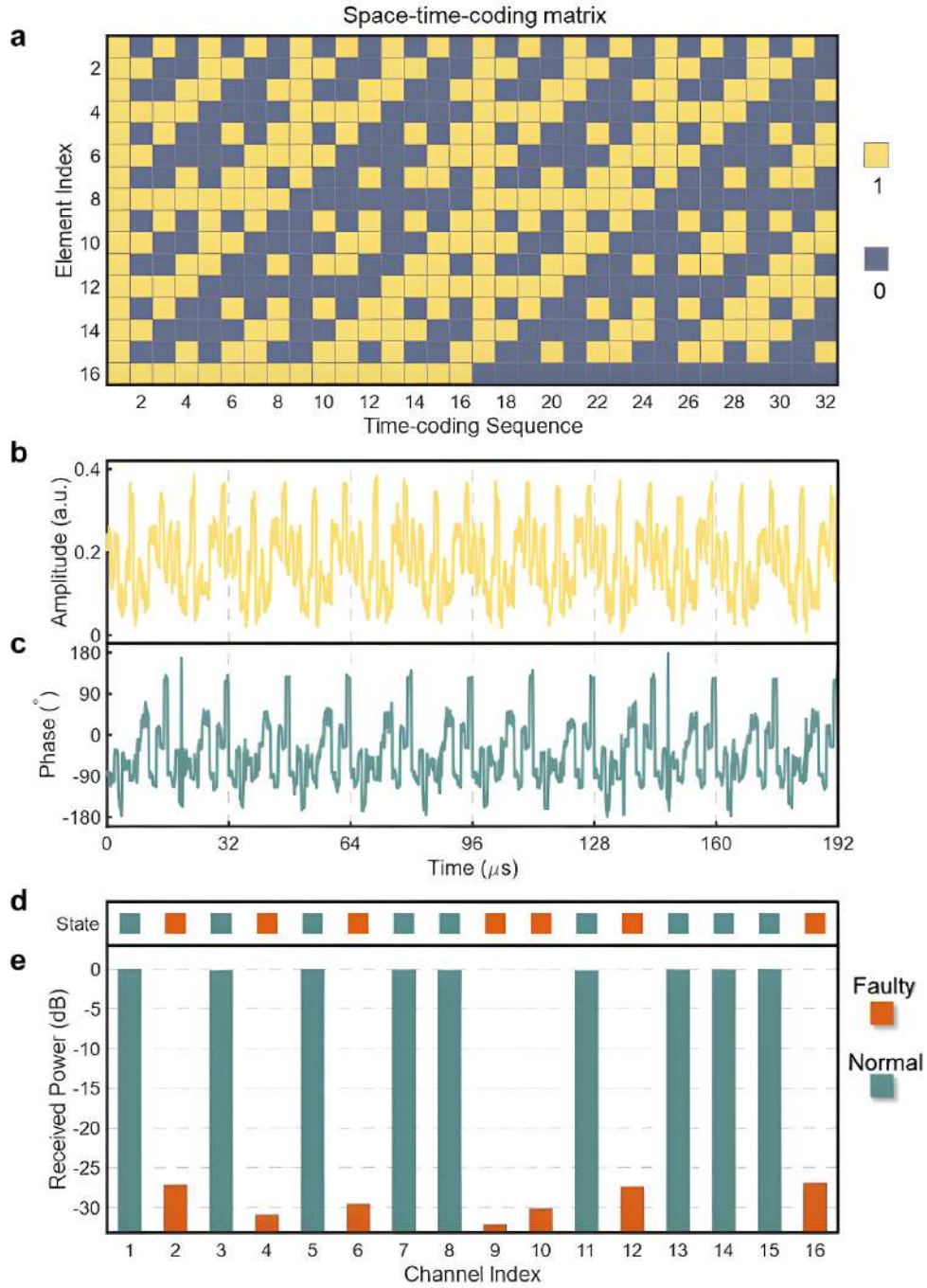


Figure 83: Simulation results for signal analysis and processing. Each column is modulated using a 32nd-order Hadamard code. **a)** STC matrix constructed from the Hadamard codes. **b,c)** Amplitude and phase, respectively, of the received time-domain signal over six modulation periods. **d)** Actual operational states of each column. **e)** Recovered power across different code channels used for fault diagnostics.

Table 2: Design parameters of RIS prototypes for fault diagnostics.

Parameter	Prototype 1	Prototype 2
Testing frequency	9.8 GHz	10 GHz
Array dimensions	16×16	8×8
Meta-atom size	$15 \times 15 \text{ mm}^2$	$15 \times 15 \text{ mm}^2$
Phase resolution	1 bit	1 bit

row excluded. To emulate faulty elements, we randomly set specific rows in the STC matrix to all zeros. By altering seven such rows, Figure 83b,c present the amplitude and phase, respectively, of the time-domain received signal across six modulation periods, while Figure 83d shows the actual operational state of each element. The received signal is processed using Equation (118) to extract the power associated with each coding channel. To mitigate the influence of noise, the mean power is computed over the six modulation periods. Figure 83e displays the recovered power for all code channels, which distinctly separates into two groups. Since the diagnostic approach is based on relative intensity rather than absolute power levels, the recovered powers are normalized with respect to the maximum observed value.

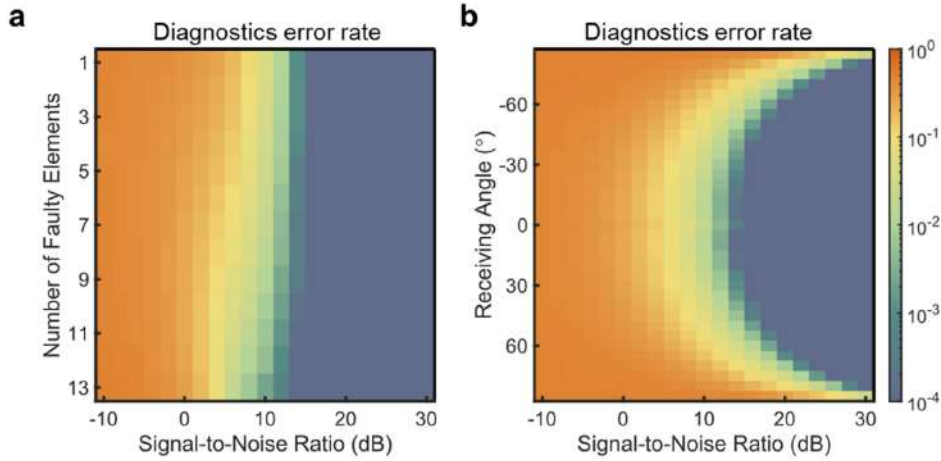


Figure 84: a,b) Simulated DER of the diagnostics under varying SNR conditions as a function of the number of faulty elements and the receiving angle, respectively.

A comparison between Figure 83e,d reveals that channels corresponding to normal elements maintain power levels close to 0 dB, whereas those associated with faulty elements show significantly attenuated values. By selecting an appropriate threshold, faulty elements can be reliably identified by their low recovered power. This outcome matches the actual fault distribu-

tion, thereby validating the effectiveness of the proposed diagnostic method. Moreover, the approach is inherently scalable to large-scale point-controlled RISs, as further discussed in [Section 5.2.3.1](#).

We further conduct simulations to evaluate the impact of SNR and the number of faulty elements N_F on the Diagnostics Error Rate (DER). The DER is defined as the average ratio of incorrectly diagnosed elements to the total number of elements, computed over 3000 simulation runs. As illustrated in [Figure 84a](#), for a fixed N_F , the DER decreases with increasing SNR. Notably, when the SNR exceeds 15 dB, the DER drops below 10^{-4} regardless of the number of faulty elements. Furthermore, as N_F increases, the DER remains nearly constant or may even slightly decrease. These results demonstrate the reliability of the proposed method, even in high fault-density scenarios.

Additionally, we analyze the DER as a function of the receiving angle θ_o , while maintaining a fixed incident angle of $\theta_i = 0^\circ$. As shown in [Figure 84b](#), smaller receiving angles provide improved DER performance under low-SNR conditions. However, for $\text{SNR} > 10$ dB, the DER remains stable within the angular range $-30^\circ \leq \theta_o \leq 30^\circ$, reinforcing the method's robustness against angular variations. Moreover, when $\text{SNR} > 20$ dB, the DER remains below 10^{-4} across the entire range $-60^\circ \leq \theta_o \leq 60^\circ$.

These results confirm both the feasibility and robustness of the proposed diagnostic method. In practical applications, achieving an SNR above 15 dB is easily attainable by increasing transmission power. As a result, the RIS diagnostic approach remains effective under a wide range of SNR levels, receiving angles, and fault densities. Additionally, it requires only a single measurement to acquire the diagnostic data, enabling rapid fault detection with minimal computational overhead.

5.2.2.2 Experimental Validation

To further validate the proposed RIS diagnostic method, we implement it on two distinct RIS prototypes.

[Figure 85a](#) presents the architecture of the diagnostic system. A pair of standard horn antennas is used for transmission and reception. One signal generator provides the monochromatic transmission signal, while a second generator supplies the local oscillator signal for down-conversion. The received signal is captured using a Universal Software Radio Peripheral (USRP) and processed by a host computer. The actual measurement setup in a realistic testing environment is shown in [Figure 85b](#).

The specific design parameters of the two RIS prototypes employed in the experiments are summarized in [Table 2](#). Prototype 1 is a 16×16 column-controlled STC-DM with a total aperture of 240×240 mm², while Prototype 2 is an 8×8 column-controlled STC-DM with a total aperture of 120×120 mm². Both RISs operate with a 1-bit phase modulation, achieved by toggling the PIN diodes soldered on each element between the "ON" and "OFF" states. Close-up photographs of the two prototypes are provided in

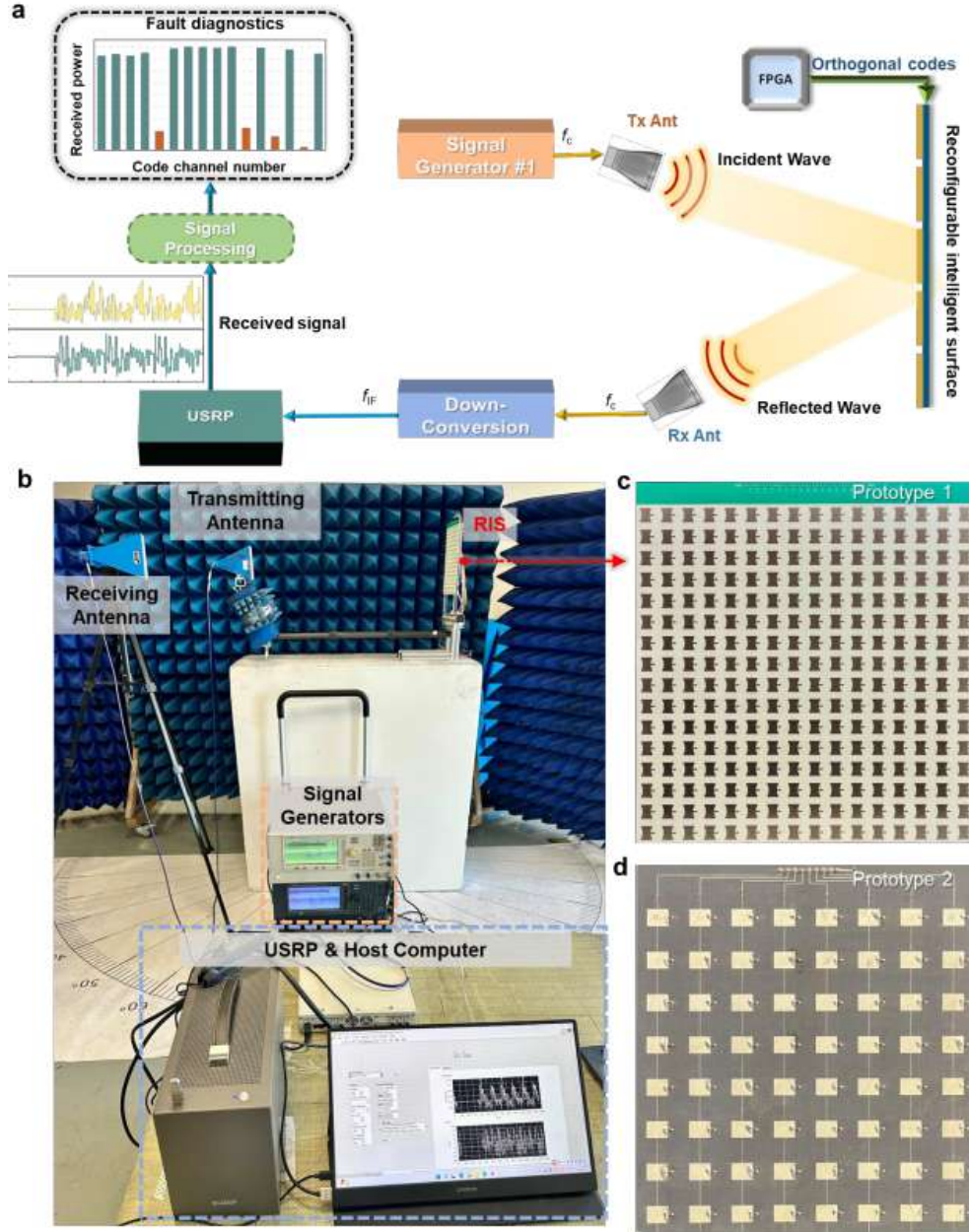


Figure 85: Measurement setup and RIS prototypes. **a)** Architecture of the proposed RIS diagnostic system. **b)** Photograph of measurement setup. **c,d)** Close-up views of RIS Prototypes 1 and 2, respectively.

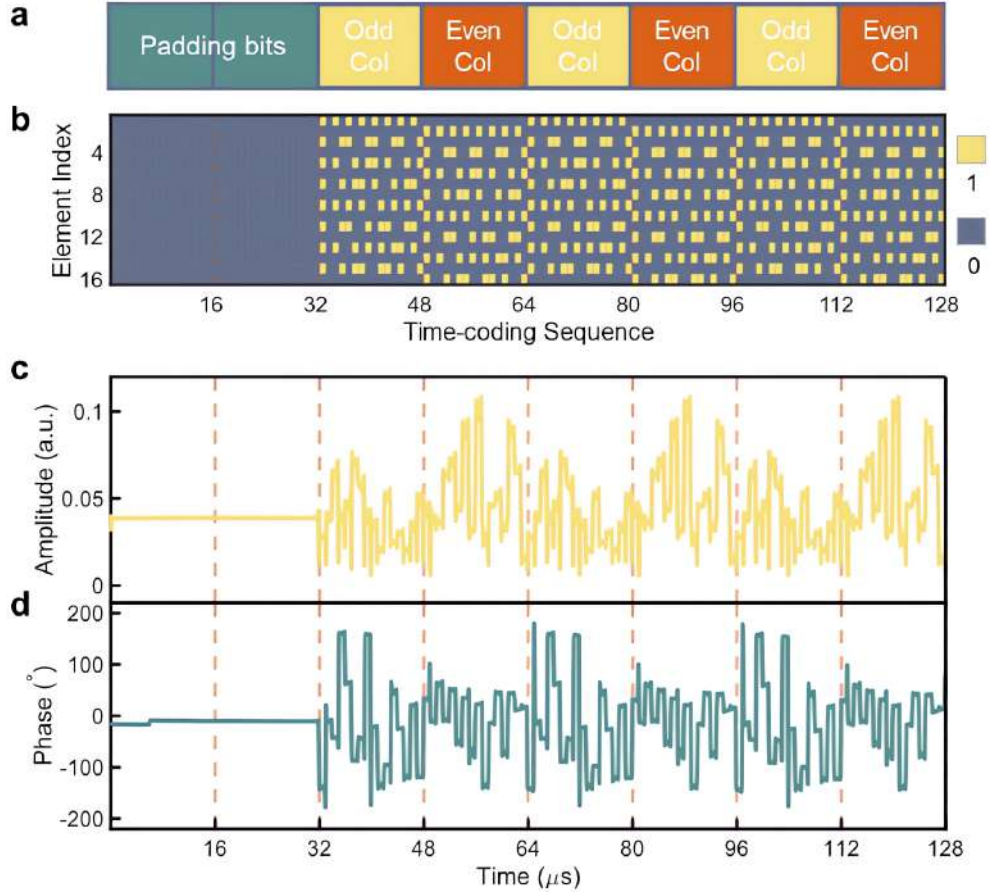


Figure 86: STC configuration in the experiment for Prototype 1. **a)** Modulation signal frame structure. **b)** Detailed STC matrix within a frame. **c,d)** Amplitude and phase, respectively, of received signal over a frame duration.

Figure 85c,d. Additional design details and discussions on the influence of inherent response imperfections are provided in Section 5.2.3.

Different coding schemes are applied to the two prototypes to accommodate variations in array scale and the effects of mutual coupling. For Prototype 2, which features a smaller array and requires only 16th-order codes, a direct coding scheme is adopted—mirroring the configuration used in the simulation setup. The STC matrix is directly constructed from 16th-order Hadamard codes. For Prototype 1, a segmented coding strategy is implemented to reduce the effective code order and mitigate the influence of mutual coupling between adjacent elements.

Figure 86a illustrates the structure of the control signal frame, which is divided into eight segments: two padding bit segments and six segments containing STC signals that alternately modulate odd and even columns. Since only half of the columns are modulated within each segment, the same set of Hadamard codes can be reused across segments, effectively reducing the required code order from 32 to 16. Moreover, by keeping adjacent elements

static while one subset is being modulated, the interference due to mutual coupling is significantly reduced. Two 16-bit padding segments filled with “0” symbols are inserted for synchronization purposes.

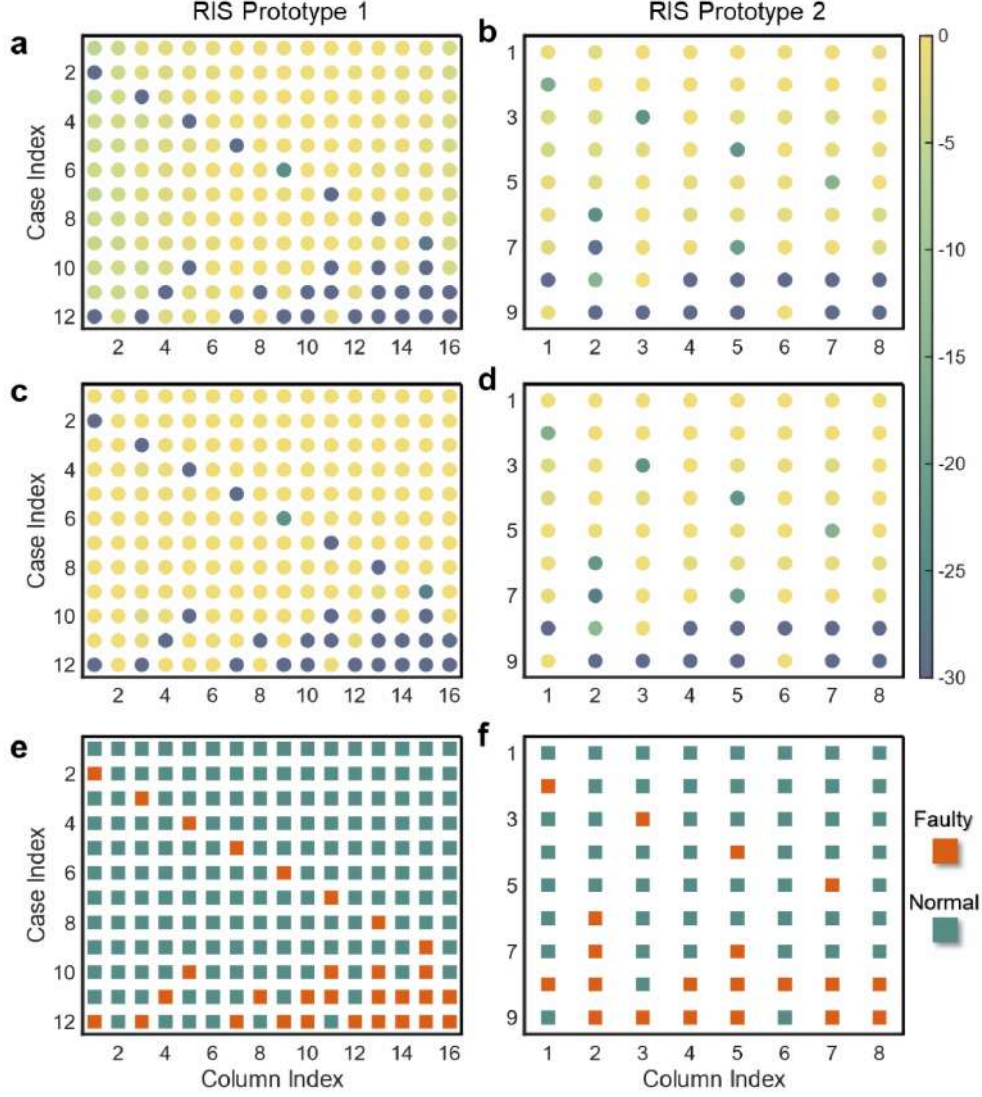


Figure 87: Experimental results of RIS diagnostics across different test cases. **a,c,e)** Recovered power, normalized power, and actual operational states, respectively, for Prototype 1 across twelve cases. **b,d,f)** Recovered power, normalized power, and actual operational states, respectively, for Prototype 2 across nine cases.

Figure 86b provides a detailed view of the STC matrix used within a control frame of 128 bits, wherein 16th-order Hadamard codes are employed. The modulation symbol duration is set to $\tau = 1 \mu\text{s}$, corresponding to a switching frequency of $f_{\text{switch}} = 1 \text{ MHz}$. Under this control scheme, the amplitude and phase of the received signal, measured over the duration of a frame, are shown in Figure 86c,d, respectively, using a sampling frequency of

$f_s = 10$ MHz. It is worth noting that a phase hop observed at the end of the padding segments—resulting from reverse symbol mapping—facilitates synchronization. Additionally, as in the simulation scenario, we emulate faulty elements in both prototypes by setting the corresponding control signals to zero. This approach avoids the risk of damaging the hardware, which would be required to reproduce all possible fault scenarios experimentally. A performance analysis under alternative fault modes is available in [Section 5.2.3.3](#).

Figure 87 presents the experimental results obtained from both RIS prototypes under various fault scenarios, each characterized by a different number of faulty elements. For Prototype 1, twelve test cases were evaluated, with Case 1 corresponding to the fault-free condition. The actual operational states for each case are depicted in Figure 87e. Figure 87a displays the received power across different columns. The results show a sharp drop in recovered power—falling below -20 dB—for columns corresponding to faulty elements. By applying an appropriate threshold, these elements can be clearly identified. In this experimental setup, a noticeable power imbalance of up to 5 dB is observed between central and edge columns. This effect is attributed to edge-related phenomena and the relatively large aperture of the metasurface, which compromises the approximation of the incident field as a plane wave. Such imbalances become more pronounced as the metasurface size increases. Despite these fluctuations, the power contrast between normal and faulty states exceeds 20 dB, rendering normalization based on the reference power from Case 1 (all operational elements) unnecessary for effective diagnostics. However, to improve clarity in result visualization, we normalize the recovered power using the reference from the fault-free case. As shown in Figure 87c, this normalization effectively stabilizes the power distribution, reducing inter-column imbalance to within 2 dB. By comparing the normalized power maps in Figure 87a,c with the actual operational states in Figure 87e, a clear distinction is observed between normal and faulty elements, confirming a power contrast of up to 20 dB and validating the diagnostic capability of the proposed method.

For Prototype 2, nine test cases were measured, as shown in Figure 87f. Figure 87b,d illustrate the received and normalized power for each case, respectively. These results demonstrate a power difference of up to 10 dB between normal and faulty elements, confirming that the proposed diagnostic method can accurately determine the operational state of RIS elements by applying an appropriate threshold. Notably, the diagnostic method remains effective even under high fault-ratio scenarios, such as Case 11 and 12 in Prototype 1 and Case 8 and 9 in Prototype 2, where more than 50% of the elements are faulty. In such cases, the sparsity assumption underlying conventional CS-based approaches no longer holds [136, 137]. Nonetheless, the proposed method continues to deliver accurate diagnostics, highlighting its robustness and confirming theoretical predictions regarding its performance in non-sparse fault distributions. Further investigation on the impact of modulation period length on diagnostic accuracy is provided in [Section 5.2.3.4](#).

The total diagnostic time comprises two main components: the communication time T_c between the RIS and the diagnostic system, and the post-processing time T_p required to analyze the acquired signal data. The communication time T_c depends on the diode switching frequency f_{switch} , the Hadamard code length L , and the number of segments per diagnostic frame N_{seg} , and is calculated as $T_c = N_{\text{seg}}L/f_{\text{switch}}$. In our experiments, with $f_{\text{switch}} = 1$ MHz, $L = 16$, and $N_{\text{seg}} = 8$, the communication time is $128 \mu\text{s}$. For RISs with fewer than 1000 elements, under the same f_{switch} and N_{seg} , the communication time remains below 8.2 ms. Thus, in most practical scenarios, the proposed method achieves millisecond-level communication times. The post-processing time T_p depends on several factors, including the performance of the processing hardware, the efficiency of the programming environment, and the size of the received data. As a result, it is difficult to provide a fixed value or general formula for T_p .

Additionally, more advanced adaptive clustering algorithms, such as K-means clustering, hierarchical clustering, or even neural network-based methods [138], can be applied to determine the working state after recovering the power of each code channel. These approaches may improve diagnostic flexibility and reliability but require increased algorithmic complexity and greater computational resources. However, in our experimental environment, the power difference between normal and faulty elements exceeds 20 dB, enabling the use of a simple fixed threshold for reliable state classification.

In summary, experimental findings validate the feasibility and effectiveness of the proposed diagnostic method across different RIS configurations and realistic operating conditions. For small-scale RISs or those exhibiting low mutual coupling, the direct coding scheme proves sufficient. Conversely, segmented coding offers a more suitable solution for large-scale arrays or those affected by significant mutual coupling. Moreover, the method operates effectively without requiring anechoic environments or multiple measurement iterations, making it highly practical for real-world applications such as factory inspection and routine maintenance. Although this study focuses on 1-bit RIS configurations using 180° phase shifts, the use of Hadamard codes can be generalized to support arbitrary phase values [139]. Therefore, the proposed approach is, in principle, extendable to multi-bit RIS systems.

5.2.3 Additional Results

5.2.3.1 Simulation Results for Diagnostics of Large RISs

We further evaluate the proposed diagnostic method on large-scale point-controlled RISs, specifically a 15×15 array (PC-RIS 1) and a 25×25 array (PC-RIS 2), each comprising $\approx 30\%$ randomly distributed faulty ele-

ments. The simulation parameters are consistent with those detailed in [Section 5.2.2.1](#).

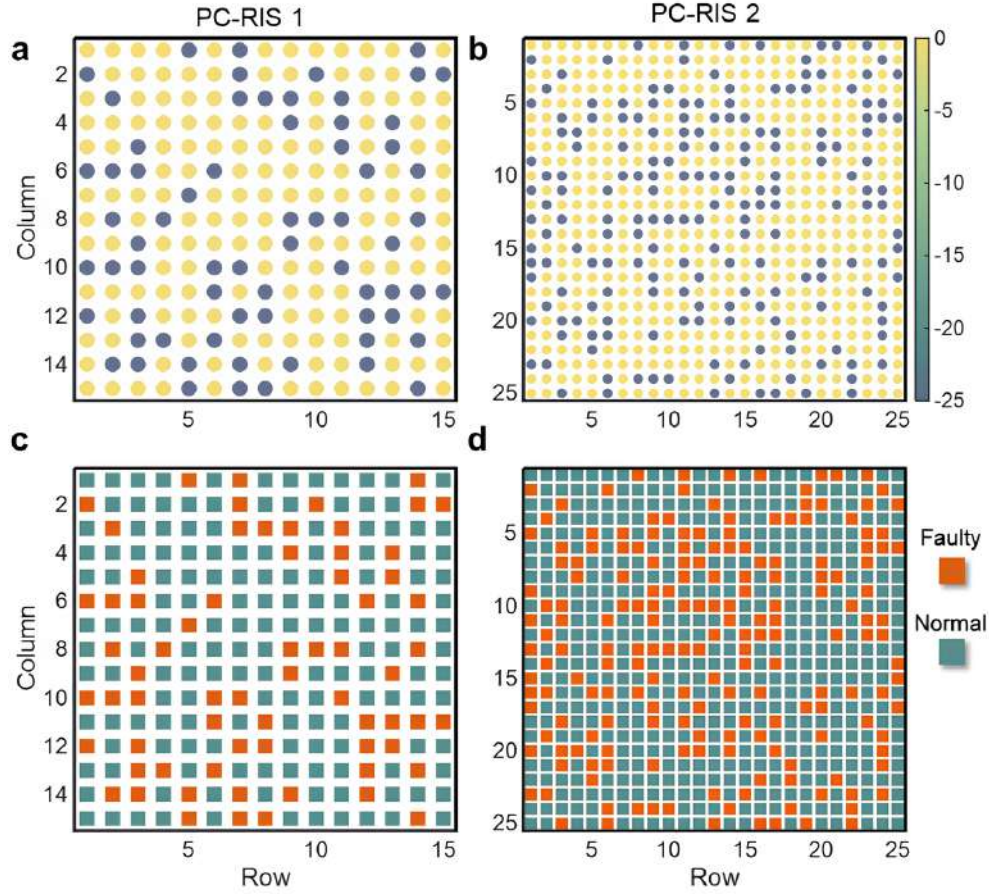


Figure 88: Simulation results for large-scale point-control RISs. **a,b)** Simulated recovered power of each coding element's channel in PC-RIS 1 and PC-RIS 2, respectively. **c,d)** Corresponding actual operational states.

For both RISs, the direct coding scheme is employed, using STC matrices derived from 256th-order and 1024th-order Hadamard codes, respectively. To assess performance, received signals over 20 modulation periods are collected and analyzed. Figure 88a,b display the average recovered power for each coding channel in PC-RIS 1 and PC-RIS 2, respectively, while Figure 88c,d show the corresponding actual operational states. The results confirm that the proposed diagnostic method accurately identifies faulty elements, as evidenced by the significantly reduced recovered power (below -15 dB) associated with these elements. This demonstrates the method's scalability and robustness, effectively extending its applicability to large-scale RISs with substantial fault ratios.

5.2.3.2 Analysis of 1-bit Phase Deviations and Reflection Amplitude Imbalance

Figure 89c–f shows that, within the operational frequency band of the RIS elements, the phase shift between the “0” and “1” states deviates from the ideal 180° , accompanied by an imbalance in the reflection amplitude. Moreover, manufacturing tolerances lead to discrepancies between the simulated and actual operational frequencies. These non-idealities affect the orthogonality of the code modulation used for RIS diagnostics. To assess the influence of these imperfections, we simulate the diagnostic performance under different phase deviations and amplitude imbalances, as depicted in Figure 89.

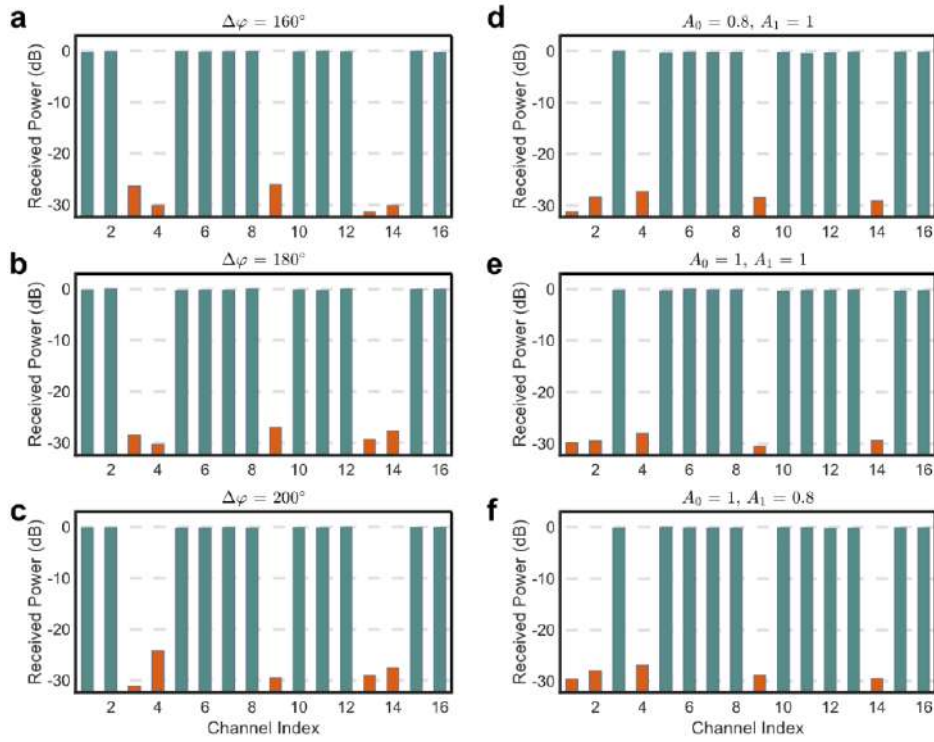


Figure 89: Simulation analysis of the impact of element response imperfections. **a–c)** Diagnostics results under varying phase differences. **d–f)** Diagnostics results under different reflection amplitude imbalances.

Figure 89a–c demonstrate that, although phase deviations cause variations in the energy distribution across code channels, the separation between functional and faulty elements remains sufficiently distinct to enable accurate diagnostics. Similarly, Figure 89d–f indicate that the diagnostic performance is only marginally affected by amplitude imbalances. Overall, these findings confirm that the proposed method is robust and maintains high accuracy even in the presence of realistic element response deviations.

Notably, at the testing frequency of 10 GHz (see Figure 87), RIS Prototype 2 exhibits a phase difference of $\approx 200^\circ$ rather than the ideal 180° expected at its nominal operational frequency of 9.8 GHz. Despite this deviation,

tion, the method successfully detects faults, further validating its tolerance to non-ideal element behaviors.

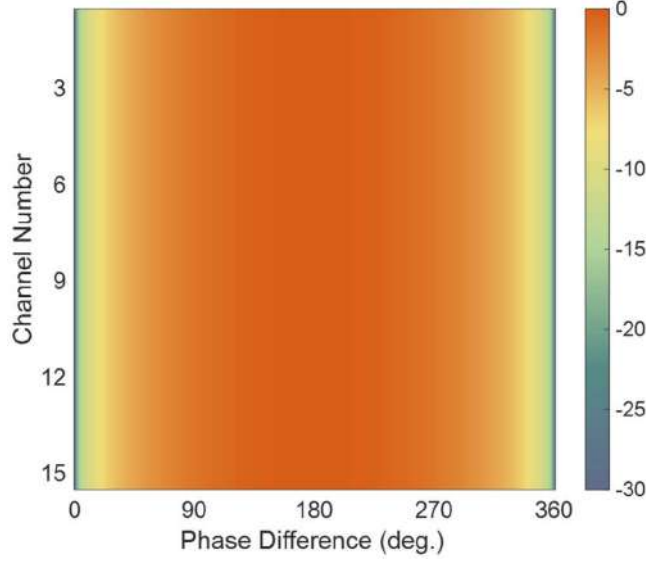


Figure 90: Simulated recovered channel power (in dB) as a function of the element's phase difference.

The diagnostic accuracy mainly depends on the orthogonality between the realized codes, which result from the space-time modulation of the RIS, and the ideal Hadamard codes used for channel power recovery. For normal elements, hardware imperfections can lead to deviations from the ideal code patterns, reducing orthogonality and slightly lowering the recovered channel power. Figure 90 presents the recovered channel power as a function of the element's phase difference, showing only minor degradation within the range of 120° to 240° . In contrast, faulty elements lack time modulation capability, leading to recovered channel power at the noise level. These results indicate that phase imperfections have limited impact on diagnostic accuracy, confirming the robustness of the proposed method.

5.2.3.3 Performance of the Proposed Diagnostics Under Alternative Fault Modes

In our experiments, it was not feasible to simulate all possible fault scenarios without risking damage to the prototypes. As a practical trade-off, we emulated faults by setting the control signals of selected elements to zero. However, other fault modes may exist. These are explored through simulations in the following analysis.

One potential fault mode involves partially degraded elements due to component aging, environmental factors like snow or dirt accumulation, or manufacturing defects. These elements may still function but exhibit reduced amplitude response and limited phase variation. Figure 91a shows simulation results for a RIS containing such partially faulty elements. Their amplitude

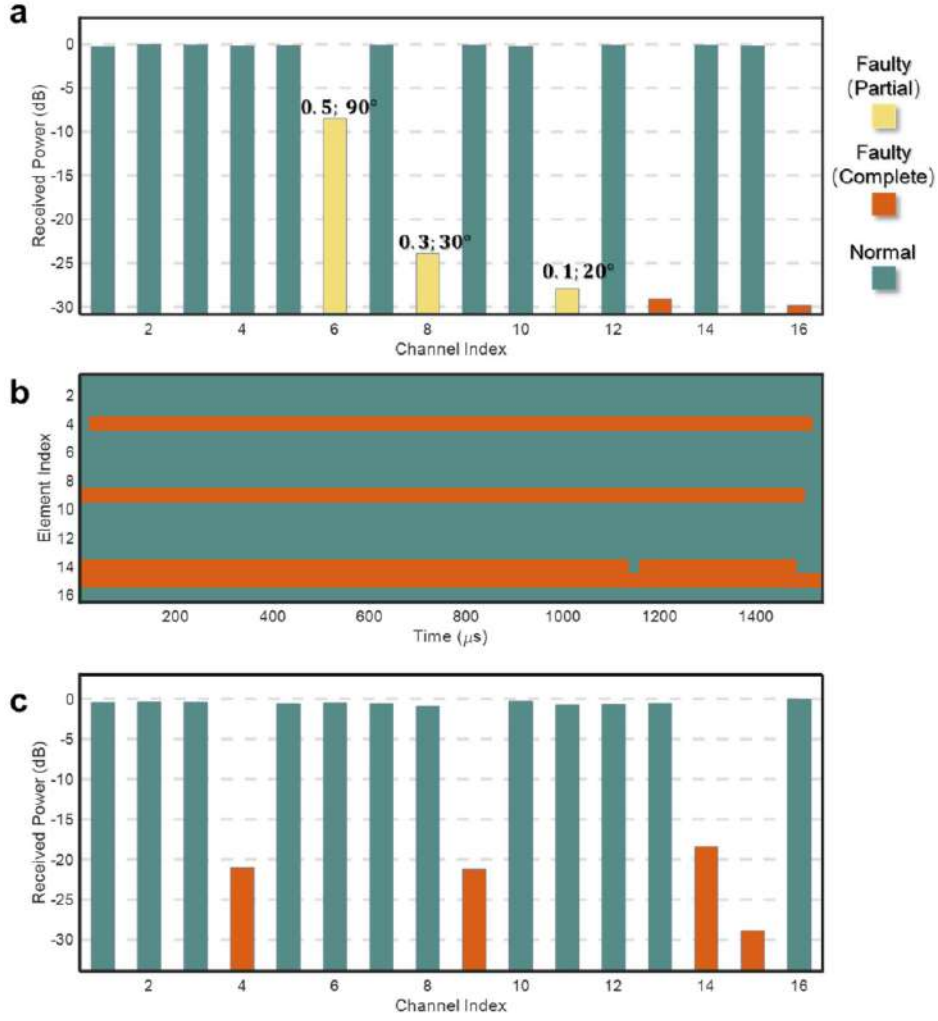


Figure 91: Simulation results for alternative fault patterns. **a)** Recovered power of a RIS containing partially faulty elements. **b)** Temporal fault pattern showing intermittently faulty elements. **c)** Corresponding recovered power of the intermittently faulty elements.

responses were set to 0.5, 0.3, and 0.1, while the phase shifts between the two states were configured as 90°, 30°, and 20°, respectively. As shown, the power of these elements (highlighted in yellow) remains sufficiently lower than that of normal elements and clearly distinguishable, although higher than completely faulty elements. This indicates that the proposed method can detect partially faulty elements under high SNR conditions.

Another fault mode involves intermittent failures, which can result from unstable soldering or loose electrical contacts. In this scenario, elements may occasionally function correctly. Figure 91b,c simulate this condition, where the 4th, 9th, and 15th elements malfunction intermittently during the red-highlighted time slots. Although their recovered power is higher than that of completely faulty elements (such as the 16th element, which remains at

–30 dB), it is still clearly distinguishable from the power of normal elements. Diagnostic reliability in such cases can be further improved by increasing the number of sampling periods, which helps reduce the average recovered power of intermittently faulty elements.

These simulations confirm that the proposed diagnostic method remains effective across a variety of realistic fault conditions.

5.2.3.4 Effects of Modulation Periods on Diagnostics Performance

In Section 5.2.2.2, the recovered power for both RIS prototypes across all test cases is derived from received signals collected within a single frame, which includes only six segments of valid modulated signal periods. To investigate how the number of modulation periods influences diagnostic accuracy, numerical simulations are performed using 1, 6, 12, and 200 periods, as illustrated in Figure 92.

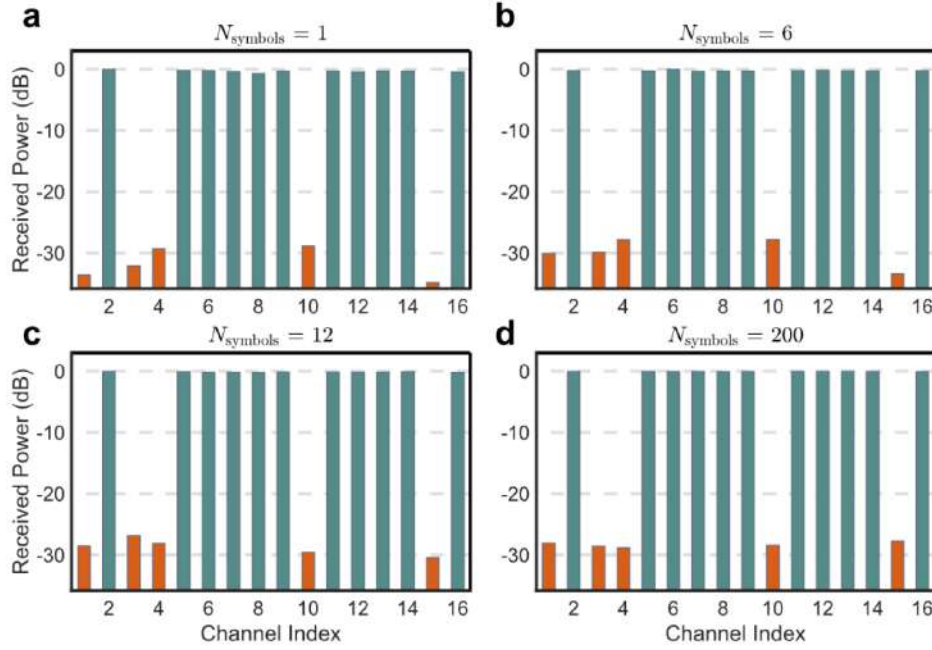


Figure 92: Simulation results of diagnostics with varying numbers of modulation periods per frame. **a–d)** 1, 6, 12, and 200 periods, respectively.

A comparison of Figure 92a–d reveals that the power associated with each code channel becomes increasingly stable as the number of modulation periods increases. Specifically, the power corresponding to normal elements tends to converge toward 0 dB, while that of faulty elements approaches the simulated noise floor (≈ -30 dB). Notably, even with a minimal number of modulation periods (Figure 92a,b), the power gap between functional and faulty elements remains clearly distinguishable. Furthermore, the incremental improvement between Figure 92c,d is minor, suggesting that increasing

the number of modulation periods beyond a certain point yields diminishing returns in terms of diagnostic performance.

Including more modulation periods within a frame helps suppress noise-induced fluctuations, thereby enhancing the consistency and reliability of the diagnostic results. However, under high-SNR conditions, even a limited number of modulation periods proves sufficient to achieve accurate fault identification. This observation reinforces the practicality of the proposed method, as it enables reliable diagnostics with minimal signal acquisition time.

5.2.3.5 *Effects of Non-Ideal Modulation Waveform*

Certain factors, such as fast switching in the RIS, can occasionally introduce impulse noise. These noise spikes occur during state transitions of RIS elements, resulting in sharp peaks, as shown in Figure 93a, and abrupt phase changes, as illustrated in Figure 93b. However, the duration of these impulses is very short compared to the symbol duration. Since multiple samples per symbol are used in the diagnostic process, the presence of such impulse noise has little impact on overall diagnostic reliability. Figure 93c,d presents simulation results based on the waveforms in Figure 93a,b, showing that the received power from faulty elements remains at the noise level, around -30 dB. Similar impulse effects are also observed in the experimental waveform shown in Figure 86. Nonetheless, the consistent accuracy of experimental results confirms that these impulse noise spikes do not compromise the reliability of the proposed diagnostic method.

5.2.3.6 *Detailed Designs and Element Responses of RIS Prototypes*

Figure 94a depicts the unit-cell design of RIS Prototype 1, which consists of a three-metal-layer structure embedded between two Rogers RO4003C dielectric substrates and a bottom FR4 layer. The programmable meta-atom integrates a PIN diode (MACOM MADP-000907-14020X), connecting a bow-tie radiating patch to a grounded rectangular patch on the top layer. The bias voltage is supplied through bottom-layer lines accessed via vertical vias. This design enables a phase shift of $\approx 180^\circ$ between the diode's "ON" and "OFF" states. Full-wave simulations performed using CST Microwave Studio® [109] confirm this behavior, as shown in Figure 94c, which displays a 1-bit phase response centered around 10 GHz with less than 1 dB of amplitude loss. Experimental measurements (Figure 94e) corroborate these results, showing a 180° phase difference at 9.8 GHz.

The unit cell of RIS Prototype 2 is illustrated in Figure 94b. This design adopts a two-metal-layer architecture built on an F4B dielectric substrate. The same PIN diode is used, bridging a central rectangular patch with ground vias, while the biasing network is implemented directly on the top layer. Simulation results (Figure 94d) and experimental data (Figure 94f) both confirm a 180° phase shift occurring at ≈ 10 GHz and 9.8 GHz, re-

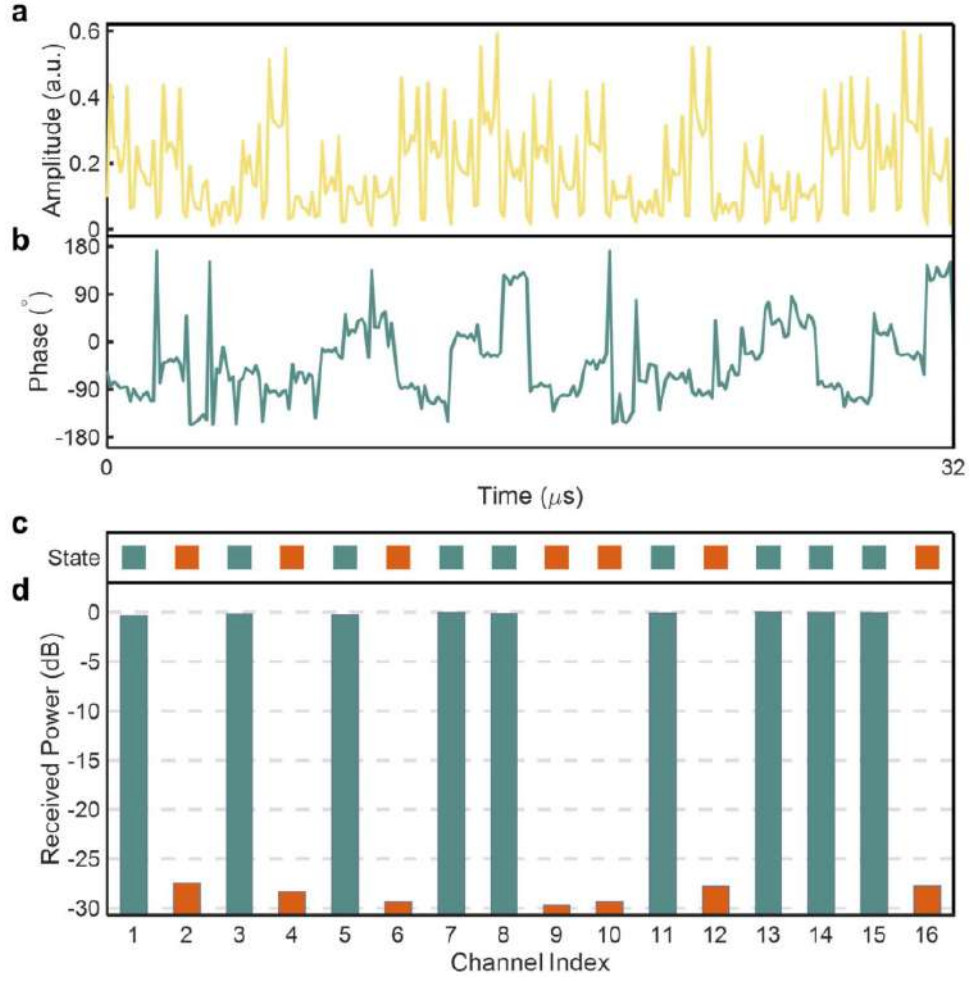


Figure 93: Simulation results for waveforms affected by impulse noise. **a,b)** Amplitude and phase, respectively, of the received time-domain signal during one modulation period. **c)** Actual operational states of each column. **d)** Recovered power of different code channels used for fault diagnostics.

spectively. Experimental results indicate a slightly higher amplitude loss of about 2 dB.

5.2.4 Experimental Details

MODELING The numerical implementation of the theoretical analysis was carried out using MATLAB®, with the far-field scattering pattern of the basic element approximated as $E(\theta, \phi) \approx \cos \theta$.

MEASUREMENTS The experimental setup used for method verification is illustrated in Figure 85a. Detailed design schematics and the corresponding measured reflection coefficients are provided in Section 5.2.3.6 (see Figure 94). A pair of standard X-band horn antennas are used as transmitter

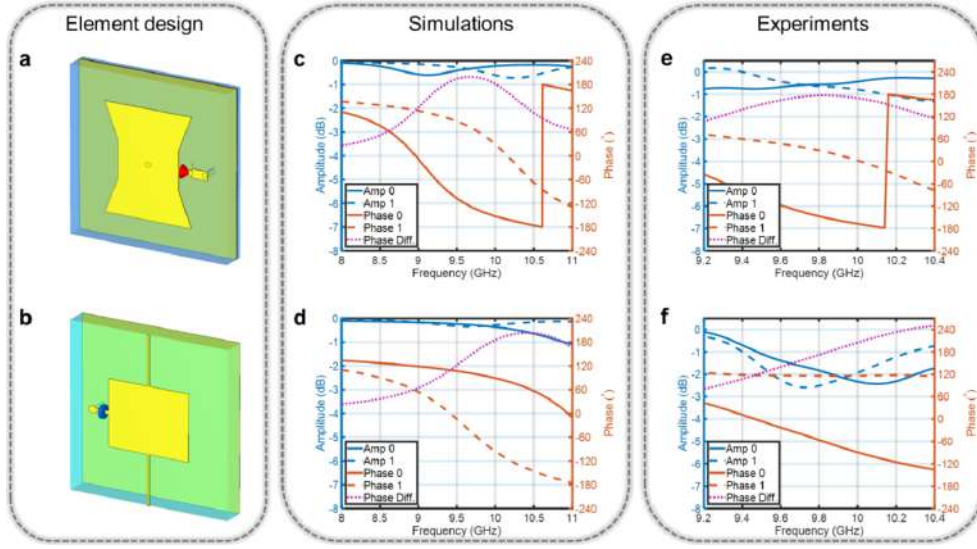


Figure 94: Designs and response characteristics of RIS prototypes. **a,b)** Element design of Prototype 1 and 2, respectively. **c,d)** Corresponding simulated responses. **e,f)** Corresponding measured responses.

and receiver. The monochromatic signal is generated by an Agilent E8257D signal generator, while reception is handled by a National Instruments NI-USRP 2943R software-defined radio, with post-processing performed on a host computer.

5.2.5 Summary and Outlook

We have proposed a diagnostic method for RISs based on a uniform STC modulation strategy, wherein each RIS element is assigned a unique orthogonal Hadamard code. Functioning elements modulate the incident signal temporally using their respective codes, while faulty elements passively reflect the signal without modulation. At the receiver, the power associated with each code channel is recovered; faulty elements are identified by a significant reduction in recovered power, enabling accurate diagnostics via a simple thresholding procedure.

To validate the proposed approach, we developed an analytical model and performed a theoretical feasibility analysis. Extensive numerical simulations confirmed the method's effectiveness, even in challenging scenarios with low SNR, high numbers of faulty elements, and various receiving angles. The results show consistent fault detection performance across all tested conditions.

Experimental validation was conducted using two distinct RIS prototypes. A direct coding scheme was applied to a small-scale prototype with low mutual coupling, while a segmented coding scheme was employed for a larger-scale RIS with more pronounced coupling effects. Both simulation and

experimental results consistently demonstrated the method's practicality, its effectiveness in high-fault regimes, and its robustness to variations in the receiving angles.

These features render the proposed method particularly suitable for practical applications requiring rapid and reliable diagnostics, such as factory testing for poor soldering or defective components, and initial fault localization during routine maintenance of outdoor RIS deployments. As RIS technology continues to advance and becomes increasingly integrated into next-generation wireless communication systems, the proposed diagnostic framework offers a valuable tool for maintaining system reliability and performance.

5.3 DIAGNOSTICS VIA NON-UNIFORM MODULATION

In Section 5.2, we introduced a uniform STC modulation strategy leveraging specific classes of orthogonal codes to enable fast diagnostics of RISs. This method effectively overcomes several limitations of conventional RIS diagnostic techniques, exhibiting robust and reliable performance. Nonetheless, it requires a coherent detection scheme, which entails the use of a local oscillator and strict synchronization. Moreover, while the method is theoretically extendable to multi-bit RISs, such an extension is non-trivial and may introduce additional complexity.

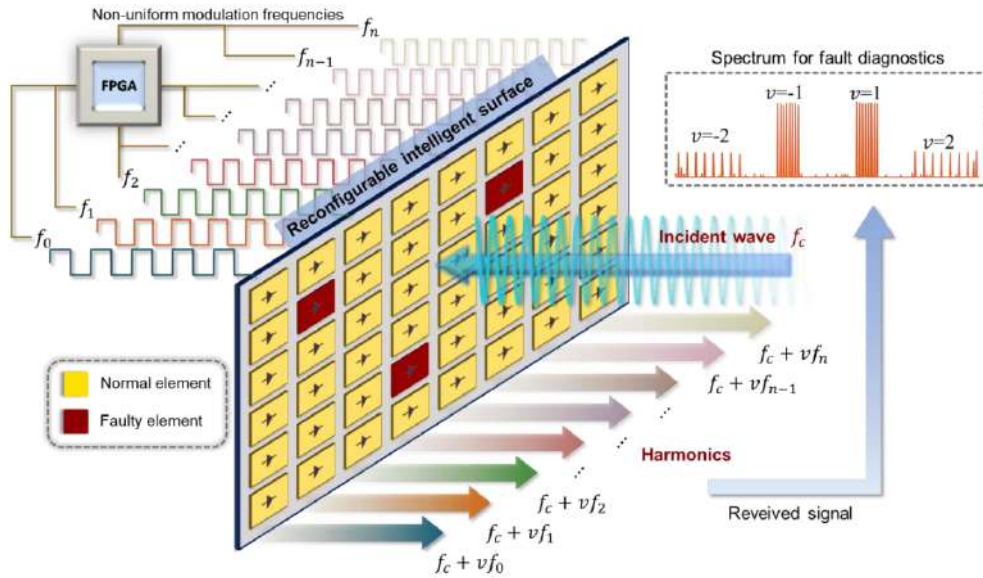


Figure 95: Conceptual illustration of the RIS diagnostics approach based on non-uniform STC modulation.

In this section, we present an alternative RIS diagnostics approach based on non-uniform STC modulation in conjunction with simple *amplitude-only* spectral measurements. This technique allows direct frequency-domain map-

ping for identifying and localizing faults, removing the need for iterative optimization, coherent detection, or extensive measurements. Consequently, it significantly lowers computational and measurement burdens. Notably, the method remains effective even when faults are not sparsely distributed—an assumption commonly required by other existing approaches. By exploiting *relative variations* in the amplitude of harmonic components, rather than their absolute values, the method also achieves high robustness to system noise and measurement variability. Furthermore, it naturally supports both 1-bit and multi-bit RIS configurations. Overall, the proposed non-uniform STC-based diagnostics method provides a practical, efficient, and real-time solution for dependable RIS fault detection. The corresponding results are also reported in Ref. [140].

5.3.1 Proposed Approach

Referring to Figure 95 for a conceptual illustration, the RIS under test is illuminated by a plane wave at the carrier frequency f_c , and each RIS element is periodically modulated by a control circuit, such as a FPGA, according to a predefined time-coding sequence. While the time-coding sequence is identical across all elements, each element is individually assigned a unique modulation frequency. This STC modulation transforms the incident monochromatic wave into multiple harmonic components. Under nominal operating conditions, the +1st harmonic components associated with each modulation frequency exhibit a uniform power distribution in the spectrum of the reflected wave. However, if one or more RIS elements are faulty, the harmonic power at their respective modulation frequencies deviates from this expected behavior. By analyzing the spectrum of the reflected wave and mapping each harmonic component to its corresponding RIS element, the operational status of each element can be accurately assessed, enabling precise fault localization across the RIS.

We begin by considering a 1-bit RIS composed of $N \times M$ elements, each incorporating a PIN diode. Each element supports two discrete reflection states ideally characterized by identical amplitude and a 180° phase difference, corresponding to logical states “0” and “1.” The RIS is normally illuminated by a linearly polarized, monochromatic plane wave at carrier frequency f_c , whose electric field amplitude is expressed as $E^{(i)} = \exp(j2\pi f_c t)$. By assigning time-coding sequences to the elements, the reflected wave in the direction normal to the surface can be expressed as follows:

$$E^{(r)}(t) = \sum_{n=1}^N \sum_{m=1}^M E^{(i)}(t) \Gamma_{nm}(t), \quad (125)$$

where $\Gamma_{nm}(t)$ is the reflection coefficient of the (n, m) -th element.

The common time-coding sequence assigned to all RIS elements is illustrated in Figure 96a. According to this sequence, each element switches between its two operating states within each modulation period, with a duty

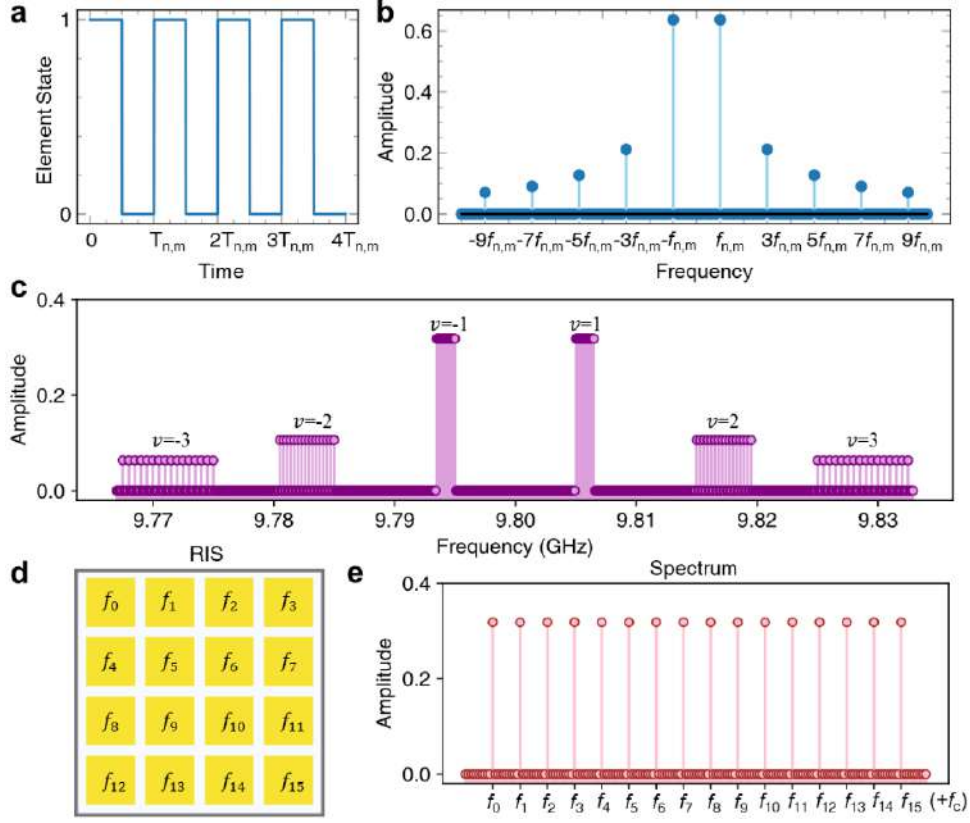


Figure 96: a) Time-coding sequence for each RIS element with 1-bit coding. b) Corresponding amplitude distribution of the Fourier coefficients derived from the periodic reflection coefficient. c) Spectrum of the reflected wave under normal incidence, assuming $f_c = 9.8$ GHz. d) Possible assignment of distinct modulation frequencies ($f_0, f_1, \dots, f_{15}$) to individual elements of a 4×4 RIS. e) Amplitudes of +1st harmonic order components for each modulation frequency.

cycle of 50%. By assigning a unique modulation frequency to each element, the reflection coefficient of the (n, m) -th element can be expressed as a periodic function:

$$\Gamma_{nm}(t) = \begin{cases} 1, & 0 \leq t < T_{nm}/2, \\ -1, & T_{nm}/2 \leq t \leq T_{nm}, \end{cases} \quad (126)$$

where T_{nm} denotes the modulation period, with the corresponding modulation frequency defined as $f_{nm} = 1/T_{nm}$. Based on Fourier transform theory, the reflected wave in the frequency domain can be written as follows:

$$\tilde{E}^{(r)}(f) = \sum_{n=1}^N \sum_{m=1}^M \tilde{E}^{(i)}(f) * \tilde{\Gamma}_{nm}(f), \quad (127)$$

where the $\sim$ identifies spectral quantities, and $*$ indicates the convolution operator. The periodic reflection coefficient can be expanded into a Fourier series

$$\Gamma_{nm}(t) = \sum_{v=-\infty}^{+\infty} a^{(v)} e^{j2\pi v f_{nm} t}, \quad (128)$$

which in the spectral domain corresponds to

$$\tilde{\Gamma}_{nm}(f) = \sum_{v=-\infty}^{+\infty} a^{(v)} \delta(f - v f_{nm}), \quad (129)$$

with $\delta(\cdot)$ denoting the Dirac delta. By specializing Equation (44) for the case $L = 2$ and setting $\{\Gamma^{(l)}\}_{l=1,\dots,L} = \{1, -1\}$, we obtain the Fourier coefficients

$$a^{(v)} = j \operatorname{sinc}\left(\frac{\pi v}{2}\right) \sin\left(\frac{\pi v}{2}\right) e^{-j\pi v}, \quad (130)$$

which represent the frequency-domain complex reflection response of the element at the v -th harmonic frequency $f_c + v f_{nm}$. Equation (127) can be further expressed as:

$$\tilde{E}^{(r)}(f) = \sum_{n=1}^N \sum_{m=1}^M \sum_{v=-\infty}^{+\infty} a^{(v)} \delta(f - f_c - v f_{nm}). \quad (131)$$

The temporal modulation transforms the incident wave into multiple harmonic components, such that the spectrum of the reflected wave $\tilde{E}^{(r)}(f)$ becomes a superposition of these harmonics. As illustrated in Figure 96b, the amplitude of the reflection coefficients exhibits a distinctive pattern: The coefficient at the fundamental frequency is zero, while the maximum amplitude occurs at $v = \pm 1$. This indicates that the applied coding sequence predominantly shifts the incident power into the ± 1 st harmonic order components at $f_c \pm f_{nm}$.

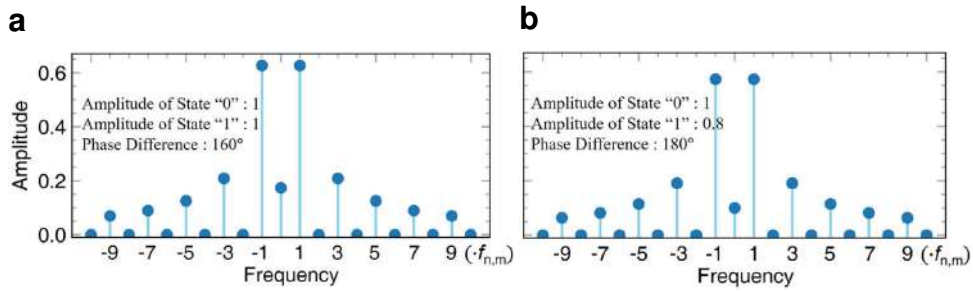


Figure 97: a) Amplitude distribution of the Fourier coefficients for a phase difference of 160° between states "0" and "1". b) Same distribution for a phase difference of 180° and reflection amplitudes of 1 for state "0" and 0.8 for state "1".

To evaluate the robustness of this spectral behavior under non-ideal conditions, we further analyze cases where the reflection responses of logical

states deviate from ideal assumptions, as commonly observed in practical RIS implementations. Figure 97a presents the case where the phase difference between the two states is reduced from 180° to 160° , with equal reflection amplitudes. Figure 97b presents the case where the reflection amplitude of state “0” is 1, the reflection amplitude of state “1” is 0.8, and the phase difference remains 180° . In both cases, the ± 1 st harmonic components remain dominant, and the fundamental component is still significantly suppressed, closely resembling the ideal spectral pattern. These results demonstrate that moderate phase shift errors or amplitude fluctuations do not significantly alter the spectral signature of the applied coding sequence.

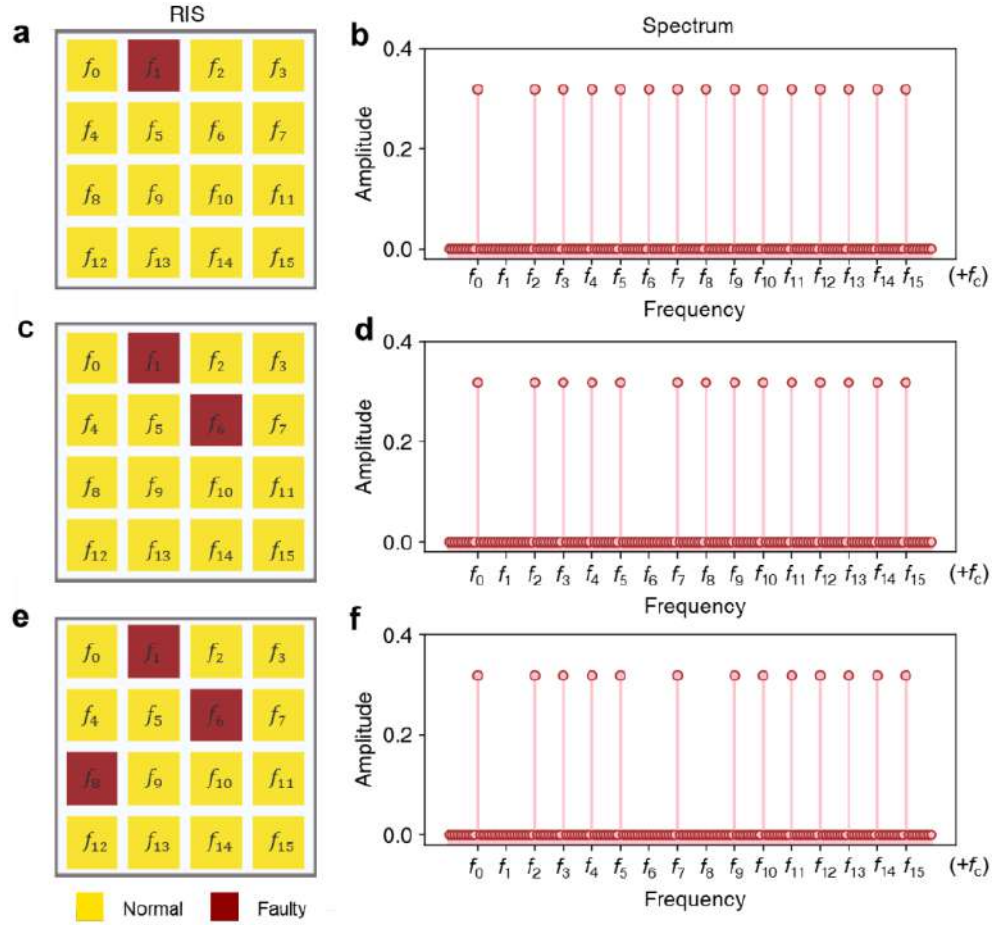


Figure 98: a,c,e) Three examples of 4×4 RISs with 1-bit coding, and different numbers of faulty elements. b,d,f) Corresponding spectral amplitude distributions of +1st harmonic order components at their respective modulation frequencies.

As a specific example, consider a 1-bit RIS with $N = 4$ and $M = 4$ (i.e., 16 elements), operating at a carrier frequency $f_c = 9.8$ GHz. Each RIS element is assigned a distinct modulation frequency ($f_0, f_1, \dots, f_{15}$), as illustrated in Figure 96d. When all elements are functioning correctly, the spectrum of the reflected wave—shown in Figure 96c—exhibits a uniform power distribution

across the +1st harmonic order components. In this scenario, the amplitudes corresponding to all 16 modulation frequencies are equal, as depicted in Figure 96e. However, if a PIN diode in one of the elements is damaged or disconnected, preventing it from toggling between states, the element becomes stuck in a fixed configuration and fails to produce the intended harmonic components. Consequently, the amplitude of the +1st harmonic at its corresponding modulation frequency is significantly attenuated relative to that of the properly functioning elements.

Figure 98 provides three examples of RIS configurations with different faulty elements, along with the associated spectral amplitude distributions of the +1st harmonic components. These examples clearly illustrate how faults manifest as drops in harmonic amplitude (e.g., at $f_c + f_1$ in Figure 98b), thereby enabling straightforward identification and localization of malfunctioning elements based on the known mapping between modulation frequencies and RIS elements.

Since the proposed RIS diagnostic method relies on analyzing the relative amplitude distribution of the +1st order harmonics for each modulation frequency, it is crucial to avoid spectral overlap with harmonics of other orders. Specifically, to prevent interference from adjacent +2nd order harmonics, the modulation frequencies must satisfy the condition $\max(\mathbb{F}) < 2 \cdot \min(\mathbb{F})$, where $\mathbb{F} = \{f_{nm}\}_{n=1,\dots,N,m=1,\dots,M}$, ensuring adequate spectral separation and accurate element identification.

Moreover, due to finite spectral resolution, the required maximum modulation frequency increases with the number of RIS elements. To address this scalability challenge, a grouping strategy can be adopted, wherein the RIS is divided into smaller subsets that are diagnosed sequentially. This approach maintains reasonable modulation frequency requirements while preserving the accuracy and reliability of the diagnostics process.

5.3.2 Results and Discussion

5.3.2.1 Experimental Validation

To validate the proposed RIS diagnostics method, two prototypes of 1-bit RISs operating in the microwave frequency range were fabricated and experimentally tested. The experimental setup, depicted in Figure 99a,b, includes a signal generator connected to a linearly polarized horn antenna, which illuminates the RIS with a monochromatic wave. A second horn antenna, connected to a spectrum analyzer, is used to capture the reflected wave and placed $\approx 20^\circ$ off the normal direction to avoid the blockage effect produced by the illuminating horn antenna.

The first prototype is a 1-bit RIS operating around $f_c = 9.8$ GHz, consisting of 16 columns, each containing 16 elements, with all elements within a given column controlled by a common voltage signal, as illustrated in Figure 99c. Each column is driven by the time-coding sequence shown in

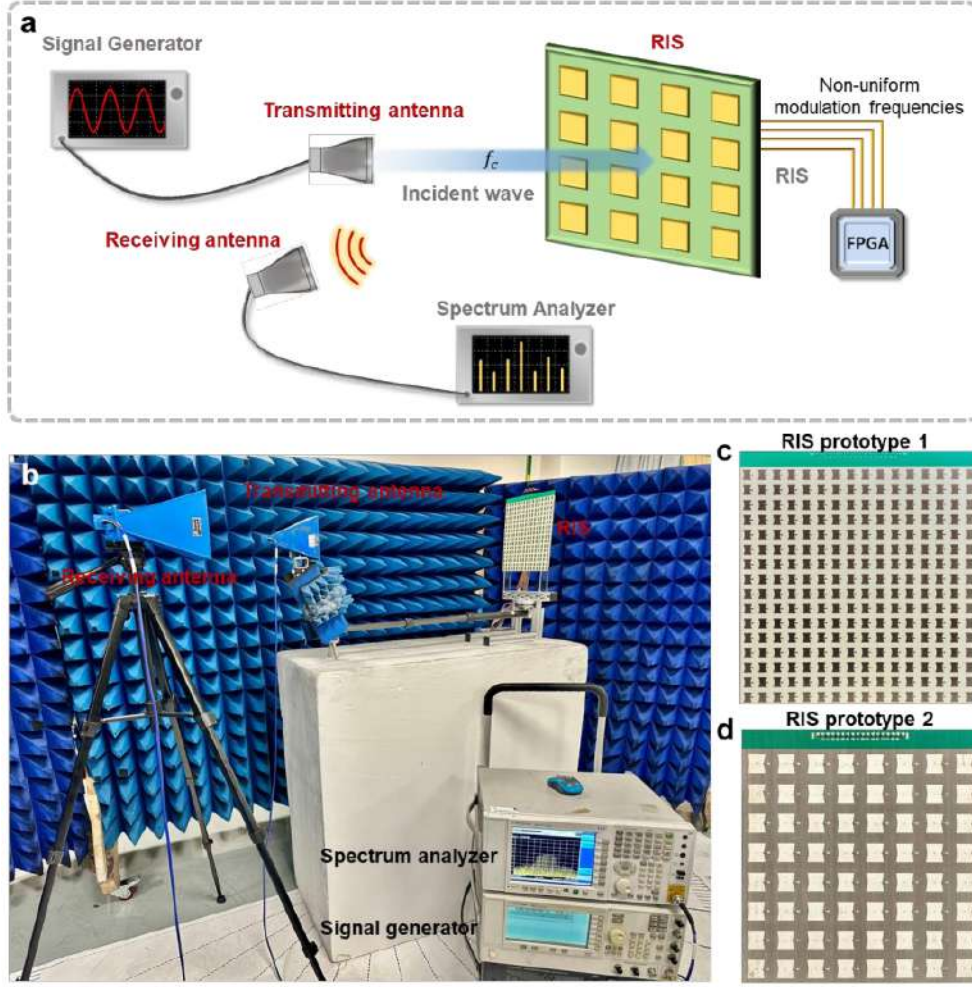


Figure 99: a) Schematic of the experimental configuration. b) Experimental setup. c,d) Two prototypes of 1-bit RISs.

Figure 99a. The modulation frequency for the n -th column is defined as $f_n = 5 + 0.1 \cdot (n + 1)$ MHz. Under normal operating conditions, the incident wave is converted into multiple harmonic components, with the majority of power concentrated in the ± 1 st harmonic orders. The resulting reflected spectrum is shown in Figure 100a, while Figure 100b illustrates that the normalized amplitudes of the $+1$ st harmonic order are uniformly distributed across the 16 modulation frequencies.

To emulate faults without physically damaging the PIN diodes, selected columns of the RIS are disabled by setting their control bias voltage to 0 V, thereby preventing proper switching between states. In these experiments, one or more columns are held in this inactive state while the remaining columns operate normally. Multiple fault scenarios are tested, and Figure 100 presents the measured spectral amplitude distributions of the reflected wave for three representative cases. The corresponding normalized

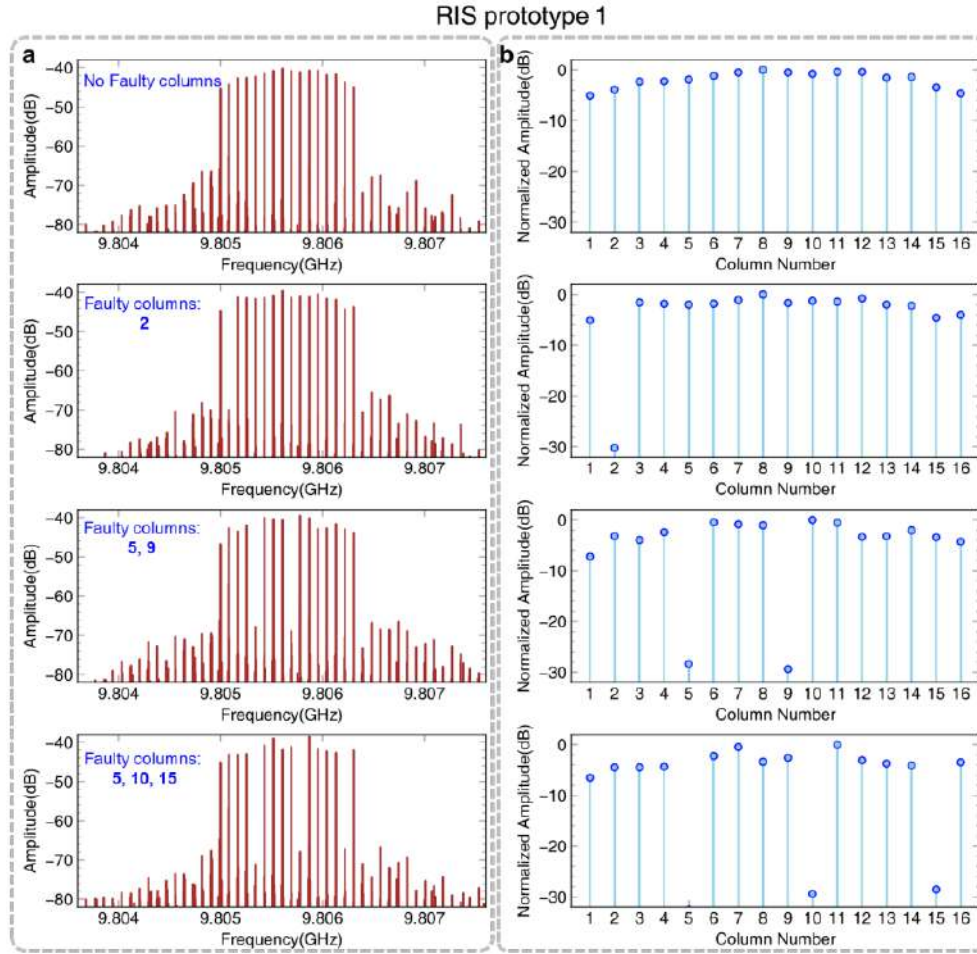


Figure 100: a) Measured spectra of the reflected wave for RIS Prototype 1, for four scenarios featuring a varying number of faults. b) Corresponding normalized amplitudes of the +1st harmonic order components across all 16 modulation frequencies.

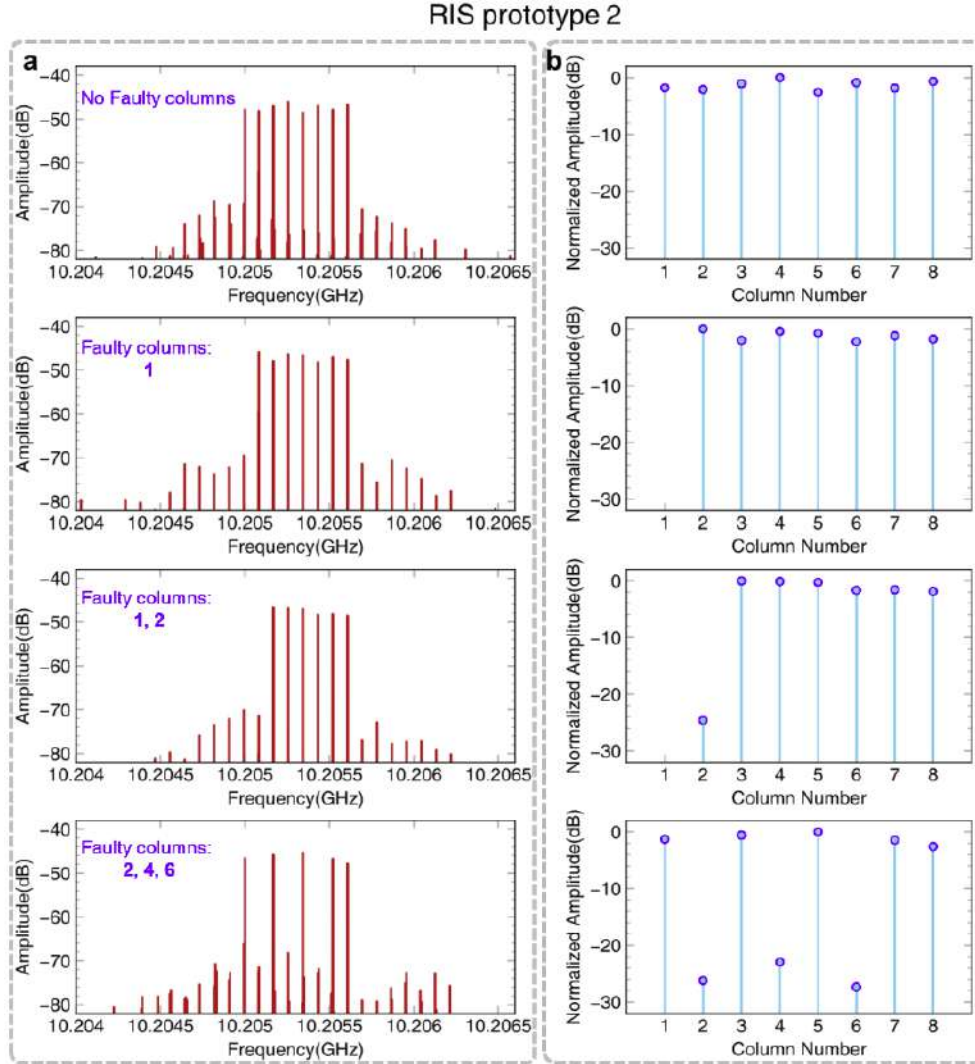


Figure 101: **a)** Measured spectra of the reflected wave for RIS Prototype 2, for four scenarios featuring a varying number of faults. **b)** Corresponding normalized amplitudes of the +1st harmonic order components across all 8 modulation frequencies.

amplitudes of the +1st harmonic order are also shown, highlighting fault conditions where columns at $n = 2$, $n = 5, 9$, and $n = 5, 10, 15$ are inactive. The results demonstrate that the amplitudes of the +1st harmonic order at modulation frequencies assigned to the faulty columns (e. g., $f_c + f_2$, $f_c + f_5$, $f_c + f_9$, or $f_c + f_5$, $f_c + f_{10}$, $f_c + f_{15}$) are significantly attenuated—typically by more than 20 dB—compared to those of normally functioning columns. This clear spectral signature confirms the effectiveness of the proposed method in detecting and localizing faults.

The second RIS prototype operates at a carrier frequency $f_c = 10.2$ GHz and comprises eight columns, with each column containing eight elements. Four experimental cases are presented in Figure 101 showing the measured spectral amplitude distributions and the corresponding normalized amplitudes at the frequency points $(f_c + f_1, f_c + f_2, \dots, f_c + f_8)$. The first case corresponds to normal operation, while subsequent cases simulate faults by disabling specific columns (at $n = 1$, $n = 1, 2$, and $n = 2, 4, 6$). The experimental results show excellent agreement with the theoretical predictions, again confirming that faults cause significant drops in the +1st harmonic order amplitudes at the affected modulation frequencies. Although the two presented prototypes operate at approximately 10 GHz, the proposed diagnostic method is not restricted to a specific frequency band and can be extended to other operating frequencies, including the millimeter-wave band.

5.3.2.2 Extension to 2-bit Configurations

The proposed diagnostic method is not limited to 1-bit RISs, and can be readily extended to multi-bit configurations. As an example, let us consider a 4×4 RIS with 2-bit coding, where each element integrates two PIN diodes (denoted as PIN1 and PIN2). Each diode can be independently switched “ON” or “OFF,” resulting in four distinct operating states: “00,” “01,” “10,” and “11.” These four states ideally exhibit equal reflection amplitude but differ in their reflection phases, typically spaced at 90° intervals, as summarized in Table 3.

Table 3: Characteristics of 2-bit RIS elements.

Operating state	PIN1	PIN2	Relative phase
“00”	OFF	OFF	0°
“01”	OFF	ON	90°
“10”	ON	OFF	180°
“11”	ON	ON	270°

Each RIS element is driven by a common time-coding sequence, as shown in Figure 102a, and is assigned a unique modulation frequency. Under normal operation, a 2-bit element transitions cyclically through the four states

("00," "01," "10," and "11") within a single modulation period, spending an equal duration (25%) in each state. Accordingly, the reflection coefficient of the (n, m) -th element can be expressed as a periodic function:

$$\Gamma_{nm}(t) = \begin{cases} 1, & 0 \leq t < T_{nm}/4, \\ j, & T_{nm}/4 \leq t < T_{nm}/2, \\ -1, & T_{nm}/2 \leq t < 3T_{nm}/4, \\ -j, & 3T_{nm}/4 \leq t < T_{nm}. \end{cases} \quad (132)$$

When this periodic function is expanded into its Fourier series, the amplitude distribution of the resulting Fourier coefficients (illustrated in Figure 102b) shows that the temporal modulation converts the incident wave into multiple harmonic components occurring at orders $\nu = 4k + 1$ with $k \in \mathbb{Z}$. Among these harmonics, the maximum amplitude is observed at the first harmonic order ($\nu = 1$).

Fault behavior in these 2-bit elements can be categorized into three distinct cases based on the status of PIN1 and PIN2. In the first case, when PIN1 is faulty and PIN2 operates normally, the element fails to switch into the "10" and "11" states. Consequently, it cycles only between "00" and "01," as illustrated in Figure 102c. The corresponding amplitude spectrum of the Fourier coefficients (shown in Figure 102d) reveals that the fundamental harmonic component remains dominant, while additional harmonic components emerge at orders $\nu = 4k + 2$ with $k \in \mathbb{Z}$.

In the second case, where PIN1 functions correctly and PIN2 is faulty, the element fails to switch into the "01" and "11" states, thus cycling over "00" and "10," as shown in Figure 102e. The amplitude spectrum in Figure 102f displays harmonic components at orders $\nu = 2k + 1$ with $k \in \mathbb{Z}$, peaking at $\nu = \pm 1$. Although the +1st harmonic component remains present, its amplitude is noticeably reduced compared to the normal operating condition.

In the third case, when both PIN1 and PIN2 are faulty, the element is stuck in a single static state, similar to the fault scenarios observed in 1-bit elements. Consequently, no harmonic conversion of the incident wave occurs, resulting in the absence of higher-order spectral components.

In particular, using the time-coding sequences shown in Figure 102a,c,e, and referring to them as Sequences 1–3, the corresponding sets $\{\Gamma^{(l)}\}_{l=1,\dots,L}$ are $\{1, j, -1, -j\}$, $\{1, j, 1, j\}$, and $\{1, 1, -1, -1\}$, respectively. By setting $L = 4$ and substituting these values into Equation (44) we obtain the following Fourier coefficients

$$a^{(\nu)} = \begin{cases} \frac{j}{2} \operatorname{sinc}\left(\frac{\pi\nu}{4}\right) \sin\left(\frac{\pi\nu}{2}\right) \zeta^{(\nu)} e^{-j\frac{3\pi\nu}{4}}, & \text{Sequence 1,} \\ \frac{1}{2} \operatorname{sinc}\left(\frac{\pi\nu}{4}\right) \cos\left(\frac{\pi\nu}{2}\right) \zeta^{(\nu)} e^{-j\frac{3\pi\nu}{4}}, & \text{Sequence 2,} \\ j \operatorname{sinc}\left(\frac{\pi\nu}{4}\right) \sin\left(\frac{\pi\nu}{2}\right) \cos\left(\frac{\pi\nu}{4}\right) e^{-j\pi\nu}, & \text{Sequence 3,} \end{cases} \quad (133)$$

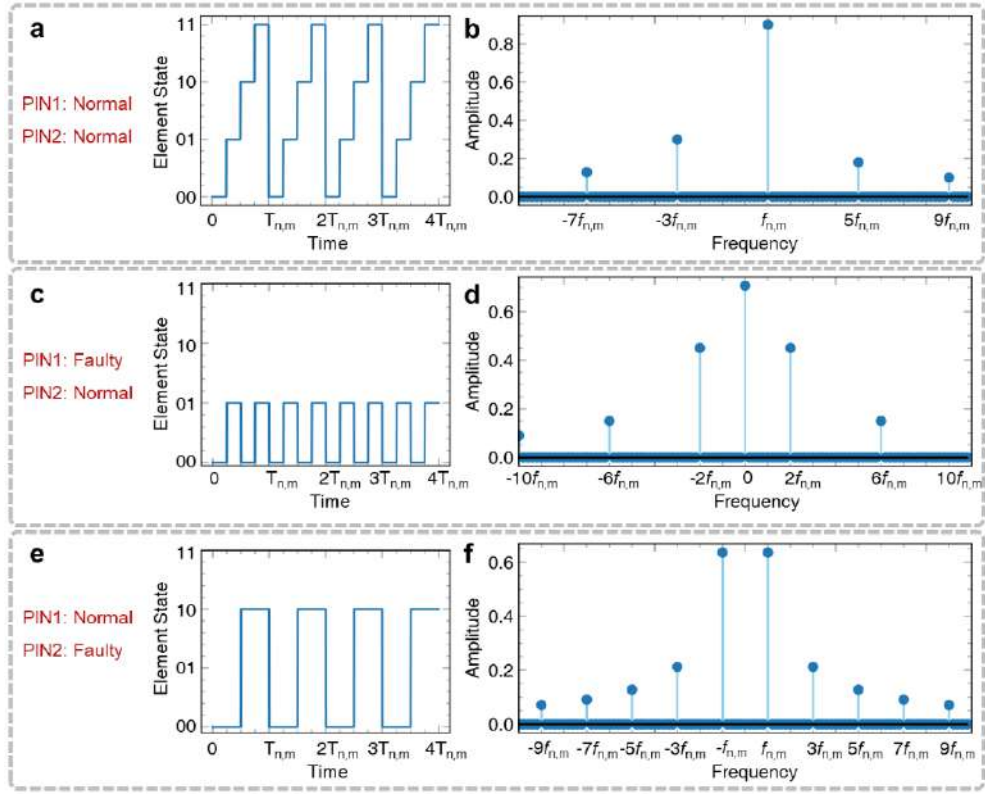


Figure 102: a) Assigned time-coding sequence for a 2-bit RIS element. b) Corresponding amplitude distributions of the Fourier coefficients. c) Actual time-coding sequence when PIN1 is faulty and PIN2 functions normally. d) Corresponding amplitude distribution of the Fourier coefficients. e) Actual time-coding sequence when PIN1 functions normally and PIN2 is faulty. f) Corresponding amplitude distribution of the Fourier coefficients.

where

$$\zeta^{(\nu)} = 1 + je^{-j\frac{\pi\nu}{2}}. \quad (134)$$

These observations demonstrate that different fault types in 2-bit RIS elements produce distinct spectral signatures. By analyzing the amplitude distribution of harmonic components—particularly at orders $\nu = \pm 1$ and $\nu = 2$ —it is possible not only to detect faults but also to infer the specific nature of the malfunction.

As an example, consider a 4×4 RIS with 2-bit coding, where each element is assigned a common time-coding sequence (shown in Figure 102a), along with a unique modulation frequency. When all elements operate correctly, the reflected wave exhibits a spectral amplitude distribution characterized by uniform +1st-order harmonic components across all modulation frequencies, with no significant components at $\nu = -1$ or $\nu = 2$ harmonic orders, as shown in Figure 103a.

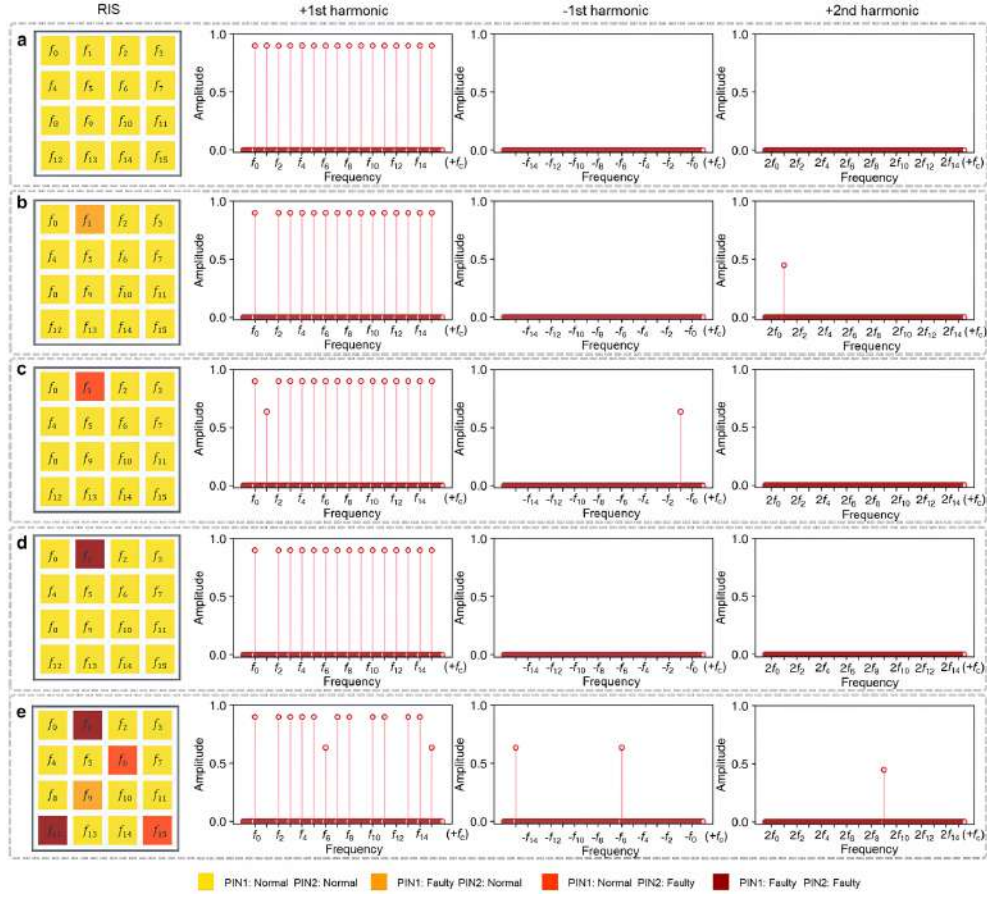


Figure 103: a) Example of an assignment of sixteen distinct modulation frequencies ($f_0, f_1, \dots, f_{15}$) to the elements of a 4×4 RIS with 2-bit coding, along with the corresponding harmonic amplitudes for $\nu = \pm 1$ and $\nu = 2$ when all elements function correctly. b–f) Four representative examples illustrating different faulty elements and fault types, along with the related harmonic amplitudes for $\nu = \pm 1$ and $\nu = 2$.

In the case of the first fault type (PIN1 faulty, PIN2 normal), the amplitude of the harmonic component at its assigned frequency $f_c + f_1$ is significantly reduced compared to the +1st harmonic components of normally operating elements. Additionally, no notable components appear at $\nu = -1$, while the amplitude at $f_c + 2f_1$ (i.e., the +2nd harmonic) is relatively high compared to the corresponding +2nd order harmonics, as illustrated in Figure 103b.

When the same element exhibits the second fault type (normal PIN1, faulty PIN2), the amplitude at $f_c + f_1$ remains lower than other +1st harmonic components. In this scenario, no harmonic components appear at $\nu = 2$, but the -1st harmonic at $f_c - f_1$ is notably stronger than those of other elements, as shown in Figure 103c.

In the third fault scenario, where both PIN1 and PIN2 are malfunctioning, the element is fixed in a single state. As a result, the amplitude at $f_c + f_1$

is again significantly reduced, with no observable harmonic components at $\nu = -1$ or $\nu = 2$, as illustrated in Figure 103d.

Figure 103e presents a composite scenario involving multiple faulty elements of various types. The corresponding spectral amplitude distributions at harmonic orders $\nu = \pm 1$ and $\nu = 2$ in the reflected wave clearly reveal fault behavior. By analyzing these spectral patterns and utilizing the one-to-one correspondence between modulation frequencies and RIS elements, it is possible not only to localize faulty elements but also to determine the specific fault type affecting each.

5.3.3 Experimental Details

PROTOTYPE FABRICATION The RIS Prototypes 1 and 2, shown in Figure 99c,d, are fabricated using standard Printed Circuit Board (PCB) technology. Each meta-atom, with dimensions of $15 \times 15 \text{ mm}^2$, incorporates a PIN diode (MACOM MADP-000907-14020X) that connects a top patch to the grounded patch. The bias voltage is supplied through bottom-layer lines connected through vias, with dynamic bias control implemented using an FPGA hardware control board (ALTERA Cyclone IV).

Prototype 1 features a three-layer metal stack separated by dielectric materials, consisting of a 1.524 mm-thick Rogers RO4003C layer (dielectric constant $\epsilon_r = 3.55$, loss tangent $\tan \delta = 0.0027$), a 0.1 mm-thick FR4 prepreg layer, and an additional 0.508 mm-thick F4B RO4003C layer. Prototype 2 employs a similar metal stack separated by a 1.5 mm-thick F4B layer (dielectric constant $\epsilon_r = 2.65$, loss tangent $\tan \delta = 0.001$), a 0.1 mm-thick FR4 prepreg layer, and an additional 0.5 mm-thick F4B layer.

MEASUREMENTS The experimental setup shown in Figure 99b comprises two linearly polarized X-band horn antennas, employed as transmitting and receiving elements, a signal generator (Keysight E8257D) and a spectrum analyzer (Keysight E4447A).

5.3.4 Summary and Outlook

In conclusion, this work introduces a novel RIS diagnostics method based on non-uniform STC modulation, enabling direct fault detection via simple amplitude-only spectral measurements. By applying a common time-coding sequence to all RIS elements and modulating each element with a distinct frequency, a direct frequency-domain mapping is established between the harmonic components of the reflected wave and individual RIS elements. This correspondence facilitates accurate identification and localization of faulty elements without requiring iterative optimization, coherent detection, or complex signal processing, thereby substantially reducing computational and measurement overhead.

The effectiveness of the proposed approach has been demonstrated through both simulations and experimental validations. Importantly, the method is applicable to both 1-bit and 2-bit RIS configurations and can be readily scaled to larger arrays by employing grouping strategies. The reliance on simple, non-coherent amplitude spectral measurements makes this technique highly practical for real-world implementations across a variety of RIS architectures.

5.4 DIAGNOSTICS VIA UNIFORM MODULATION

This section explores a variant of the approach that adopts a uniform modulation strategy across the entire RIS surface. Specifically, instead of assigning element-specific orthogonal codes, all RIS elements are modulated using a globally consistent time-domain pattern, repeated periodically across the structure. This uniformity allows the RIS to be viewed as a spatially structured, temporally modulated system, where local anomalies (e.g., faulty or misbehaving elements) induce predictable distortions in the global spectral signature.

The proposed diagnostic method exploits these distortions by analyzing the spectral content of the reflected signal, enabling the identification and localization of faults through a simple frequency-domain interpretation. Compared to Hadamard-based coding, this approach trades off spatial resolution for hardware simplicity and reduced control complexity, as fewer unique control sequences are required. Moreover, it maintains compatibility with non-coherent detection schemes and does not rely on precise synchronization, making it particularly appealing for cost-sensitive or time-constrained inspection scenarios.

5.4.1 Theory and Modeling

To accurately model the system's behavior under this uniform modulation scheme, it is essential to describe the control states of each RIS element in terms of the underlying physical parameters. Each element is controlled by two PIN diodes, whose voltages, V_{d1} and V_{d2} , determine the element's reflection state. By assigning logical values to the diode states, i.e., -1 for the "ON" condition and 1 for "OFF," we obtain a convenient representation that maps the electronic control signals to the element's modulation pattern. This mapping is summarized in [Table 4](#).

Therefore, the coding state $\Gamma(t)$ of an element can be expressed as a function of the two diode voltages as follows:

$$\Gamma(t) = s_0[V_{d1}(t)V_{d2}(t) + jV_{d1}(t)], \quad \text{where } s_0 = \frac{1}{\sqrt{2}}e^{-j\frac{\pi}{4}}. \quad (135)$$

Table 4: Mapping of the logical control states (“ON/OFF”) to the coding states of a 2-bit RIS element.

PIN1	PIN2	Coding state
OFF	OFF	1
OFF	ON	j
ON	OFF	-1
ON	ON	$-j$

Now, we can introduce the faulty states s_1 and s_2 , which can assume the value 0 or 1, denoting a faulty state, or a normal functioning state, respectively. By doing this, we can express the two diode voltages as:

$$V_{d1}(t) = s_1 V_1^{(\text{id})}(t) + (1 - s_1) V_1^{(\text{err})}, \quad (136a)$$

$$V_{d2}(t) = s_2 V_2^{(\text{id})}(t) + (1 - s_2) V_2^{(\text{err})}, \quad (136b)$$

where $V_{1,2}^{(\text{id})}$ and $V_{1,2}^{(\text{err})}$ are the ideal and error voltages, respectively, for the two PIN diodes. As a result, considering a RIS composed of N elements, the state of the n -th element can be described as

$$\Gamma_n(t) = s_0 \left[s_{1,n} s_{2,n} V_{1,n}^{(\text{id})}(t) V_{2,n}^{(\text{id})}(t) + (V_{a,n} + j s_{1,n}) V_{1,n}^{(\text{id})}(t) + V_{b,n} V_{2,n}^{(\text{id})}(t) + V_{0,n} \right], \quad (137)$$

where

$$V_{a,n} = s_{1,n} (1 - s_{2,n}) V_{2,n}^{(\text{err})}, \quad (138a)$$

$$V_{b,n} = (1 - s_{1,n}) s_{2,n} V_{1,n}^{(\text{err})}, \quad (138b)$$

$$V_{0,n} = (1 - s_{1,n}) V_{1,n}^{(\text{err})} \left[j + (1 - s_{2,n}) V_{2,n}^{(\text{err})} \right]. \quad (138c)$$

At this point, considering the Fourier series of $\Gamma_n(t)$, the complex-valued equivalent currents are

$$a_n^{(\nu)} = s_0 \left[(V_{a,n} + j s_{1,n}) A_n^{(\nu)} + V_{b,n} B_n^{(\nu)} + s_{1,n} s_{2,n} C_n^{(\nu)} \right], \quad \nu \neq 0, \quad (139)$$

where $A_n^{(\nu)}$, $B_n^{(\nu)}$, and $C_n^{(\nu)}$ are the Fourier coefficients of $V_{1,n}^{(\text{id})}(t)$, $V_{2,n}^{(\text{id})}(t)$, and $V_{1,n}^{(\text{id})}(t) V_{2,n}^{(\text{id})}(t)$, respectively. As a consequence, the far-field scattering pattern corresponding to the ν -th harmonic order can be expressed as

$$F^{(\nu)}(\theta) = \sum_{n=1}^N a_n^{(\nu)} e^{j k d (n-1) \sin \theta}, \quad (140)$$

and the far-field measurements are computed for a fixed angle $\theta = \theta_0$

$$\mathcal{M}^{(\nu)}(\theta_0) = \left| F^{(\nu)}(\theta_0) \right|^2. \quad (141)$$

Note that this formulation reduces to the 1-bit case when

$$s_0 = -j, \quad (142a)$$

$$s_{2,n} = 0 \quad \forall n, \quad (142b)$$

$$V_{d2}(t) = 0. \quad (142c)$$

To enable rapid diagnosis of the RIS based on far-field measurements, we optimized the STC matrix using a Genetic Algorithm (GA) to establish a mapping between the diagnostic state and the corresponding complex-valued equivalent currents.

Specifically, the 1-bit case allows for a straightforward mapping: When the PIN diode is functioning normally, the equivalent currents excite a pair of symmetric harmonic orders, while a faulty PIN diode results in no harmonic orders being excited. Since the coding alphabet is $\{1, -1\}$, the amplitudes of the equivalent currents will be symmetric with respect to ν . In this case, we can define the following set to identify the labeled harmonic orders:

$$\mathcal{A} = \{\nu \in \mathbb{Z} \mid 1 \leq \nu \leq N\}. \quad (143)$$

In the 2-bit case, instead, we implemented the following diagnostic mapping:

- Case 1: When both PIN diodes of the n -th element are normally operating, i. e., $s_{1,n} = s_{2,n} = 1$, the equivalent currents excite only a positive harmonic order. In particular, we considered the first N consecutive harmonic orders ($\nu = 1, \dots, N$) given by the following set:

$$\mathcal{B}_1 = \mathcal{A}. \quad (144)$$

- Case 2: When PIN1 is normally operating and PIN2 is faulty, i. e., $s_{1,n} = 1$ and $s_{2,n} = 0$, the equivalent currents excite symmetric harmonic orders in the range $\nu \in \{\pm 1, \dots, \pm N\}$, defined by the set:

$$\mathcal{B}_2 = \{\nu \in \mathbb{Z} \mid 1 \leq |\nu| \leq N\}. \quad (145)$$

In this case, the n -th element excites only the harmonic orders $\nu = \pm n$.

- Case 3: When PIN1 is faulty and PIN2 operates normally, i. e., $s_{1,n} = 0$ and $s_{2,n} = 1$, the equivalent currents excite a pair of harmonic orders that are symmetric with respect to $\nu = 0$. These orders are shifted by N compared to those in Case 2 ($\nu \in \{\pm(N+1), \dots, \pm 2N\}$) and are given by the set:

$$\mathcal{B}_3 = \{\nu \in \mathbb{Z} \mid N+1 \leq |\nu| \leq 2N\}. \quad (146)$$

In this case, the n -th element excites only the harmonic orders $\nu = \pm(N+n)$.

5.4.2 Optimization Criterion

Referring to Equation (135), the generic element of the coding sequence is

$$\Gamma_n^{(l)} = \begin{cases} V_{1,n}^{(l)}, & \text{1-bit case,} \\ \begin{cases} s_0 (V_{1,n}^{(l)} V_{2,n}^{(l)} + j V_{1,n}^{(l)}), & \text{Case 1,} \\ s_0 (1 + j) V_{1,n}^{(l)}, & \text{Case 2,} \\ s_0 (V_{2,n}^{(l)} + j), & \text{Case 3,} \end{cases} & \text{2-bit case,} \end{cases} \quad (147)$$

where $V_{1,n}^{(l)}$, $V_{2,n}^{(l)}$ describe the coefficients of the sequences associated with $V_{d1}(t)$ and $V_{d2}(t)$, respectively. These sequences share the same functional expression as shown in Equation (38). To achieve the harmonic mapping discussed so far, we optimized the coding sequence $\{\Gamma_n^{(l)}\}_{l=1,\dots,L}$ using a discrete GA. This optimization was performed to enforce the constraint $\Gamma_n^{(l)} \in \{1, j, -1, -j\}$, minimizing the following fitness function:

$$\mathcal{F}_n^{(\bar{v})} = \begin{cases} \frac{\|\alpha_n\|_2}{|a_n^{(\bar{v})}|}, & \text{1-bit case} \\ \frac{1}{3} \sum_{m=1}^3 D_{nm}^{(\bar{v})}, & \text{2-bit case} \end{cases}, \quad n = 1, \dots, N, \quad (148)$$

where $\|\cdot\|_2$ is the L_2 -norm, $\bar{v}$ is the chosen harmonic order to map the n -th element, and auxiliary terms are defined as:

$$\alpha_n = \left\{ a_n^{(v \neq \bar{v})} \right\}_{v \in \mathcal{J}, v \in \mathcal{A}}, \quad (149a)$$

$$D_{nm}^{(\bar{v})} = \frac{\|\beta_{nm}\|_2}{|a_n^{(\bar{v})}|}, \quad (149b)$$

with

$$\beta_{nm} = \left\{ a_n^{(v \neq \bar{v})} \right\}_{v \in \mathcal{J}, v \in \mathcal{B}_m}, \quad (150)$$

where $\mathcal{J}$ is the set of harmonic orders considered to compute the synthetic measures, namely:

$$\mathcal{J} = \begin{cases} \{v \in \mathbb{Z} \mid 1 \leq v \leq N\}, & \text{1-bit case,} \\ \{v \in \mathbb{Z} \mid 1 \leq |v| \leq 2N\}, & \text{2-bit case.} \end{cases} \quad (151)$$

The terms $D_{nm}^{(\bar{v})}$ in the definition of the fitness function in the 2-bit case arise because the final coding sequence of the n -th element requires three joint optimizations. These optimizations are necessary to fulfill the mapping described in Cases 1–3, which in turn requires the coefficients $a_n^{(v)}$ to be computed accordingly.

5.4.3 Results and Discussion

Before proceeding to the analysis, it is essential to clearly define the concept of a fault state. A diode is in a fault state when it is no longer capable of switching dynamically over time, remaining permanently locked in one of its two possible operational states: “OFF” or “ON”. For the purposes of this discussion, the term faulty diode will specifically refer to a non-switching diode that is locked in the “OFF” state.

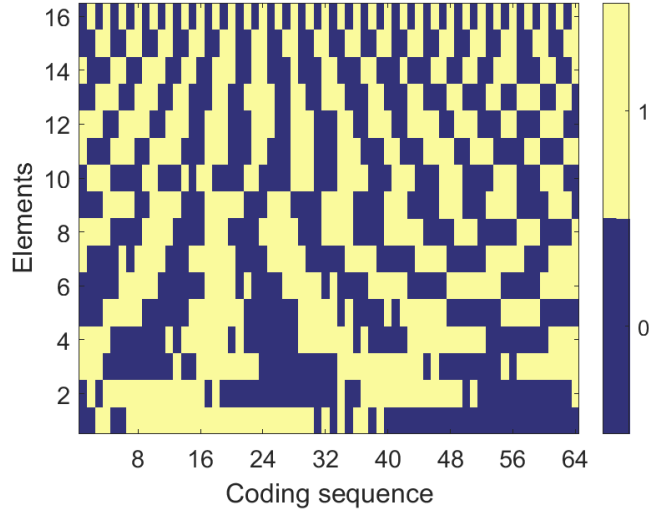


Figure 104: Optimized STC matrix for the 1-bit diagnostics scenario, pertaining to a RIS featuring $N = 16$ elements, with a time-coding sequence of length $L = 64$. The blue and yellow pixels represent the “0” and “1” digits of the 1-bit coding, respectively.

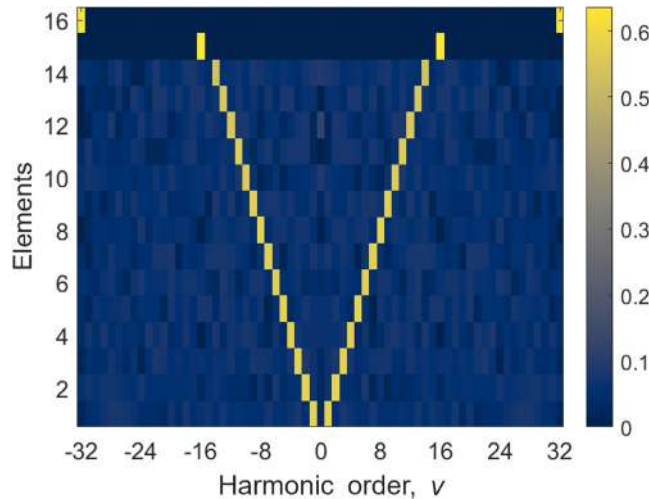


Figure 105: Complex-valued equivalent currents pertaining to the optimized STC matrix in Figure 104.

Now, according to the adopted diagnosis mapping, and considering a RIS of $N = 16$ elements, the results of the GA-based optimization for the STC matrices are reported in Figures 104 and 106, for a time-coding sequence of length $L = 64$ and $L = 128$, respectively, corresponding to the 1-bit and 2-bit cases. The corresponding mappings between the elements and the complex-valued equivalent currents are shown in Figures 105 and 107 for the 1-bit and 2-bit case, respectively. Note that in the 1-bit case, the equivalent currents from the last two elements ($n = 15, 16$) excite the orders $\pm 16, \pm 32$. This choice is based on the fact that by alternating the coding sequence between states “0” and “1” with periods of 1 and 2, we obtain sequences such as “0101...” and “0011...”; resulting in $a_n^{(v)} \propto \tilde{\delta}_{nv}$, with $n = v \in \{\pm 16, \pm 32\}$, respectively, thus avoiding the optimizations typically required for such coding sequences.

Exploiting this mapping, we aim to train a regression model to identify the faulty state of elements in the RIS by analyzing the far-field measurements described in Equation (141). Specifically, the input dataset is exhaustive and includes all possible binary fault configurations $\mathbf{s} \in \mathbb{R}^{N_d}$, where N_d denotes the total number of diodes. In the 1-bit case, each element integrates only one diode, so $N_d = N$ and $\mathbf{s} = \mathbf{s}_1$. In the 2-bit case, instead, each RIS element integrates two diodes, leading to $N_d = 2N$. The binary vector $\mathbf{s}$ is thus partitioned into two halves: $\mathbf{s}_1$, corresponding to the first diode in each element, and $\mathbf{s}_2$, corresponding to the second. The dataset considers up to $N_F = 4$ faulty elements, as illustrated in Figure 108a,b. Given these input datasets, the corresponding output datasets are computed using Equation (140), which are illustrated in Figure 108a,b.

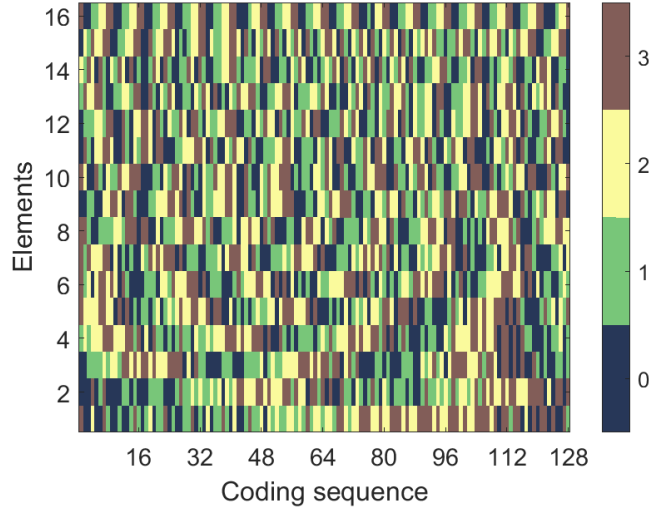


Figure 106: Optimized STC matrix for the 2-bit diagnostics scenario, pertaining to a RIS featuring $N = 16$ elements, with a time-coding sequence of length $L = 128$. The blue, green, yellow, and brown pixels represent the “0,” “1,” “2,” and “3” digits of the 2-bit coding, respectively.

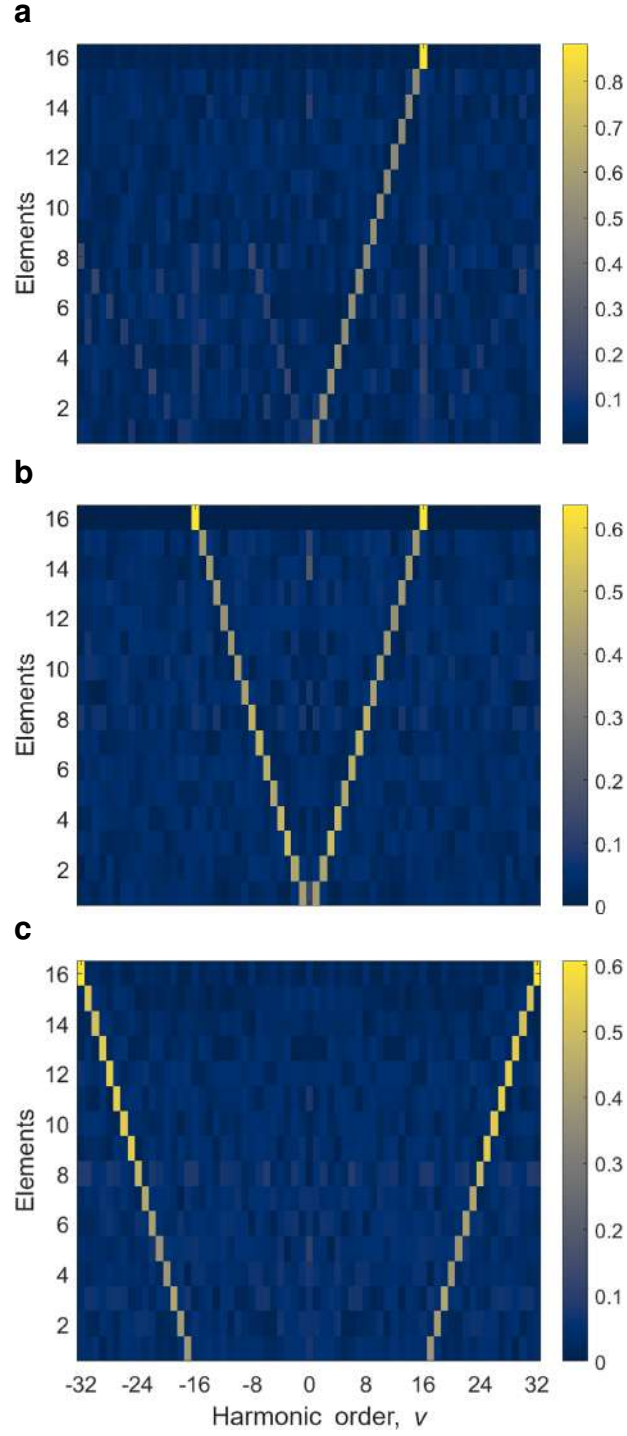


Figure 107: Complex-valued equivalent currents pertaining to the diagnosis mapping for a 2-bit RIS featuring $N = 16$ elements and a time-coding sequence of length $L = 128$. **a–c)** Spectral behaviors corresponding to Case 1–3, respectively.

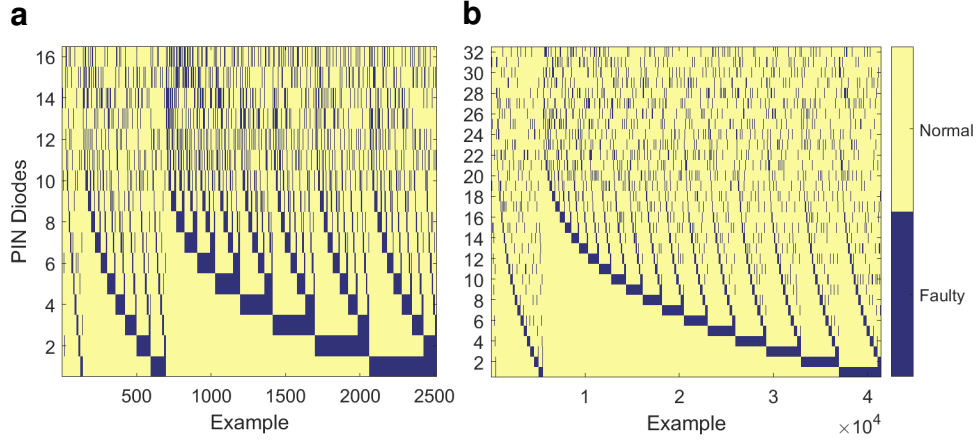


Figure 108: Input dataset composed by all the binary faulty sequences with a number of faulty elements $N_F = 1, \dots, 4$. **a)** 1-bit case. **b)** 2-bit case.

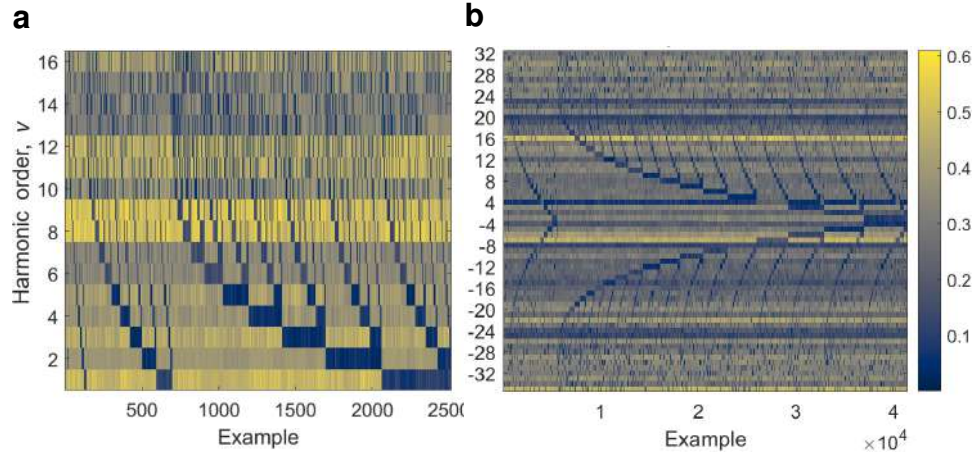


Figure 109: Output dataset composed by all the harmonic far-field scattering pattern measures corresponding to the input dataset, computed in correspondence of the observation angle $\theta_0 = 40^\circ$. **a)** 1-bit case. **b)** 2-bit case.

The training performance of the regression model is presented in Figure 110a,b, where the training loss is plotted over the iterations, for both 1-bit and 2-bit case. Furthermore, Figure 111 shows the DER, as a function of the SNR, for different values of the number of faulty elements N_F . This error rate has been computed as

$$\text{DER}(N_F) = \frac{N_{\text{err}}}{N_{\text{comb}}(N_F)}, \quad (152)$$

where

$$N_{\text{err}} = \|\hat{\mathbf{s}} - \mathbf{s}\|_1, \quad (153a)$$

$$N_{\text{comb}}(N_F) = \binom{N_d}{N_F} = \frac{N_d!}{N_F!(N_d - N_F)!}. \quad (153b)$$

It is worth noting that the difference $\hat{\mathbf{s}} - \mathbf{s}$, used to define the number of errors committed by the regression model, also provides information on the nature of the detection error, i. e., we have that

$$\hat{s}_n - s_n = \begin{cases} +1, & \text{Missed detection,} \\ 0, & \text{Correct detection,} \\ -1, & \text{False alarm.} \end{cases} \quad (154)$$

In particular, the *missed detection* state occurs when the regressor fails to detect a faulty element, meaning a fault exists but the regressor incorrectly predicts that the element is functioning properly. The *correct detection* state refers to the case where the regressor successfully identifies whether an element is functioning properly or is faulty, matching the true state of the element. Finally, the *false alarm* state refers to the case where the regressor incorrectly classifies a functioning element as faulty. This represents a false positive where a non-defective element is mistakenly marked as defective.

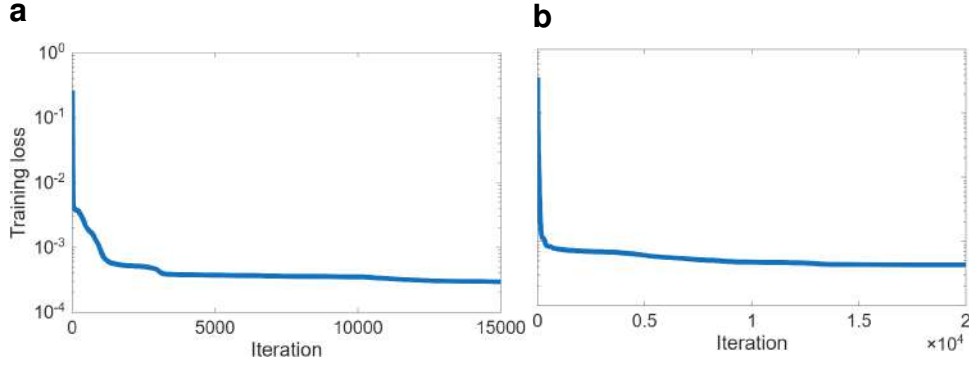


Figure 110: Training loss of the regression model. **a)** 1-bit case. **b)** 2-bit case.

To plot the DER as a function of the SNR, Gaussian noise components $n_r, n_i \sim \mathcal{N}(0, \sigma^2)$ were added to both the real and imaginary parts of the harmonic scattering pattern described by Equation (140). These noisy components were then used to compute the synthetic measurements shown in Figure 109a,b. Specifically, for a signal power P_s and a fixed noise power $P_n = 2\sigma^2$, where σ^2 denotes the noise variance affecting the measures defined in Equation (141), we defined the SNR as

$$\text{SNR} = \frac{P_s}{P_n} = \frac{1}{2\sigma^2} \sum_{n=1}^N \sum_{v \in \mathcal{J}} |a_n^{(v)}|^2. \quad (155)$$

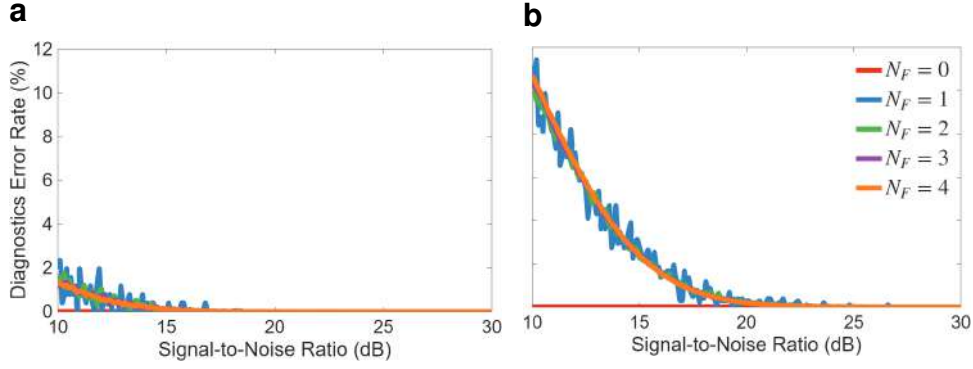


Figure 111: DER of the trained regression model, displayed as a function of the SNR, for different values of N_F . **a)** 1-bit case. **b)** 2-bit case.

As can be observed from Figure 111a,b, the DER increases as the SNR decreases, exhibiting a clear degradation trend. In particular, the DER starts to significantly deteriorate below ≈ 16 dB in the 1-bit case and 20 dB in the 2-bit case, for all fault configurations except $N_F = 0$, which corresponds to the no-fault condition. The overall worse performance exhibited by the regressor in the 2-bit case can be attributed to the increased complexity of the problem—since the number of unknowns is doubled compared to the 1-bit case—as well as to a less effective convergence of the GA during the optimization of the corresponding STC matrix.

5.4.4 Additional Results

In this section, we discuss and compare the results obtained both numerically and experimentally, using the same theoretical model that has been discussed so far, but with a different temporal coding strategy. Specifically, instead of optimizing the sequences via GA to achieve a specific spectral behavior of the equivalent complex amplitudes, Pseudorandom Noise (PN) sequences were employed.

5.4.4.1 Pseudorandom Noise Sequences

In communication systems, PN sequences [141] are widely used for applications like spread spectrum modulation, cryptography, and error detection. These sequences, though appearing random, are generated by deterministic algorithms and can be reproduced if the generating parameters are known. A common method for generating PN sequences is using Linear Feedback Shift Registers (LFSRs) [142], which are characterized by feedback polynomials.

In what follows, we utilize the MATLAB[®]comm.PNSequence [143] system object, which allows for the generation of PN sequences based on specified parameters. The sequence is produced using a combination of feedback poly-

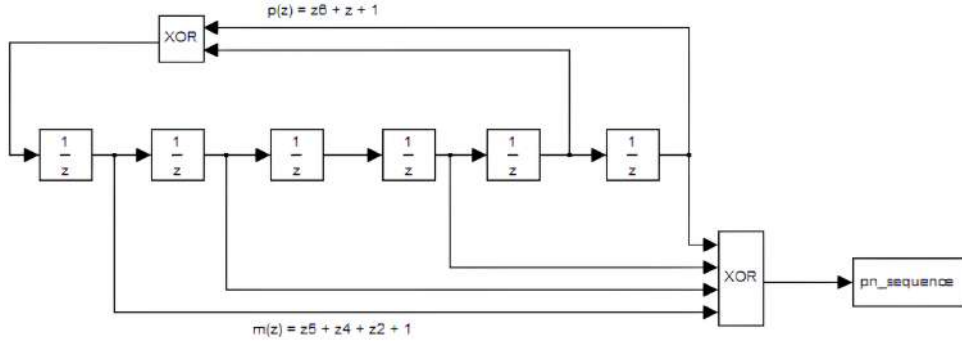


Figure 112: Schematic representation of a LFSR used as a PN sequence generator, characterized by the generator polynomial $p(z) = z^6 + z + 1$ and the mask polynomial $m(z) = z^5 + z^4 + z^2 + 1$ [143]. These polynomials are specific to the example presented in this figure and have been chosen as a proof of concept to demonstrate the PN sequence generation.

nomials—one representing the generator polynomial, denoted as $p(z)$, and another representing the mask polynomial, $m(z)$. The generator polynomial defines the feedback taps of the LFSR, while the mask polynomial is applied to the output sequence to alter its characteristics, enhancing its randomness and ensuring that it meets specific system requirements.

Figure 112 illustrates the LFSR structure, which, driven by the generator and mask polynomials, outputs a sequence of bits that are pseudo-random, yet reproducible under the same conditions. To characterize the PN sequence generator, the following polynomials were chosen:

$$\begin{aligned} p(z) &= z^h + z + 1, \\ m(z) &= 1, \end{aligned} \quad (156)$$

where

$$h = \lceil \log_2(NL) \rceil. \quad (157)$$

This allows for deriving the STC matrix from a single frame of length NL , ensuring that the sequences exhibit good pseudo-random characteristics.

5.4.4.2 Results for PN Sequences

The experimental setup shown in Figure 113 is identical to that in Figure 99b. The transmitting and receiving antennas are positioned at 0.85 m and approximately 2.5 m away from the RIS, respectively. The RIS under test, as shown in Figure 99c, has an element periodicity of 15 mm, a diode switching frequency of 4 MHz, a modulation frequency $f_0 = 0.0625$ MHz, and is illuminated by a plane wave with a central frequency $f_c = 9.8$ GHz.

The equipment in the foreground includes a signal generator and a spectrum analyzer, which are essential for generating test signals and analyzing the frequency response of the system, respectively. The white grid and angular markings on the floor allow precise control and adjustment of the mea-

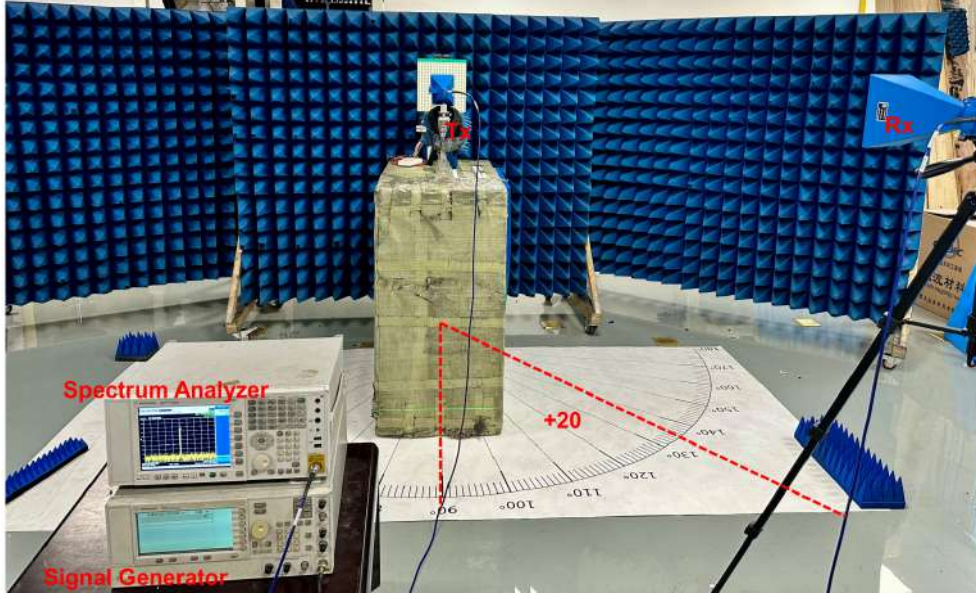


Figure 113: The experimental setup, conducted within an anechoic chamber, is designed for testing a RIS. The signal generator and spectrum analyzer are positioned in the foreground to enable signal transmission analysis at a frequency of 9.8 GHz. The transmitting antenna is aligned with the RIS, while the receiving antenna is positioned at an angle of $\theta_0 = 20^\circ$, swept relative to the normal of the RIS. The floor grid aids in adjusting the measurement angle.

surement angle. For these tests, the measurement angle used was $\theta_0 = 20^\circ$. This controlled experimental environment, with the use of specialized equipment and the anechoic chamber, ensures accurate testing of the RIS's performance without interference from external noise or reflections.

To perform experimental tests, 30 simulation scenarios were considered, with a number of faults $N_F = 0, \dots, 4$. However, for brevity, 4 significant examples are presented and discussed to illustrate the issues that arose from these tests. In particular, Figures 114a, 115a, 116a, and 117a show four test configurations considered, along with the corresponding STC matrices. As noted, when an element is faulty, the corresponding row of the STC matrix does not vary over time, presenting a constant coding. Configuration 1 is the case where all N elements of the 1-D RIS are functioning normally. This case was included as it serves as a reference point for interpreting the subsequent results. Configurations 2, 3, and 4 feature one, two, and three faulty elements, respectively. Figures 114b, 115b, 116b, and 117b show the corresponding spectral behaviors of the complex-valued equivalent amplitudes.

During the test, we found that the harmonic spectrum is sensitive to the observation angle due to the PN distribution. A deviation of the observation angle by $1\text{--}2^\circ$ leads to a slight difference in harmonic distribution.

Figures 114c, 115c, 116c, and 117c show the comparison between the numerically derived synthetic harmonic measurement of the far-field scattering

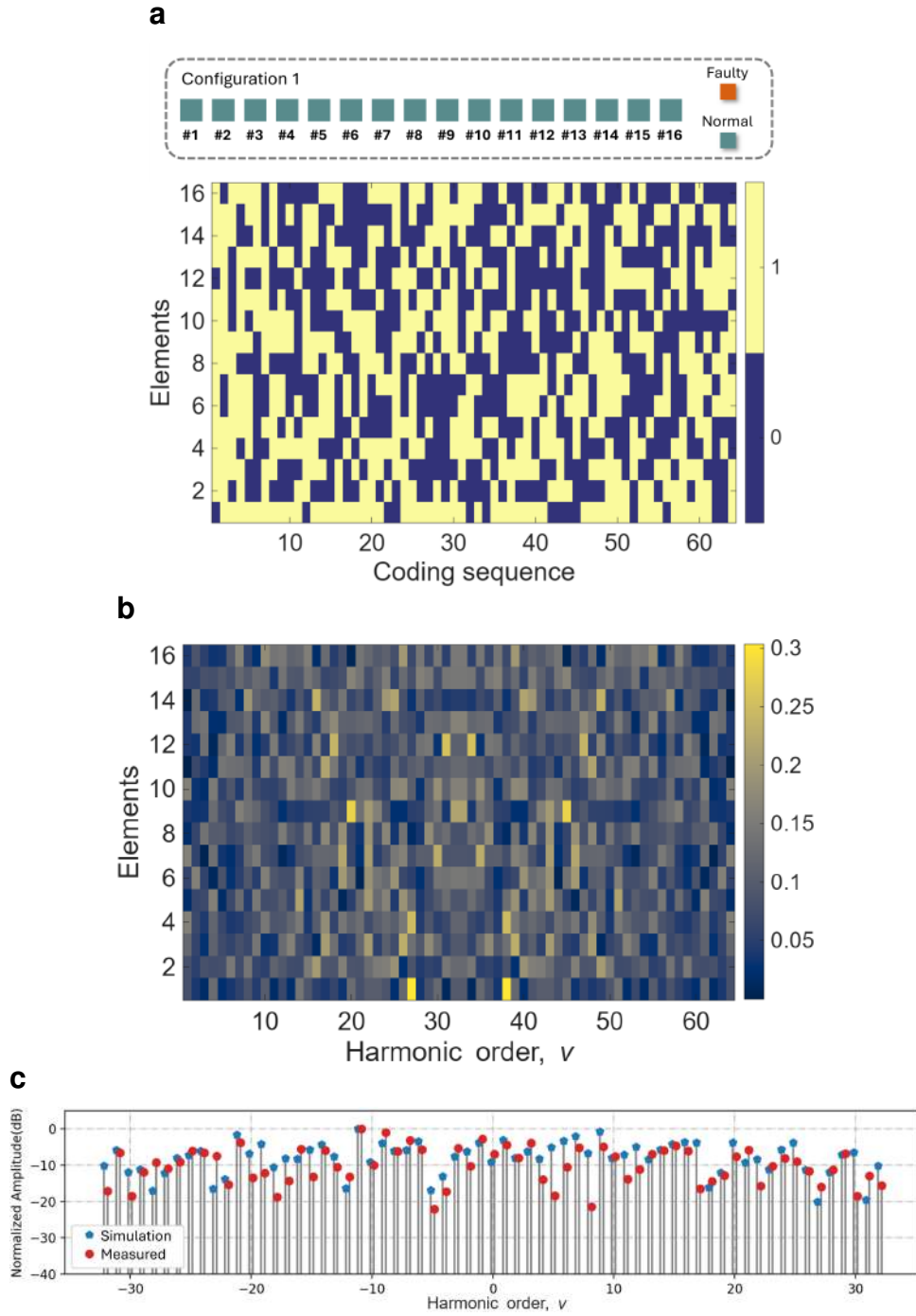


Figure 114: **a)** STC matrix for the 1-bit diagnostics using PN sequences, corresponding to a RIS with $N = 16$ elements, all of which are operating normally, and a time-coding sequence of length $L = 64$. **b)** Corresponding spectral behavior of the complex-valued equivalent amplitudes. **c)** Comparison between the numerically-derived synthetic harmonic measurement of the far-field scattering pattern and the actual experimental measurement for $\theta_0 = 20^\circ$.

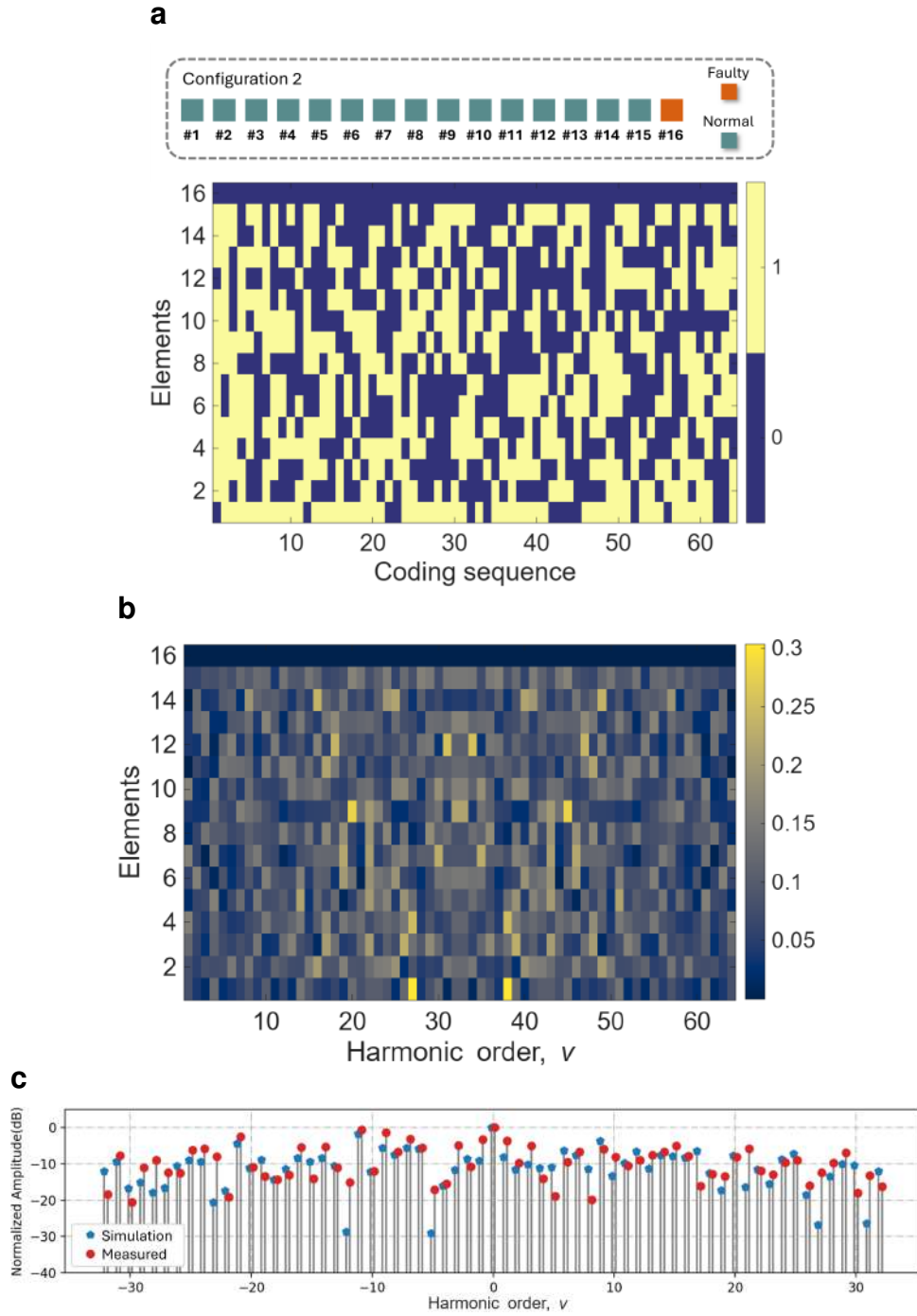


Figure 115: **a)** STC matrix for the 1-bit diagnostics using PN sequences, corresponding to a RIS with $N = 16$ elements, one of which is faulty, and a time-coding sequence of length $L = 64$. **b)** Corresponding spectral behavior of the complex-valued equivalent amplitudes. **c)** Comparison between the numerically-derived synthetic harmonic measurement of the far-field scattering pattern and the actual experimental measurement for $\theta_0 = 20^\circ$.

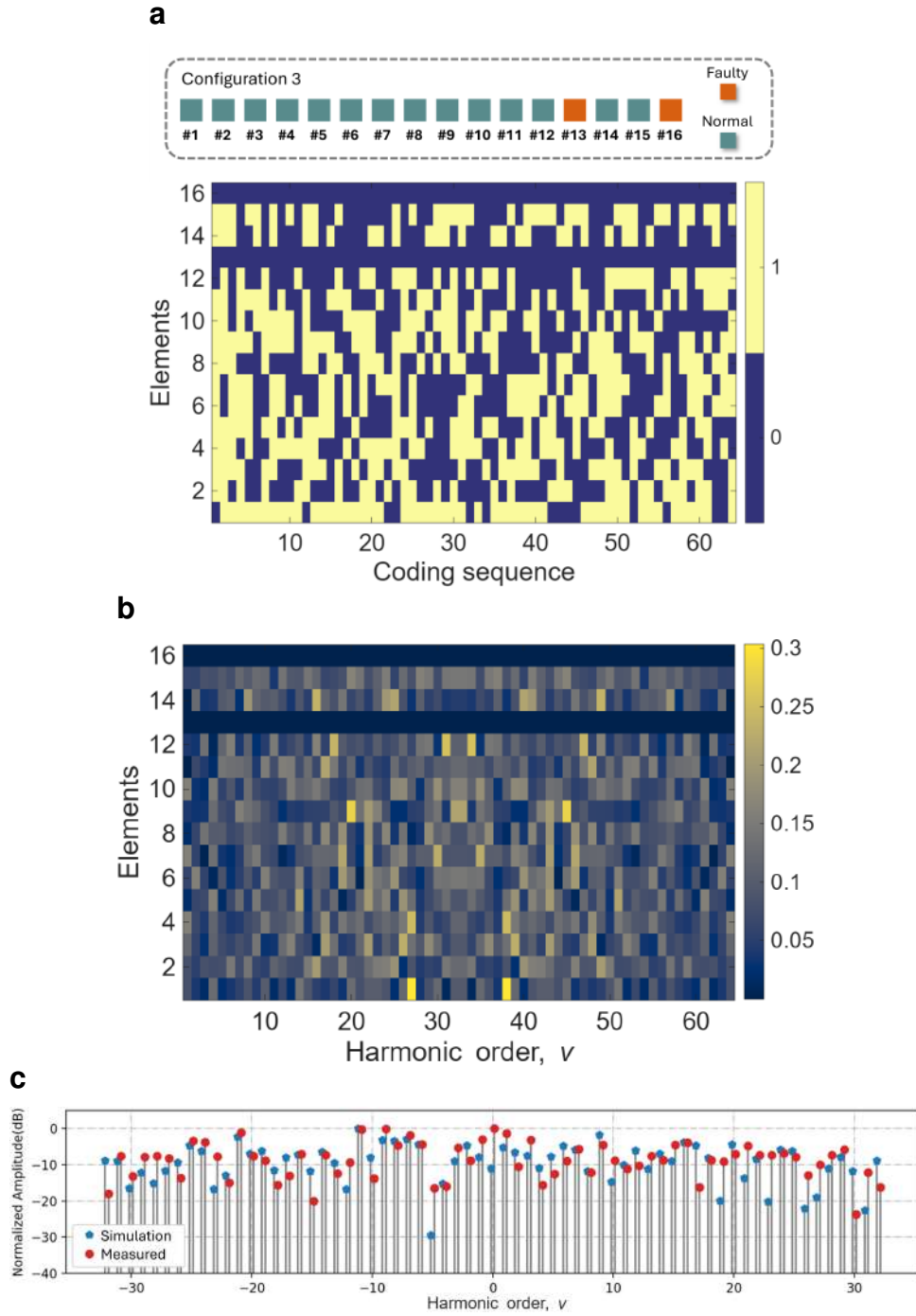


Figure 116: **a)** STC matrix for the 1-bit diagnostics using PN sequences, corresponding to a RIS with $N = 16$ elements, two of which are faulty, and a time-coding sequence of length $L = 64$. **b)** Corresponding spectral behavior of the complex-valued equivalent amplitudes. **c)** Comparison between the numerically-derived synthetic harmonic measurement of the far-field scattering pattern and the actual experimental measurement for $\theta_0 = 20^\circ$.

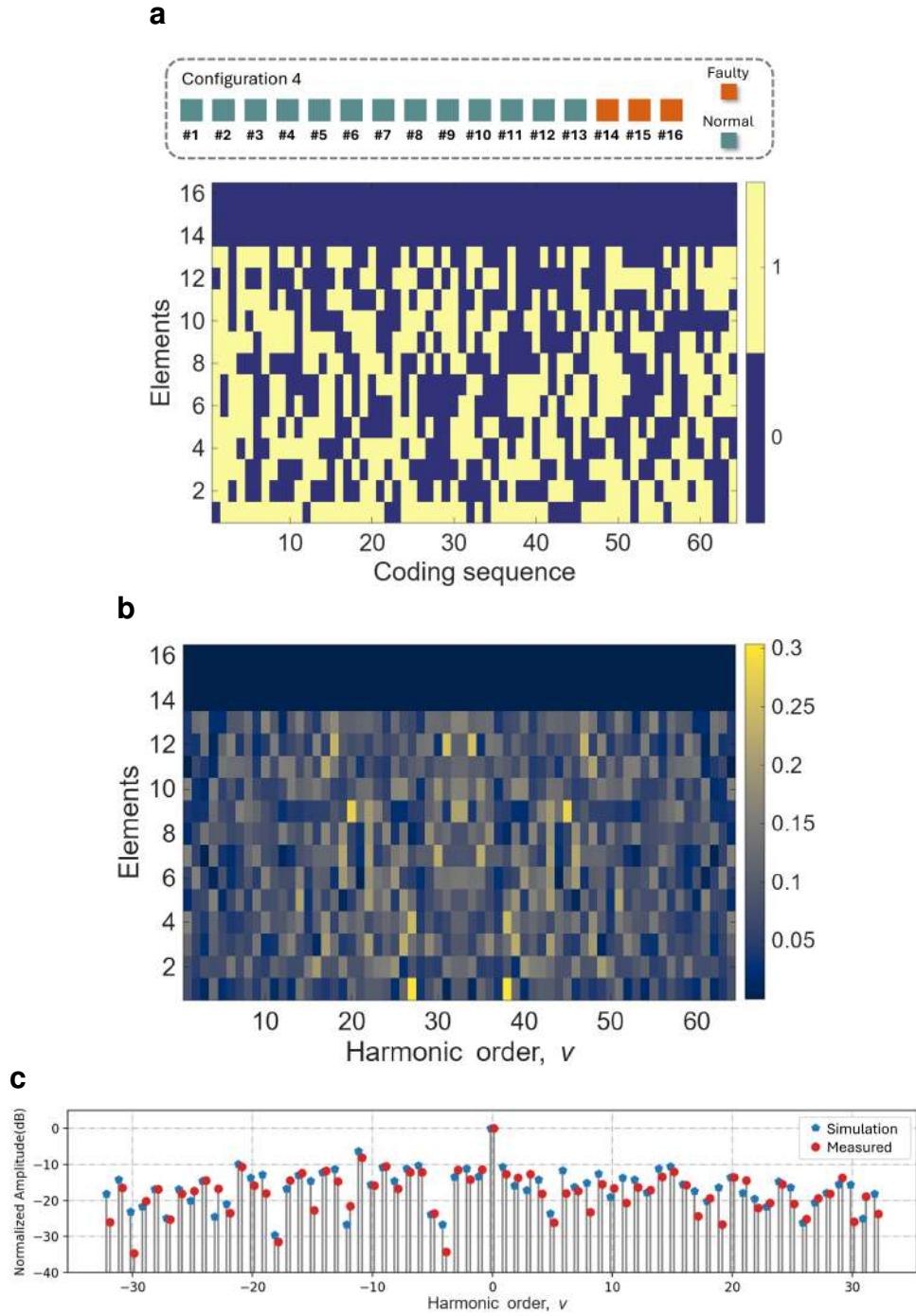


Figure 117: **a)** STC matrix for the 1-bit diagnostics using PN sequences, corresponding to a RIS with $N = 16$ elements, three of which are faulty, and a time-coding sequence of length $L = 64$. **b)** Corresponding spectral behavior of the complex-valued equivalent amplitudes. **c)** Comparison between the numerically-derived synthetic harmonic measurement of the far-field scattering pattern and the actual experimental measurement for $\theta_0 = 20^\circ$.

pattern and the actual experimental measurement. As can be observed, the harmonic distribution of simulation and experiment is similar to some extent, but there are no obvious characteristics and trends. This may be due to testing errors and the approximation of the metasurface model. Therefore, the accuracy of using the test results to validate the CNN model trained on the simulation data remains uncertain. Furthermore, the variation in harmonic distribution for different STC matrix sequences, observed at the same observation angle, is not particularly significant. At present, it is uncertain whether these differences can be accurately and robustly characterized using the CNN model.

The input dataset, shown in Figure 108a, is used to generate the corresponding output dataset, which is illustrated in Figure 118.

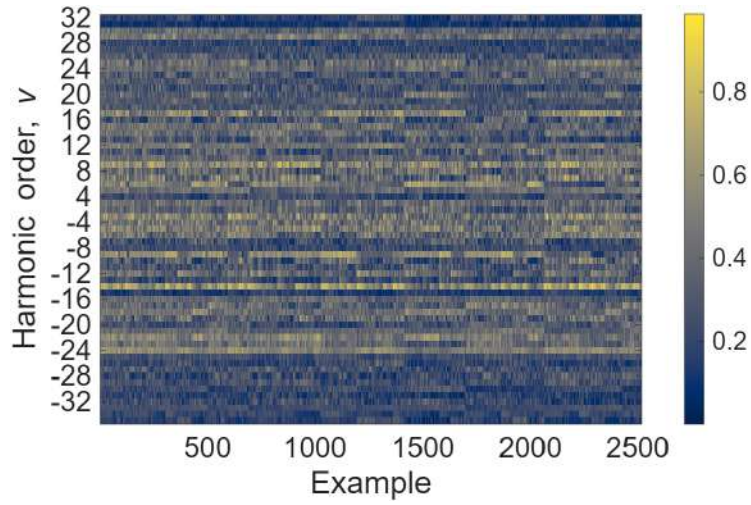


Figure 118: The output dataset consists of the harmonic far-field scattering pattern measurements corresponding to the input dataset, computed at the observation angle $\theta_0 = 20^\circ$ using the PN sequences.

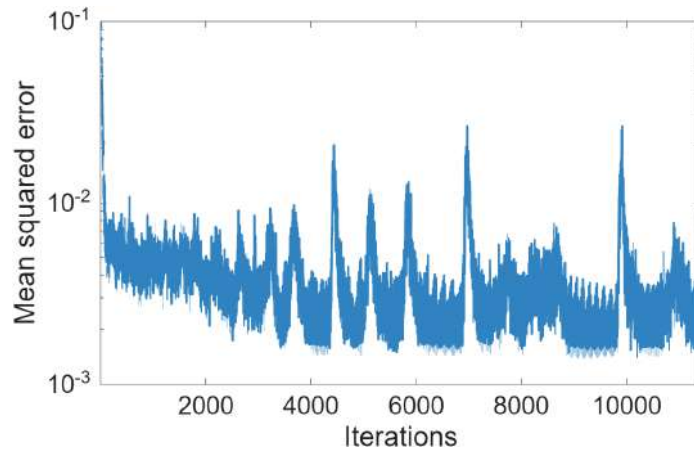


Figure 119: Training loss of the inverse CNN model.

The training performance of the inverse CNN model is presented in Figure 119, where the training loss is plotted over the iterations. A comparison of the training performance shown in Figure 119 with that in Figure 111a clearly reveals that the CNN model exhibits stronger oscillations during training. While these oscillations in the learning curve of the inverse CNN, particularly in a tandem architecture, are not unusual, they do not necessarily indicate problems with the overall convergence of the model. These fluctuations are typically associated with the complexity of the network architecture, the non-linear nature of the activation functions, and the optimization algorithm used.

It is important to emphasize that these oscillations do not impede the network's ability to converge to an optimal MSE. In fact, in many cases, the final error value remains stable, and further training does not lead to significant improvements. As the network stabilizes over several epochs, the oscillations tend to decrease, and the model ultimately converges to a point where the error cannot be significantly reduced any further, signaling that the optimal solution has been reached.

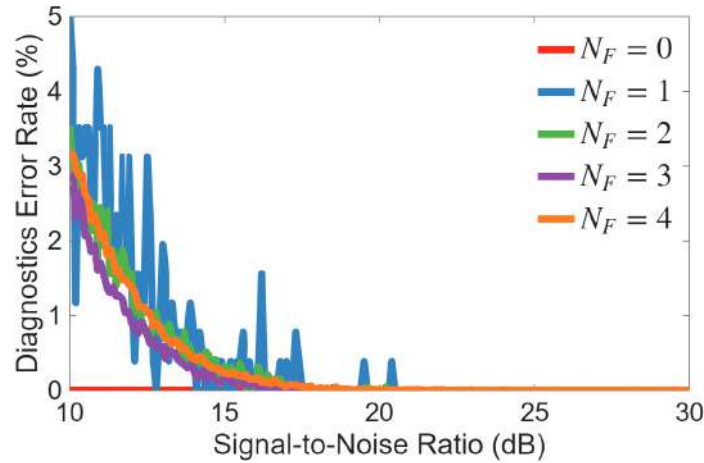


Figure 120: DER of the trained inverse CNN model, displayed as a function of the SNR, for different values of N_F .

Furthermore, Figure 120 shows the DER, as a function of the SNR, for different values of the number of faulty elements N_F . As can be observed from Figure 120, the DER increases as the SNR decreases, exhibiting a clear degradation trend. In particular, the DER starts to deteriorate below ≈ 18 dB, for all fault configurations except $N_F = 0$.

By using PN sequences, the obtained results are comparable to those shown in Figure 111a in terms of DER performance. However, experimental evidence revealed a strong sensitivity of the far-field harmonic measurements to factors such as noise and measurement inaccuracies, even for small variations in the observation angle, on the order of a few degrees. Furthermore, it is important to note that the experimental tests were conducted well

before the Fraunhofer region, meaning that the curvature of the wavefront is non-negligible.

These tests highlighted clear issues with the sensitivity of the measurements to noise and other system parameters, stemming from the pseudo-random nature of the used coding sequences. Consequently, under these conditions, the proposed approach is not well-suited for solving diagnostic problems that aim to infer faults in RIS based on far-field harmonic measurements.

5.4.5 *Experimental Details*

MODELING The numerical implementation of the theoretical analysis was carried out using MATLAB[®], with the far-field scattering pattern of the basic element approximated as $E(\theta, \phi) \approx \cos \theta$.

REGRESSION MODEL IMPLEMENTATION The neural network regression model was implemented using the `fitrnet` library in MATLAB[®] [144]. The network consists of a single hidden layer with $8N_d$ neurons. Therefore, for the 1-bit scenario, resulting in $8N$ neurons, while for the 2-bit scenario, the network contains $16N$ neurons. The input to the network is a feature vector extracted from the measurement data, while the output is the predicted diagnostic vector. The model employs the ReLU activation function in the hidden layer and has been trained using the default settings of the `fitrnet` function, using Limited-Memory Broyden–Fletcher–Goldfarb–Shanno Quasi-Newton Algorithm (LBFGS) [145] as the training solver and the MSE loss function.

TANDEM MODEL IMPLEMENTATION The tandem CNN scheme was implemented and tested using the functionalities provided by the `dlnetwork` library in MATLAB[®] [113]. The sigmoid activation function used in the final layers is $f_\sigma(x) = 1/(1 + \exp(-x))$. The architecture, training algorithm, and dataset partitioning follow the same setup as described in the experimental section of Section 4.3.5. The layers are listed in Table 5 in the same order as they appear in the network, with the inverse model placed after the direct model.

5.4.6 *Summary and Outlook*

This framework presented a diagnostic method for RIS-based systems utilizing uniform STC modulation. By exploiting the predictable distortions in the spectral signature caused by faulty or misbehaving elements, we demonstrated how a frequency-domain analysis could effectively identify and localize faults. The approach offers a trade-off between spatial resolution and hardware simplicity, providing a viable solution for cost-sensitive and time-constrained scenarios. Unlike more complex Hadamard-based cod-

Table 5: Architecture of the inverse model using a CNN.

Layer	Description
Fully connected	256 neurons
Reshape	16×16 feature map
Hidden ($5\times$)	Conv [†] , BatchNorm [‡] , ReLU activation
Fully connected	512 neurons, sigmoid activation
Output	N features, sigmoid activation

[†] Convolutional layer

[‡] Batch normalization layer

ing schemes, this method leverages fewer control sequences, making it compatible with non-coherent detection techniques, thus reducing the overall system complexity.

The optimization of STC matrices using the GA for both 1-bit and 2-bit RIS configurations revealed the potential of this approach in accurately diagnosing faulty RIS elements. The results also highlighted the impact of increasing the number of diodes per element on the diagnostic complexity, particularly in the 2-bit case, where the problem complexity doubled compared to the 1-bit case. Additionally, the analysis of the DER as a function of the SNR showed a clear degradation in performance as the SNR decreased, with noticeable performance thresholds for both 1-bit and 2-bit configurations.

Looking forward, the proposed method opens up several avenues for further improvement. Future work could focus on enhancing the robustness of the diagnostic method under challenging environmental conditions, including higher levels of noise or signal interference. Furthermore, the integration of advanced machine learning models for automated classification of faulty states could complement the current system, allowing for real-time diagnosis in more dynamic and large-scale RIS environments. Additionally, exploring alternative modulation schemes or hybrid techniques may offer further optimization opportunities in terms of both diagnostic accuracy and computational efficiency.

Ultimately, the proposed diagnostic framework could serve as a cornerstone for the next generation of intelligent, self-aware RIS systems, paving the way for more reliable, cost-effective, and scalable wireless communication networks.

CONCLUSIONS

This Thesis has addressed two fundamental aspects of STC-DMs: multi-frequency synthesis and diagnostics. Both challenges are crucial to enabling reliable and scalable deployment of such platforms in advanced EM applications, ranging from wireless communications to sensing and imaging.

On the synthesis side, a novel ML-based strategy has been proposed to enable multi-frequency scattering pattern design through a tandem DNN architecture. By mapping target equivalent-current distributions directly onto meta-atom coding sequences, the method decouples the multi-frequency synthesis problem and allows for precise tailoring of magnitude and phase distributions. This approach successfully integrates ML-driven and semi-analytical techniques, enabling the design of complex scattering responses with computational complexity that remains essentially independent of the electrical size of the metasurface. Experimental validation with an X-band prototype has confirmed the potential of this approach for advanced functionalities in smart radio environments, as well as in sensing and imaging. These results are reported in Ref. [101].

On the diagnostics side, two space-time modulation strategies have been developed to detect and localize faulty elements in RISs. The first relies on orthogonal coding, assigning distinct code channels to each element and enabling accurate diagnostics through simple signal processing of the recovered power distribution. The second leverages a non-uniform modulation scheme that establishes a one-to-one correspondence between harmonic amplitudes and RIS elements, allowing fault detection with amplitude-only spectral measurements and eliminating the need for coherent detection or complex retrieval procedures. Both methods have been extensively validated through theoretical analysis, simulations, and proof-of-principle experiments, demonstrating their robustness, scalability, and practicality across different operating conditions. These results are detailed in Refs. [134, 140].

Overall, the contributions presented in this Thesis advance the state of the art in both the synthesis and diagnostics of STC-DMs. They provide computationally efficient and experimentally validated solutions that broaden the functional range of digital metasurfaces while addressing the critical issue of reliability in large-scale deployments. Future research directions include extending the proposed synthesis strategies to higher frequency regimes by exploiting fast-switching mechanisms based on emerging materials, and further integrating diagnostic frameworks with system-level control to ensure seamless operation in next-generation EM platforms.

Part IV

APPENDICES

ML is becoming an integral part of EM systems. Antennas, as a core component of such systems, are evolving to become more advanced and versatile, driving the need for new techniques to improve their performance. As a branch of Artificial Intelligence (AI), which seeks to replicate what comes naturally to humans, learning from experience, ML is a data analysis method that automates the building of analytical models with minimal human intervention. Its potential to solve complex, unpredictable, and nonlinear problems has drawn significant interest from researchers in the field of EM, particularly in antenna and antenna-based systems. Today, ML techniques are being widely applied to tasks such as antenna simulations, synthesis, radiation pattern recognition, and nonlinear inverse scattering-based object identification.

Although the accuracy of ML algorithms heavily depends on the availability of sufficient data and expert handling of models and hyperparameters, it is increasingly becoming the preferred solution for researchers looking for cost-effective options that do not require excessive time investment. In this context, this Appendix aims to provide an overview of ML and its applications in EM. It offers an in-depth discussion on recent advancements in intelligent algorithms for antenna design, synthesis, analysis, and EM inverse scattering. Furthermore, it highlights the limitations of this emerging field of study.

A.1 BACKGROUND AND GENERALITIES

The genesis of DL dates back to the 1940s, undergoing an evolution of nomenclature before finally settling on the current popular designation [146]. Initially, some of the initial learning algorithms were conceived to create computational models of biological learning—in essence, simulating how knowledge acquisition occurs within the human brain. This conceptual link is why the field was referred to as Artificial Neural Networks (ANNs) from the 1980s, coinciding with the resurgence of AI research largely catalyzed by the *connectionism* movement [147]. A critical turning point in this era was the seminal 1986 publication by David Rumelhart, Geoffrey Hinton, and Ronald Williams [148]. In this work, the researchers refined and highlighted the power of the original back-propagation algorithm, significantly accelerating the learning process compared to earlier techniques and enabling the solution of previously intractable problems. The contemporary phase of neural network investigation was ushered in by a 2006 breakthrough: Hinton, the scientist who later coined the term DL, demonstrated a powerful training

approach for DNNs called greedy layer-wise pretraining [149]. Following this, DL achieved widespread prominence. Researchers could now successfully train significantly deeper neural architectures than before, confirming the theoretical significance of a deep model architecture. Driven by the exponentially expanding volume of data available, DL remains the primary catalyst for the current AI explosion, consistently delivering performance that vastly outperforms predecessor models, often achieving or exceeding human benchmarks [150].

The integration of DL—or, more precisely, ANNs as they were known then—into the domain of EM research began in the 1980s, aligning with the second major cycle of AI development. This cross-disciplinary effort was largely driven by the microwave engineering community, where ANNs became a valuable computer-aided design tool. Their primary utility was the rapid prototyping of microwave devices and radio-frequency circuits [151]. The efficacy of ANN-based design methodologies stemmed from employing the most fundamental neural network structure: The feedforward neural network composed of multiple fully connected layers. Within this architecture, every node in one layer maintains a distinct weighted connection to every node in the subsequent layer, hence the alternative designation, the *fully connected neural network*. Initially, the deployment of ANNs in EM applications was quite simple. The core design challenge was usually reframed as training an ANN to establish a mapping between an input parameter set and an output characteristic set. This paradigm allowed ANNs to successfully address numerous microwave design challenges, encompassing components such as transmission lines [152], filters [153], amplifiers [154], and antennas [155].

In microwave and photonic design, the deployment of ANNs typically operates under the well-established supervised learning framework. The fundamental goal here is to determine a reliable mathematical relationship linking an input set, X , to a corresponding output set, Y . The optimization phase involves adjusting the internal model parameters to accurately replicate the intended $X \rightarrow Y$ mapping. This procedure is performed on a pre-assembled training dataset consisting of numerous input examples X and their ground-truth labels Y . Beginning with arbitrary values, this data-centric learning methodology progressively refines the model's parameters across the training data. The adjustments are dictated by minimizing a designated loss function, which quantifies the discrepancy against the true labels Y . This iterative process continues until the model converges, resulting in a generalized function capable of making accurate predictions on previously unseen data. This methodology starkly contrasts with traditional optimization techniques, whether relying on gradients or being gradient-free. Such conventional methods lack pre-existing input-output pairs; instead, the optimization towards specific targets is governed by tailored, case-specific operational rules. To facilitate the efficient training of ANNs, the back-propagation algorithm was a pivotal development. Comprehensive details regarding the fundamental

ANN architecture and the operation of the back-propagation algorithm are provided in [Section A.1.1](#) and [Section A.1.2](#), respectively.

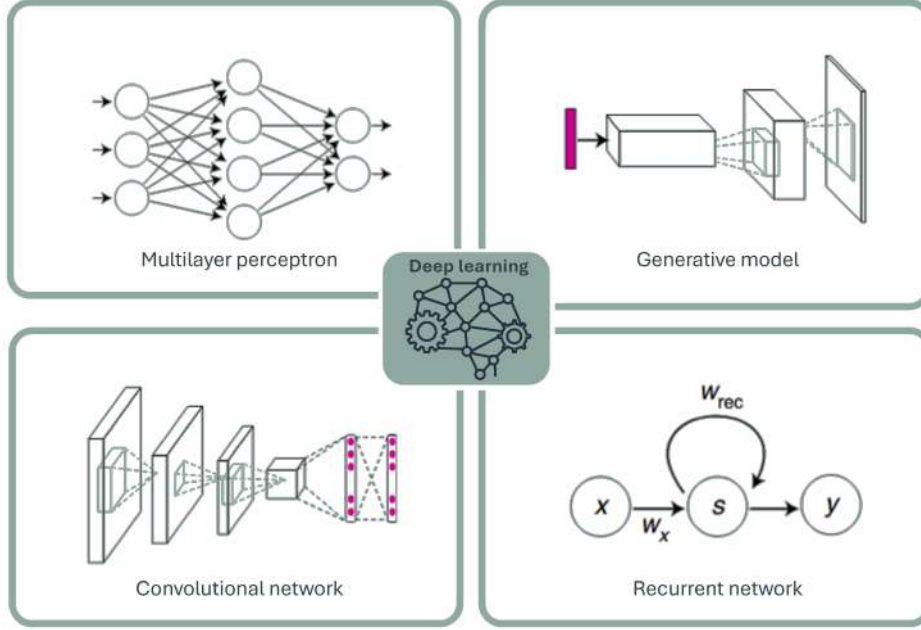


Figure 121: Illustrations of four basic neural network architectures: MLP, CNN, RNN, and GAN.

The last ten years have seen DL rise to prominence, profoundly impacting numerous fields of research. With the invention of new training and regularization techniques such as ReLU activation [156], dropout [157] and batch normalization [158], it is now possible to design and train increasingly DNNs capable of leveraging larger datasets with improved performance. Meanwhile, advanced model architectures have been proposed or improved to solve tasks in specific fields of ML and pattern recognition, such as the Convolutional Neural Network (CNN) [159] for image recognition, Recurrent Neural Network (RNN) [160] for natural language processing, Generative Adversarial Network (GAN) [161] and Variational Autoencoder (VAE) [162] for image generation. The EM community also benefited from the rapid advances in DL techniques. Unlike rule-guided optimization that explores the design space by following certain strategies case by case, DL, as a data-driven method, aims to describe the design space holistically, using the training data as samples. Therefore, with generalization ability within a given design space, DL can produce fast and accurate designs without the need for case-by-case, time-consuming numerical calculations. Properly trained DL models are capable of directly mapping photonic device designs to their optical properties, as well as performing the inverse mapping. In addition, DL can interact with traditional optimization methods to improve the algorithm performance.

A.1.1 Fully Connected Neural Network

A fully connected neural network comprising L layers is structured with a single input layer, one output layer, and $L - 2$ intervening hidden layers. As illustrated in Figure 122, every neuron within a given layer forms a link with all neurons in the subsequent layer, yet no connections exist between non-adjacent layers. The core function of the neurons is to process the input data sequentially, layer by layer, utilizing a set of tunable elements known as learnable parameters, specifically weights and biases.

To detail this operation, consider $w_{pq}^{(\ell)}$ as the weight linking the q -th neuron in layer $\ell - 1$ to the p -th neuron in layer ℓ , and $b_p^{(\ell)}$ as the bias associated with the p -th neuron in layer ℓ . The final output of the p -th neuron in layer ℓ , denoted by $y_p^{(\ell)}$, is calculated by first performing a weighted summation of the outputs from the preceding layer, $z_p^{(\ell)}$, followed by the application of a nonlinear function $f_\sigma(\cdot)$. This operation is expressed as:

$$y_p^{(\ell)} = f_\sigma(z_p^{(\ell)}) = f_\sigma\left(\sum_{q=1}^{N^{(\ell-1)}} w_{pq}^{(\ell)} y_q^{(\ell-1)} + b_p^{(\ell)}\right), \quad p = 1, \dots, N^{(\ell)}. \quad (158)$$

Alternatively, this calculation can be more concisely represented using the vector-matrix notation. This compact format allows the calculation for all neurons in a layer to be performed simultaneously:

$$\mathbf{y}^{(\ell)} = f_\sigma(\mathbf{z}^{(\ell)}) = f_\sigma(\mathbf{W}^{(\ell)} \mathbf{y}^{(\ell-1)} + \mathbf{b}^{(\ell)}), \quad (159)$$

where

$$\begin{aligned} \mathbf{y}^{(\ell)}, \mathbf{z}^{(\ell)}, \mathbf{b}^{(\ell)} &\in \mathbb{R}^{N^{(\ell)}}, \\ \mathbf{W}^{(\ell)} &\in \mathbb{R}^{N^{(\ell)} \times N^{(\ell-1)}}, \end{aligned} \quad (160)$$

are the weight matrix and bias vector of the ℓ -th layer, respectively, and $N^{(\ell)}$ represents the total neuron count within that layer.

The nonlinear function $f_\sigma(\cdot)$, known as the *activation function*, is indispensable. Its role is to enable the network to capture highly complex data representations, preventing the model from degenerating into a mere linear transformation. Popular choices for this function include the sigmoid, hyperbolic tangent, and the ReLU. Consequently, during the forward pass, input information propagates from one layer to the next, undergoing a sequence of linear summation and nonlinear activation at each stage, mimicking the flow of electrical signals across biological neurons and synapses.

To gauge the efficacy of the neural network, a cost function $C(\cdot)$ must be defined. This function quantifies the error by measuring the disparity between the final network output and the desired output. Operating on the outputs of the last layer and the true labels, the cost function yields a scalar error value that the learning process aims to minimize. For the parameters to be correctly adjusted, the learning algorithm calculates the gradient of the

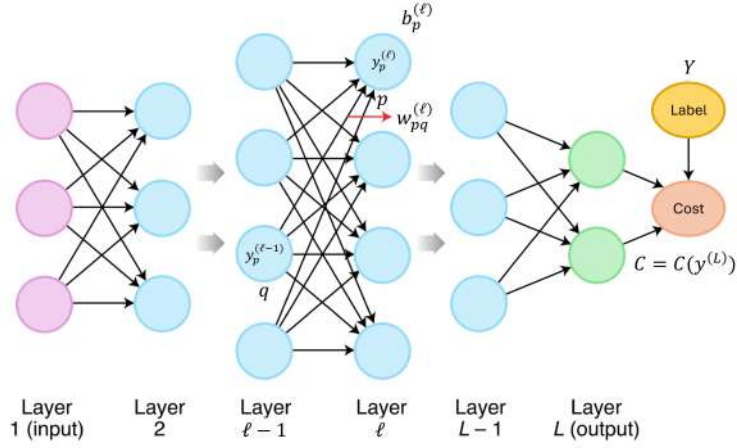


Figure 122: Architecture of a fully connected neural network.

cost function with respect to every weight and bias. This gradient signals the precise rate at which the error would change in response to a minuscule alteration of the parameter. Subsequently, each parameter is updated by moving in the opposite direction of its computed gradient, thus reducing the overall error.

A.1.2 Back-Propagation Algorithm

The process of training a neural network relies on the back-propagation algorithm, which fundamentally involves a single pass executed backwards from the final (output) layer to the initial (input) layer. This procedure is essentially a highly practical application of the chain rule for computing the derivatives of multivariable functions. To illustrate the mechanics of back-propagation, it is common to first define an intermediate quantity, the error vector $\delta^{(\ell)} = (\partial C / \partial z_1^{(\ell)}, \partial C / \partial z_2^{(\ell)}, \dots)$. This vector captures the partial derivative of the overall cost C with respect to the weighted input z at layer ℓ . For the final layer, L , this error vector is computed directly as:

$$\delta^{(L)} = (\nabla_{\mathbf{y}} C) \odot f'_\sigma(\mathbf{z}^{(L)}), \quad (161)$$

where $\odot$ denotes the Hadamard product (element-wise multiplication) between two vectors. Since the functional forms of both the cost function $C(\mathbf{y}^{(L)})$ and the activation function $f_\sigma(\mathbf{z}^{(L)})$ are known analytically, the error value $\delta^{(L)}$ at the last layer can be determined immediately. By recursively applying the chain rule, the error vector for any preceding layer ℓ can be derived from the error computed in the subsequent layer:

$$\delta^{(\ell)} = \left[\left(\mathbf{W}^{(\ell+1)} \right)^T \delta^{(\ell+1)} \right] \odot f'_\sigma(\mathbf{z}^{(\ell)}). \quad (162)$$

Equations (161) and (162) encapsulate the core mechanism of back-propagation. The error, which originates from the disparity between the

network's output and the target, is systematically transmitted backward through the network, layer by layer, while storing the crucial intermediate error values at each stage.

The quantities required to update the network's parameters—the partial derivatives $\partial C / \partial w_{pq}^{(\ell)}$ and $\partial C / \partial b_p^{(\ell)}$ —are directly related to these calculated errors:

$$\frac{\partial C}{\partial w_{pq}^{(\ell)}} = y_q^{(\ell-1)} \delta_p^{(\ell)}, \quad (163a)$$

$$\frac{\partial C}{\partial b_p^{(\ell)}} = \delta_p^{(\ell)}. \quad (163b)$$

Once the partial derivatives of the cost function C with respect to every learnable parameter (the weight matrices $\mathbf{W}^{(\ell)}$ and bias vectors $\mathbf{b}^{(\ell)}$ across all layers, $\ell = 1, \dots, L$) are computed, the model's training can proceed using Stochastic Gradient Descent (SGD). In the SGD procedure, the entire training dataset is first randomized and then segmented into multiple batches, each containing only a small subset of the total data. Each training step consists of several phases: feeding a batch of data through the network, computing the resulting outputs and errors, using back-propagation to determine the average gradient for that specific batch, and finally, adjusting the weights and biases based on this gradient.

Specifically, a hyper-parameter known as the learning rate, α_{lr} , must be defined. This parameter governs the magnitude of the modification applied to the parameters relative to their computed gradients with respect to the batch cost. For a batch containing N_B training samples, the update rules for the weights and biases, derived from Equations (163a) and (163b), are as follows:

$$\mathbf{W}^{(\ell)} \rightarrow \mathbf{W}^{(\ell)} - \frac{\alpha_{lr}}{N_B} \sum_{n=1}^{N_B} \delta^{(n,\ell)} \left(\mathbf{y}^{(n,\ell-1)} \right)^T, \quad (164a)$$

$$\mathbf{b}^{(\ell)} \rightarrow \mathbf{b}^{(\ell)} - \frac{\alpha_{lr}}{N_B} \sum_{n=1}^{N_B} \delta^{(n,\ell)}. \quad (164b)$$

This entire training process is repeated for numerous iterations until the cost function's value ceases to significantly decrease. Post-training, the model's performance is rigorously assessed using a separate test set—samples the model has never encountered during training—to evaluate its capacity for generalization.

A.2 REPRESENTATIVE EXAMPLES

A.2.1 Multilayer Perceptron

Although the architecture is relatively simple, the Multilayer Perceptron (MLP) has been theoretically established as a universal approximator, capable

of representing any continuous function [163]. In contemporary DL models with sophisticated, task-specific architectures, the MLP often acts as a bottleneck layer, extracting meaningful features as a compact representation of high-dimensional data such as images. Even prior to the remarkable success of DL in computer vision [159], researchers had explored MLP applications in photonics, following design paradigms adapted from the microwave community. The effectiveness of these early models was largely constrained by immature training strategies, limited data availability, and consequently shallow architectures.

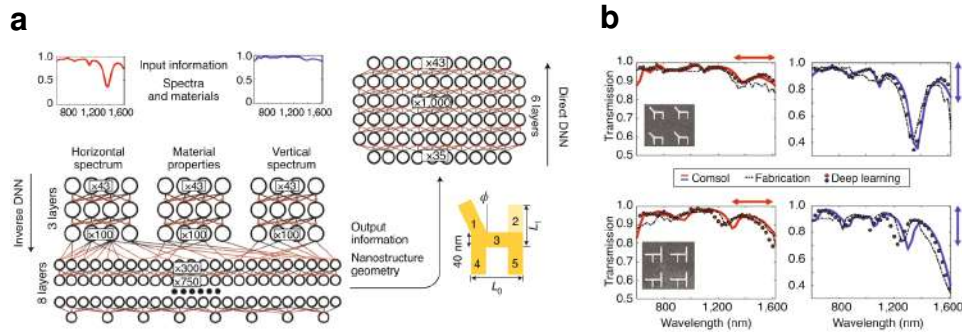


Figure 123: **a)** A bidirectional MLP model for design of H-shaped plasmonic nanostructures. **b)** Predicted (by DL model), simulated and measured spectra of retrieved H-shaped structures under different polarization conditions [97].

In 2018, Malkiel *et al.* reported a bidirectional MLP-based DL model for the design of plasmonic nanostructures [97]. Building upon earlier efforts, the authors were able to capture the complex physical relationship between photonic structures and their optical responses by employing a “deeper” network architecture. As illustrated in Figure 123, the structure under study is an H-shaped metallic configuration defined by eight parameters: three continuous parameters (length and rotation angle of the arms) and five binary parameters (presence or absence of specific arms). The design objectives comprise two reflection spectra under illumination with light polarized along horizontal or vertical directions. Each spectrum is discretized into 43 points, while the material properties (e. g., permittivity of the indium tin oxide adhesion layer and hosting materials) are represented as a 25-dimensional vector. A geometry-predicting network with eight grouped layers and a spectrum-predicting network with six layers were trained on $18 \cdot 10^3$ samples, enabling the model to function simultaneously as a fast simulator and an inverse design tool. The scale of the dataset, the complexity of the inputs, and the network depth demonstrate that, from a data-driven perspective, deeper models trained on larger datasets offer a promising alternative approach for solving photonic design problems.

When trained on large datasets, MLP-based models for inverse design often encounter convergence issues due to the existence of multiple designs

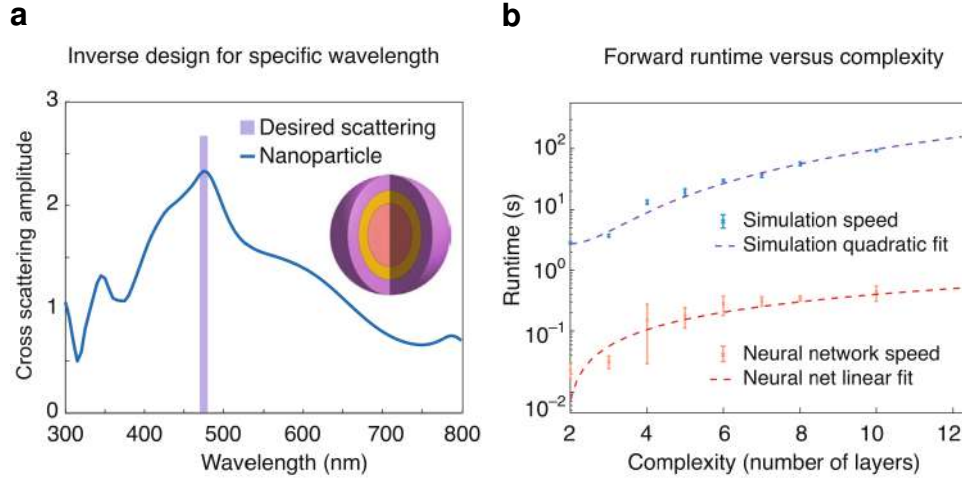


Figure 124: **a)** DL model as an optimization tool by providing analytical gradients, which can realize single-peak high scattering by a multilayer nanoparticle. **b)** Runtime with a speed two orders of magnitude faster than that of the conventional numerical optimization [164].

that satisfy similar requirements. To address this, Liu *et al.* proposed a tandem training method for inverse design [96]. The approach involves first training a forward model that maps design parameters to optical responses. This pretrained forward network is then connected to the output of the inverse design network, using the forward prediction error as the supervision signal. By training indirectly in this tandem configuration, the inverse design outputs are guided to converge toward a unique solution, effectively mitigating the data inconsistency problem arising from the inherent non-uniqueness of the inverse design task.

Beyond input-output mapping, MLP-based models, as universal function approximators, enable analytical gradient computation. Peurifoy *et al.* employed a single neural network to approximate light scattering by multilayer nanoparticles, using the analytical gradient from the model for structural optimization according to spectral requirements [164]. They demonstrated that, guided by the analytical gradient from the MLP, the single-band high scattering effect in core-shell nanoparticles could be efficiently optimized (Figure 124). Additionally, they compared the performance of a DNN with interior-point methods, a conventional numerical inverse design algorithm for nanoparticles. Under the same error threshold, the runtime of the neural network was two orders of magnitude shorter than that of the numerical method. More notably, as shown in Figure 124, as the design complexity increases (i.e., the number of nanoparticle layers), the neural network exhibits a linear increase in runtime, whereas numerical methods show a polynomial increase with an exponent of 4.5. These results highlight that, for the same level of accuracy, fully trained neural networks outperform conventional nu-

merical inverse design methods by a significant margin in computational efficiency.

Following these first forays into applying data-driven methodology in photonics research, deep MLP models and their variants have been extensively studied for many photonic design tasks, including topological photonics [165], integrated silicon photonics [166], color generation from nanostructures [167, 168], metamaterials [169–171], plasmonic structures [172] and photonic crystals [173]. For instance, in the work shown in Figure 125, the authors propose to use DL to design silicon-on-insulator-based 1×2 power splitters with various target transmission ratios at two ports (T_1 and T_2), in which the device structure is represented by a matrix of binary variables, indicating whether the silicon is etched or not at certain coordinates [166].

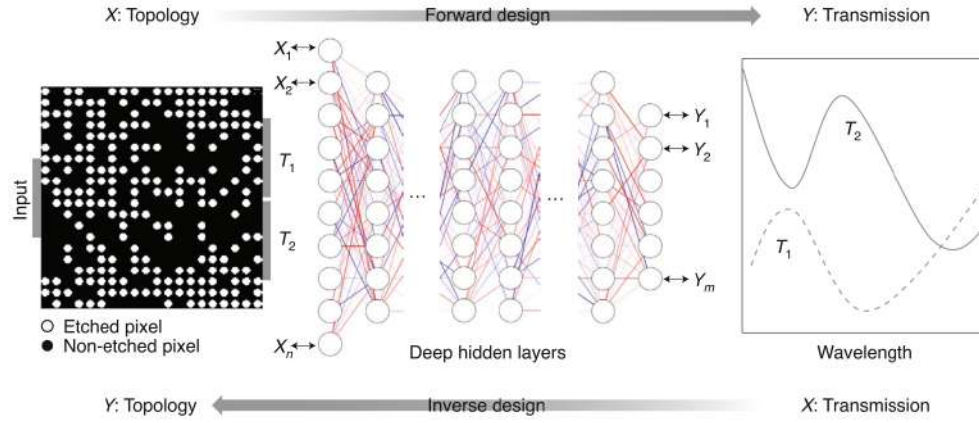


Figure 125: DNN to model silicon-on-insulator-based 1×2 power splitters. T_1 and T_2 denote transmission ratio of port 1 and port 2, respectively [166].

In addition to design problems, researchers have also developed sophisticated MLP models to tackle other problems in photonics, such as modal classification and effective refractive index retrieval in waveguides [174, 175], dimension reduction to reveal underlying physics [176] and near-field manipulation of plasmonic nanoantennas [177]. As the plain form of the DL model, MLP requires vectorized input and output. By parameterizing model inputs and outputs into vectors composed of several discrete and dimensionless elements, the MLP model can be adapted to a broad range of photonic applications regardless of the underlying physics.

A.2.2 Convolutional Neural Networks

MLPs have proven effective in metamaterial design; however, they face challenges when handling 2-D images and 3-D structures due to their lack of inherent spatial processing capabilities and the large number of parameters

required, which can lead to overfitting and high computational costs. To overcome these limitations, CNNs have been employed.

In DL, a CNN is a class of networks designed to extract features from input data through convolution operations at each layer. Thanks to the nature of convolution, a CNN can capture local spatial correlations in images. The fundamental architecture of CNN-based generative models enables the transformation of data between high-dimensional images and low-dimensional feature representations. Convolutional layers recognize hierarchical spatial structures, making CNNs particularly suitable for image-based data. Each neuron in a convolutional layer is connected only to a small region of the input, which reduces the total number of parameters while enhancing the network's capacity to capture spatial and temporal patterns. The central principle of CNNs is the convolution operation, which allows the network to automatically and adaptively learn spatial hierarchies of features from the input. This is achieved by applying a set of filters (or kernels) across the input, producing feature maps that encode the presence of specific patterns at various spatial locations. Mathematically, the convolution operation for a single filter $\mathbf{K}$ and a 2-D input $\mathbf{X}$ can be expressed as

$$\mathbf{Y}(p, q) = \sum_{m=1}^M \sum_{n=1}^N \mathbf{K}(m, n) \cdot \mathbf{X}(p + m - 1, q + n - 1), \quad (165)$$

where $\mathbf{Y}(p, q)$ is the output feature map at position (p, q) , $\mathbf{K}(m, n)$ represents the filter weights, and $\mathbf{X}(p + m - 1, q + n - 1)$ is input pixel value at position $(p + m - 1, q + n - 1)$. M and N are the height and width of the filter, respectively. The convolution operation involves sliding a filter over the input image and computing the dot product between the filter and local regions of the input. This mechanism enables CNNs to capture spatial relationships and hierarchical patterns, which are crucial for tasks such as image recognition and classification. As a result, CNNs have demonstrated outstanding performance in metamaterial design, emerging as a primary tool due to their efficiency and effectiveness in handling complex and nuanced designs compared to traditional approaches [170, 178–183].

CNNs have been applied to a wide range of problems in optics and photonics, including the inverse scattering problem [184], wavefront correction [185], digital coding metasurfaces [119, 186], and the prediction of optical properties in complex photonic and material systems [187, 188]. The convolution operation in a CNN is translationally invariant, making it particularly well-suited for modeling periodic photonic structures such as photonic crystals and metamaterials. These structures, typically represented as 2-D images, inherently exhibit translation invariance under periodic boundary conditions used in numerical simulations. For instance, in 2018, Asano *et al.* employed a CNN-based neural network to approximate the Q -factor of photonic crystals [173]. Using back-propagation, the positions of nanocavities were optimized, resulting in a substantial improvement of the Q -factor from $3.8 \cdot 10^8$ to $1.6 \cdot 10^9$, as illustrated in Figure 126.

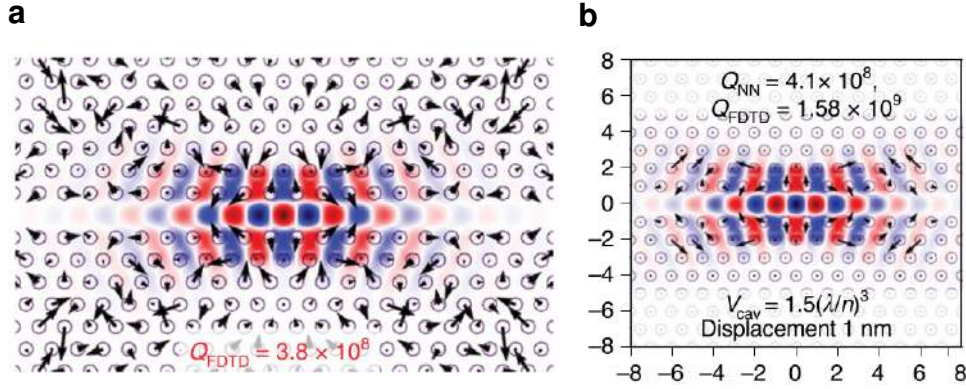


Figure 126: Optimization of the Q -factors of photonic crystals using CNNs [173]. **a)** Initial photonic crystals with a Q -factor of $3.8 \cdot 10^8$. **b)** Optimized photonic crystal with an improved Q -factor of $1.58 \cdot 10^9$. Q_{FDTD} denotes the quality factor calculated via finite-difference time-domain simulations, Q_{NN} is the quality factor predicted by the neural network, and V_{cav} represents the modal volume.

In addition to the basic CNN architectures, advanced network structures such as residual networks [189] and inception networks [190] have been developed to improve both the performance and capability of the models. These architectures can be employed to train surrogate models for simulating complex photonic structures with a large number of degrees of freedom [191].

A.2.3 Recurrent Neural Networks

RNNs are designed to handle sequential data, such as sentences or audio signals. As illustrated in Figure 121, RNN architectures process sequential inputs one element at a time, incrementally generating new sequences. In photonic design, RNNs are well-suited for modeling optical signals or spectra in the time domain, where specific line shapes arise from various resonance modes. RNNs have been employed to analyze optical signals and mitigate noise in high-speed fiber transmission [192].

When combined with CNNs, RNNs have also been used to improve the approximation of optical responses of nanostructures represented as images [191]. Figure 127 depicts random silver nanostructures used for simulation. Leveraging RNNs, the network is able to predict the absorption spectra of these structures over the 800–1700 nm range, demonstrating excellent agreement with full-wave simulations (Figure 127).

The performance of RNNs can be further enhanced by employing advanced variants such as Long Short-Term Memory (LSTM) [160] and Gated Recurrent Units (GRUs) [193]. Hybrid architectures combining CNNs and

RNNs represent a promising approach for modeling and designing photonic devices that manipulate complex spatiotemporal properties of light.

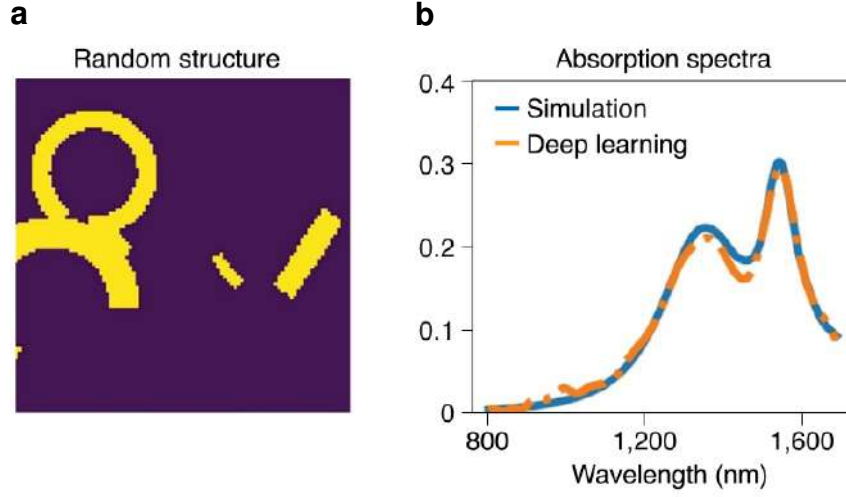


Figure 127: Prediction of the absorption of nanostructures using a combination of CNNs and RNNs [191]. **a)** Randomly generated silver nanostructures (yellow region). **b)** Absorption predicted by the CNN-RNN model compared with the actual absorption of the nanostructures.

A.2.4 Deep Generative Models

Generative models, instead of determining the conditional distribution and corresponding decision boundaries, describe the joint distribution of inputs and outputs to optimize a given objective in a probabilistic manner. Unlike deterministic models, generative models can handle one-to-many mappings and produce structured outputs according to specified requirements, as illustrated in Figure 121. Empowered by DL techniques, deep generative models can generate data that closely replicate or resemble the training dataset.

An Autoencoder (AE) comprises an encoder and a decoder. The encoder maps the training data into a reduced-dimensional space, known as the latent space, while the decoder reconstructs the data from the latent representation. AEs are commonly used to reduce the dimensionality of the design space and the optical response features of photonic devices, providing essential insights into light-matter interactions for device optimization [176].

Building on the AE, a VAE introduces probabilistic perturbations in the latent space [162], as illustrated in Figure 128. Because the latent space provides a continuous representation of the training data, new designs can be generated by sampling this space [194]. Exploiting the reduced dimensionality and continuity of the latent space, Ma *et al.* employed a VAE-based generative model to encode the meta-atoms of double-layered chiral meta-materials and their optical responses. This approach enables exploration of

the complex structure-performance relationship without requiring extensive data collection [195]. Figure 129 illustrates a target circular dichroism spectrum with a sharp peak at 60 THz. The VAE-based framework successfully identifies a double-layered metamaterial that reproduces the desired spectrum (right panel). More importantly, the model addresses the one-to-many inverse problem, generating distinct metamaterial structures that satisfy the same chiroptical response requirements.

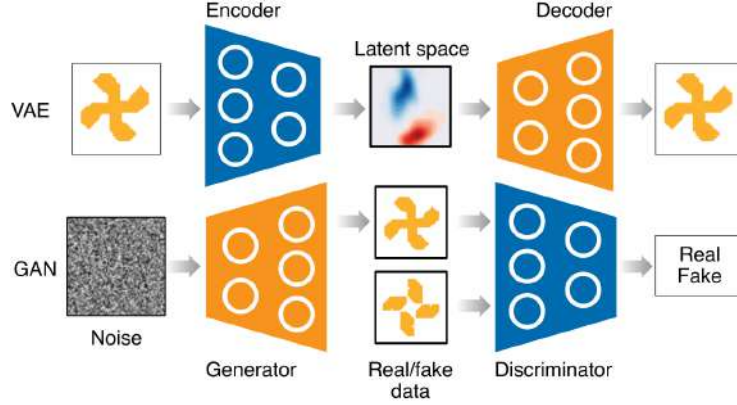


Figure 128: Illustrative representation of the architectures of a VAE and a GAN.

The GAN, illustrated in Figure 128, is another class of deep generative models, composed of a generator and a discriminator [161]. The generator is trained adversarially to produce samples that ideally form a distribution indistinguishable from the training dataset. Due to their capability to generate a large number of nanostructures in a short time, GANs have been employed for the stochastic design and optimization of both dielectric and metallic metasurfaces [196].

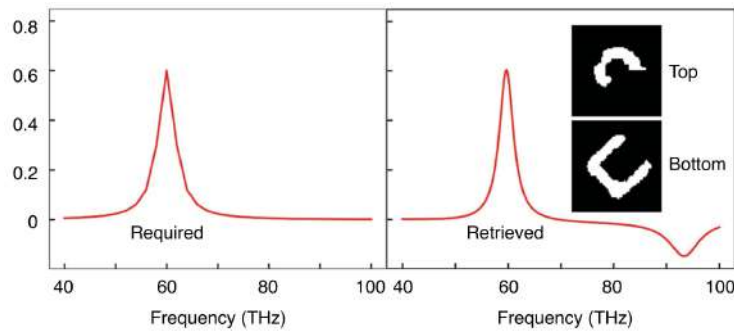


Figure 129: Design of a chiral metamaterial using a VAE combined with a semi-supervised learning strategy. The required circular dichroism spectrum is shown, along with the retrieved patterns of the top and bottom layers of the chiral meta-atom (insets) and the corresponding resulting spectrum [195].

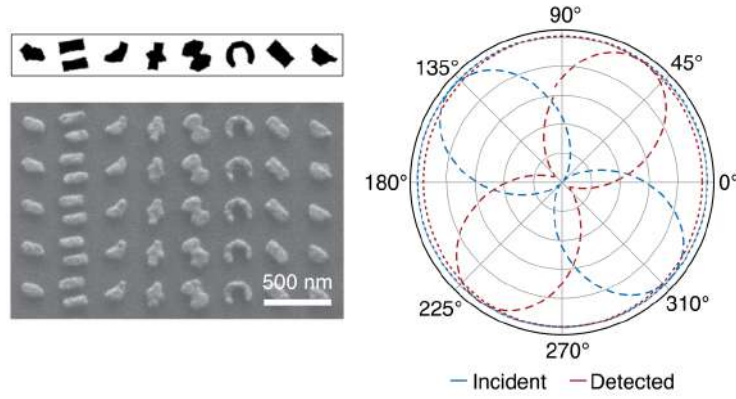


Figure 130: GAN-based design of spatially varying meta-molecules. Designed meta-atoms convert left-circularly polarized light to the cross polarization, with the measured polarization states [197].

In addition to the standard architectures of GANs and VAEs, other varieties of generative models have also been implemented for the discovery and design of photonic structures. For example, Compositional Pattern-Producing Networks (CPPNs) have been employed as generators within the GAN framework to produce high-quality nanostructure patterns for the inverse design of metasurfaces with multiple meta-atoms per unit cell [197]. Figure 130 illustrates a gradient metasurface generated by a CPPN-GAN, which partially converts LCP incident light to its cross-polarization while maintaining a constant phase gradient. The figure also presents the polarization states of incident and diffracted light measured after a quarter-wave plate, confirming the polarization conversion.

Other examples of generative models applied in photonics have also been reported. For instance, U-net, a DNN originally developed for image segmentation, has been used to accurately predict the near-field optical responses of arbitrary 3-D nanostructures, enabling the approximation of various far-field responses [198]. Moreover, by combining traditional optimization techniques with a generative model, global optimization of a 1-D meta-grating can be efficiently achieved for multiple wavelengths and deflection angles [199].

A.2.5 Optimization Leveraging Deep Learning Algorithms

Recent advances in material optimization, coupled with progress in nanofabrication techniques, have enabled the use of nanophotonic devices to address global challenges in quantum information [200], space exploration [201], and secure communication [202]. However, these inherently complex problems require highly constrained, multiparametric device optimization. Traditionally, adjoint, topology, and genetic optimization methods have been employed to tackle a wide range of inverse problems in photonics. These

approaches typically demand substantial computational resources and time, with requirements increasing further as the dimensionality of the optimization space and the number of constraints grow. Such resource intensity significantly limits the applicability of conventional methods to problems that involve optimization over extended parametric spaces, including both geometrical and material domains. By combining conventional optimization techniques with advanced DL algorithms, it becomes possible to leverage the strengths of both approaches and substantially reduce the computational burden.

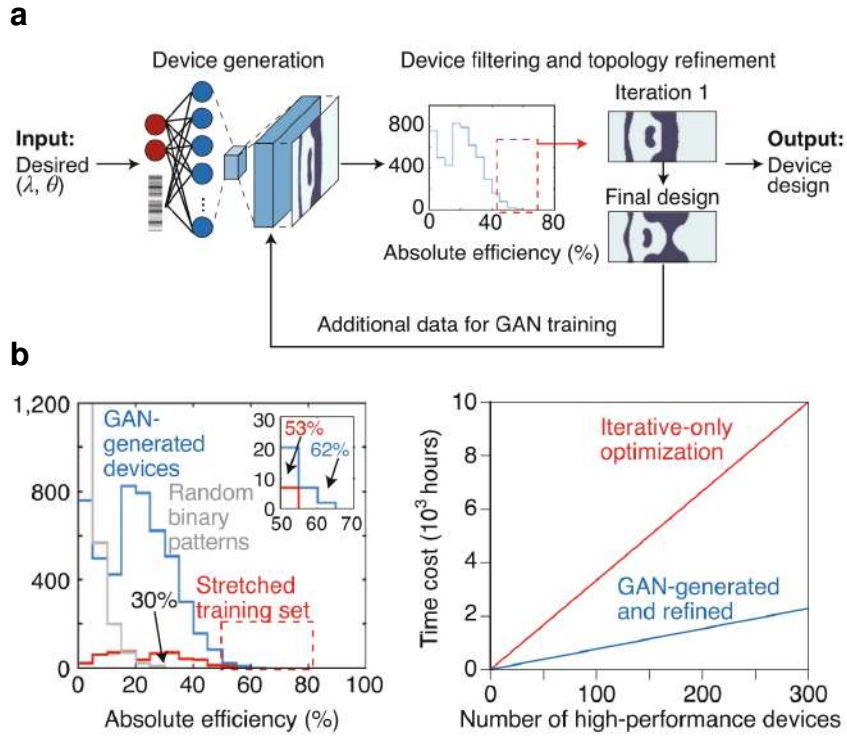


Figure 131: GAN-based development of metasurface designs [203]. **a)** Conditional GAN, composed of a generator and a discriminator, trained on topologically optimized designs. **b)** Efficiency histograms of meta-gratings generated by the trained GAN, with the time cost of device creation using iterative-only optimization compared to GAN generation and refinement.

The integration of deep generative networks, such as GANs and various types of AEs, with an adjoint optimization framework has been explored to enhance both speed and performance. As schematically illustrated in Figure 121, this approach leverages the strong correlation between device topology (geometrical shape) and its optical response. During training, the generative network learns characteristic geometrical features from pre-optimized device designs and constructs a compressed representation of the design space.

Recent studies show that conditional GANs can be efficiently trained on labeled, topology-optimized meta-grating designs to rapidly generate a large set of high-performance devices [203] (Figure 131a). The GAN captures and generalizes the main features of the meta-gratings, allowing the generation of devices with operating parameters (wavelength and deflection angle) beyond those in the training set. As shown in Figure 131b, GAN-generated designs match the performance of direct topology-optimized devices while producing five times more high-performance designs within the same time-frame.

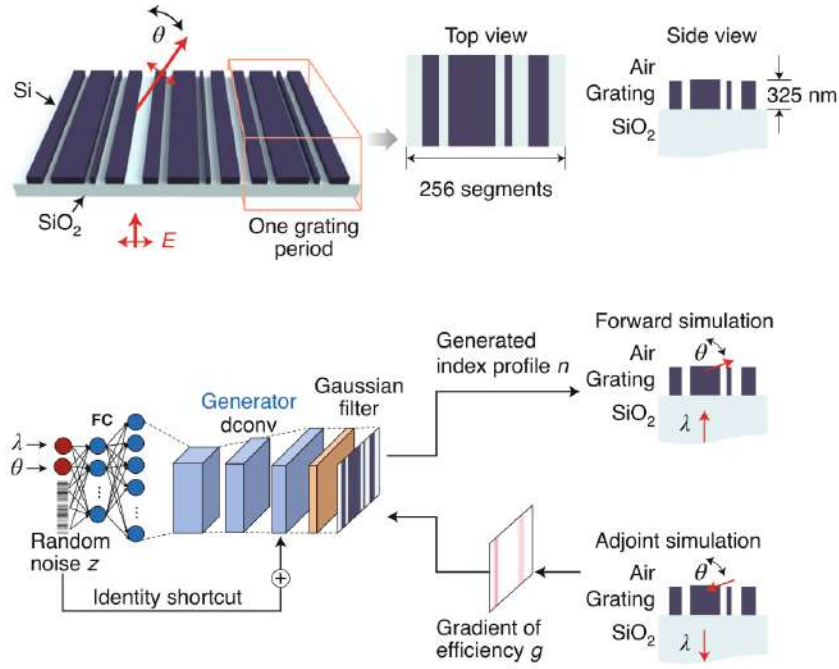


Figure 132: Global optimization driven by a generative neural network [199].

Furthermore, generative networks can be directly combined with topology optimization by replacing the discriminator with a direct EM solver [199]. Using the topology optimization of a dielectric meta-grating as an example (Figure 132), Jiang *et al.* showed that this approach allows continuous optimization of the device distribution until convergence to a cluster of high-efficiency designs [199]. Physics-based gradients from forward and adjoint EM simulations are used for error back-propagation during generator training, establishing a direct link between the compressed latent space of the network and improved device efficiency.

This strategy also eliminates the need to generate a separate training dataset, as the generative network learns the physical relationship between device geometry and optical response directly through EM simulations. The loss function is designed to assign higher weight to devices with superior efficiency, while reducing the influence of low-efficiency designs that may be trapped in local optima. Consequently, the compressed latent space of

the generative network is effectively narrowed to the sub-domain of highly efficient designs. Remarkably, this method requires only approximately 10% of the computational cost compared to conventional adjoint-based topology optimization.

Optimization in high-dimensional parametric spaces can be efficiently achieved by employing dimension-reduction techniques. Recently, linear Principal Component Analysis (PCA) has been applied to map and characterize the multiparameter design space of photonic systems [204]. As illustrated in Figure 133a, the proposed approach consists of three main steps:

1. Construction of a sparse set of pre-optimized designs using conventional optimization methods;
2. Identification of a lower-dimensional sub-space of highly efficient designs by applying a dimensionality reduction algorithm to the high-dimensional parametric space of pre-optimized designs;
3. Exhaustive mapping of the design sub-space to determine the continuous region of highly efficient designs.

This methodology has been applied to the multiparametric optimization of a vertical fiber grating coupler (Figure 133b). It has been shown that dimensionality reduction dramatically decreases the problem complexity, allowing the continuous sub-space of grating couplers with comparable fiber coupling efficiencies to be mapped [205]. Such a global analysis not only enables efficient multiparametric optimization but also reveals the inherent performance or structural limitations of the given design configuration.

Another strategy has been developed to address one of the primary bottlenecks of conventional optimization techniques: The computational cost of direct EM simulations. Data-driven methods have been proposed to accelerate Maxwell’s equation solvers involved in the optimization process. The feasibility of using data-driven approaches for solving Partial Differential Equations (PDEs) has been investigated recently, demonstrating the learning of “optimal” approximations of derivatives for time-domain simulations [206], as well as the use of neural networks for solving PDEs [207, 208]. In particular, data-driven methods have enabled the solution of nonlinear PDEs in the time domain at resolutions 4 to 8 times coarser than those required by standard finite-difference methods. Using CNNs, the generalized minimal residual algorithm for solving frequency-domain Maxwell’s equations has been accelerated [209], achieving an order-of-magnitude reduction in the number of iterations required when trained on datasets of wavelength-splitting gratings. This approach can be directly integrated within gradient-descent-based optimization of photonic devices, where multiple simulations with similar material distributions are required.

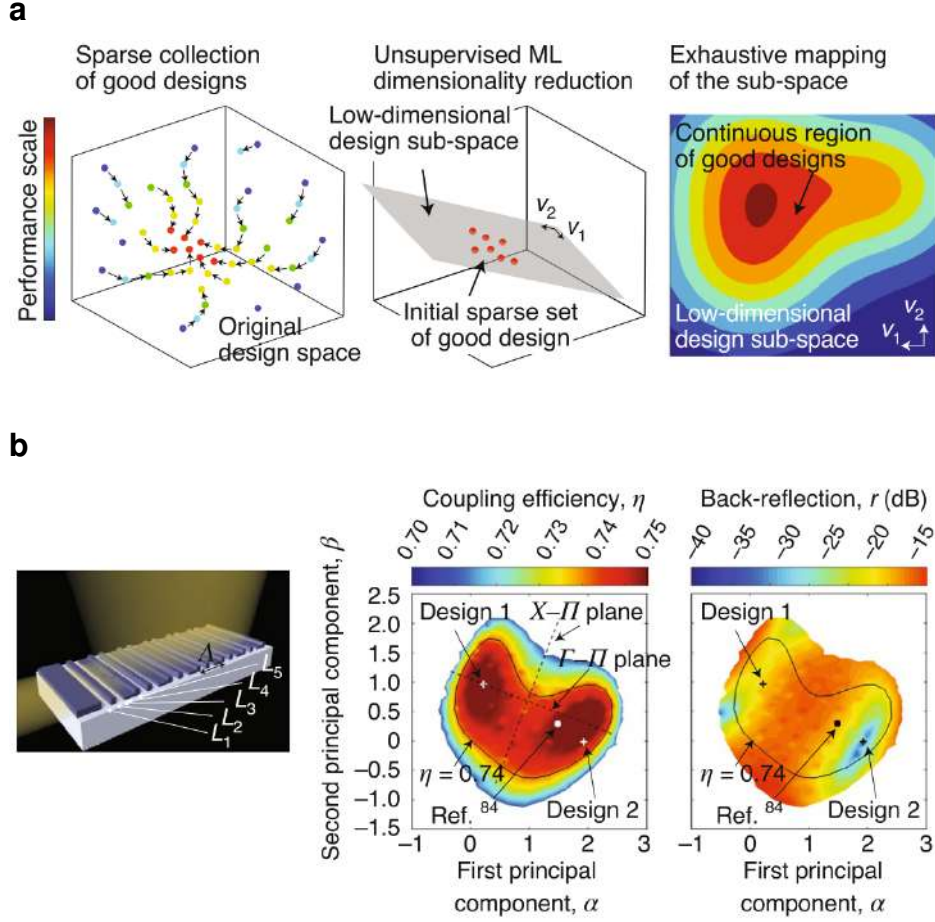


Figure 133: High-dimensional analysis and dimensionality reduction approach applied to the design of a nanophotonic component [204]. **a)** High-parametric space analysis using dimensionality reduction to identify the sub-space of efficient designs. **b)** Schematic representation of the grating coupler structure. By reducing the five-dimensional parametric space to the two principal component coefficients, α and β , exhaustive mapping of the sub-space of high-performance designs becomes feasible with modest computational resources. Two designs with comparable coupling efficiency are highlighted, alongside the design reported in Ref. [205]. Γ -II and X -II denote two orthogonal hyperplanes, both orthogonal to the first two principal components, α and β .

A.3 SOME REMARKS

DATA COLLECTION DNNs typically contain thousands to millions of learnable parameters, necessitating large amounts of labeled data for effective training. However, generating such datasets often requires extensive physical simulations or experimental measurements, which may not always be practical. In these cases, unsupervised and semi-supervised learning strategies can be employed to reduce the burden of data collection. Unlike supervised learning, unsupervised methods—such as PCA, AEs, VAEs, and GANs—require few or no labels and are commonly used for data clustering and dimensionality reduction. The processed data from these unsupervised approaches can be analyzed to identify the structural parameters of photonic devices that have the greatest impact, thereby enabling neural networks to capture essential design information without redundant data collection [210, 211].

The need for massive data collection can also be alleviated by integrating DL models with the underlying physics of the system. For instance, instead of training a model on enormous datasets to approximate the bidirectional relationship between physical structures and their responses, DL can be employed as an intermediate tool to efficiently solve the PDEs governing the physical system [208, 209]. Alternatively, learning the governing laws underlying the physical phenomena can further reduce the reliance on large datasets [207]. In this approach, DL is typically used to extract features from limited data for the regression of a small number of parameters that parametrize the system’s models or equations. Moreover, physics-informed DL models can generate solutions that inherently satisfy certain PDEs [212]. In other scenarios, when a neural network is well-trained to approximate specific physical processes, transfer learning can be applied to transfer the learned knowledge to similar simulation tasks, significantly reducing the amount of additional data required [213].

ADVANTAGES OF DEEP LEARNING From the standpoint of efficiency, a well-trained DL model can perform orders of magnitude faster than traditional optimization algorithms [164], provided that sufficient training data have been collected in advance. However, data collection for DL does not conflict with traditional optimization strategies [214], particularly global optimization algorithms that extensively explore the solution space. The key difference lies in how the data are utilized. In DL, the model actively accumulates training data that holistically describe the problem, with the quantity and quality of these pre-collected data directly influencing performance. In contrast, traditional numerical algorithms passively generate data during iterative optimization steps, primarily to monitor progress and determine when to adjust the strategy. While the amount of data required for DL is typically larger than that generated in conventional optimization, this collection represents a one-time cost, unlike the ongoing computational expense

of ab initio numerical methods. This characteristic renders DL particularly advantageous for photonic design tasks involving periodic structures, such as metamaterials or photonic crystals, where a large number of unit cells must be optimized to form a device or system.

Balancing training time and inference time for a DL model requires efficient exploitation of data, potentially in an online fashion, where data are continuously supplied as the model evolves for different tasks. Traditional optimization methods explore the design space guided by gradient information (e.g., level-set methods) or evolutionary processes (e.g., genetic algorithms).

In combination with traditional numerical optimization, a fast DL surrogate can be constructed by saving simulation data from the beginning of the optimization process [215, 216]. In such cases, the neural network also captures the gradients of the objective function with respect to input parameters. When the network is sufficiently reliable, performing local gradient descent using gradients predicted by the surrogate is equivalent to traditional adjoint methods. Furthermore, DL can assist in locating global minima. Its fast inference speed enables global search algorithms for highly complex structures, while latent variable and generative models can capture the underlying physics, reducing unnecessary exploration of the solution space. In practical scenarios, traditional optimization and data-driven methods can be jointly integrated into the same design pipeline: DL identifies solutions near the global minimum, and conventional optimization refines performance. Even if a true global optimum is challenging to find in complex photonic design tasks, these approaches, including DL, typically produce results that significantly surpass empirical designs. In cases where only limited data can be collected in a reasonable timeframe, incorporating statistical learning or other ML techniques can further enhance the design strategy. Nevertheless, since DL and ML models require sufficient training datasets, data-driven methods may not be suitable as a starting point in extreme situations where collecting a single data point takes hours or days. In such cases, direct optimization techniques, such as Bayesian optimization, may be more appropriate before applying DL models.

A.4 SUMMARY AND OUTLOOK

The concept of DL has evolved far beyond a mere computational analogy of biological neural networks, establishing itself as a powerful tool capable of tackling highly complex problems through the construction of multilevel abstractions from large-scale data. Within the EM community, DL and other AI techniques are currently revolutionizing EM design, integration, and measurement processes. These methods have already shown remarkable potential in designing EM structures, optimizing architectures, selecting materials, and even enhancing entire optical systems, while continuing to offer

unprecedented acceleration in optical measurements and the discovery of novel optical phenomena.

In this Appendix, we review a range of model structures applied to EM design, from individual plasmonic nanoparticles to metamaterials composed of arrays of meta-atoms, and up to integrated photonic devices. These impressive advancements have emerged over the past few years, and further progress is expected as researchers from diverse backgrounds continue to contribute to this rapidly growing field. Cross-disciplinary approaches combining AI, EM, and materials science are anticipated to enable large-scale EM designs with unique functionalities, as well as innovative methods for optical characterization. This, in turn, will pave the way for high-speed super-resolution imaging, real-time detection and manipulation, efficient energy conversion systems, and transformative developments in quantum measurements and metrology.

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